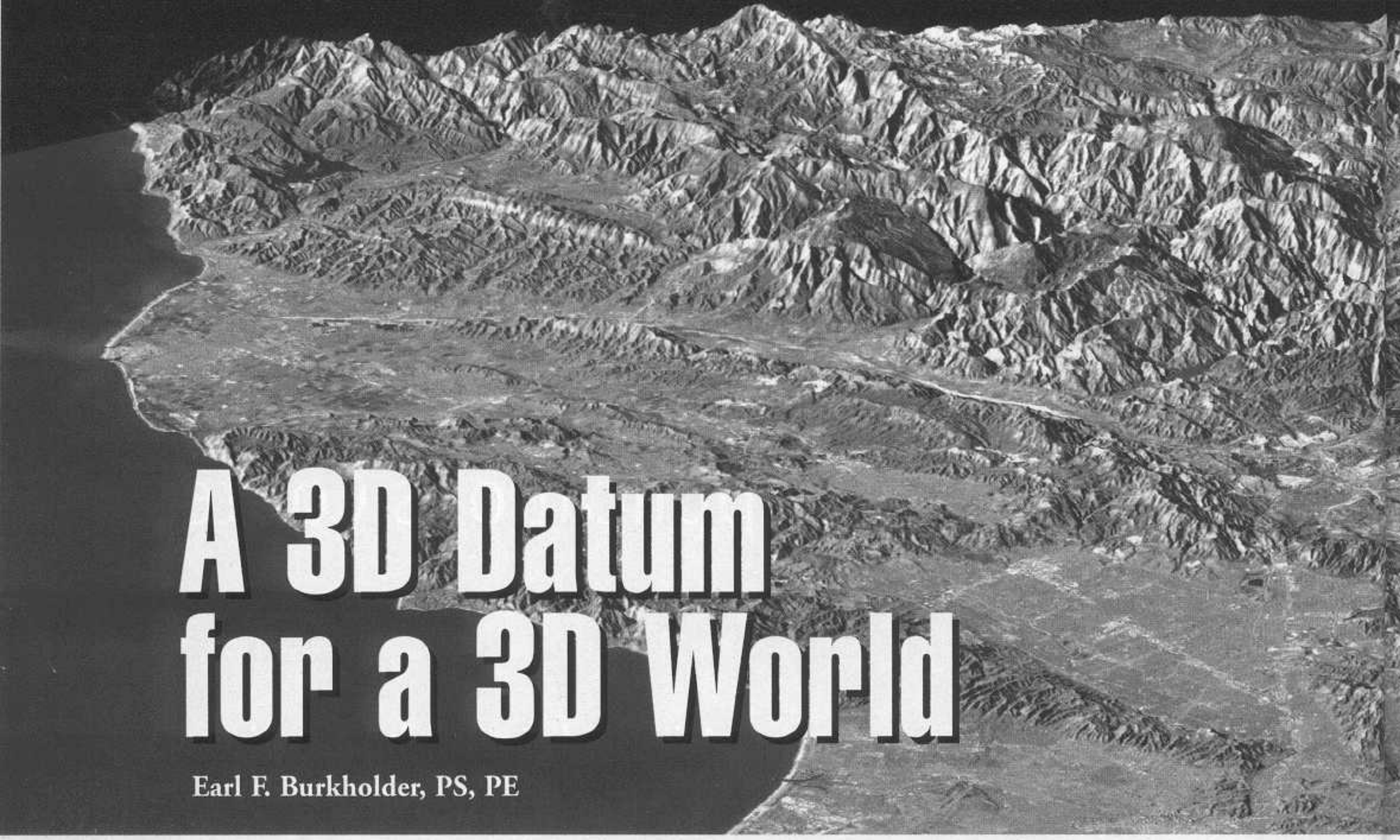

A 3D Datum for a 3D World

Earl F. Burkholder, PS, PE

The world is not flat and neither are spatial data. To provide undistorted spatial representations of our world, we need a new 3D datum that allows any user to view spatial data from any perspective.

We live in a three-dimensional (3D) world in which the process of collecting data and making maps has become largely automated. Spatial data are collected using 3D tools such as GPS, photogrammetric mapping, and total-station instruments. GIS organizes and routinely displays and plots spatial data as maps or 3D renderings using various data visualization routines. And larger storage devices and faster computers have markedly improved productivity and efficiency of handling vast amounts of data.

Despite such progress, however, standards for generating and using digital spatial data are established on the basis of an obsolete geospatial data model. Surveyors and mappers continue to use traditional models for handling spatial data that make a distinction between horizontal and vertical datums. That is, because map projections are strictly two-dimensional mathematical models, a separate model (typically elevation above sea level) must be used for the third dimension. Some GIS software packages use elevation as an attribute of location, whereas other programs simply assign a Z value — the third component for rectangular X, Y, and Z coordinates. But these are only pseudo 3D systems because they reference a flat X–Y plane for the third dimension instead of the Earth's true curved surface.

The pseudo 3D system works just fine, so long as one does not apply the rules of solid rectangular geometry beyond the limit one can safely assume a flat Earth. But, eventually, spatial data will have to be routinely referenced to a true 3D geospatial model that has a single origin. Consequently, a comprehensive 3D global spatial data model (GSDM) has been defined (Burkholder 1997) that readily accommodates digital spatial data, uses one set of equations worldwide (no projection zones or constants), provides a concise definition for spatial data accuracy, and offers simple standard procedures for data handling. Most importantly, the GSDM permits the user to view the digital world (or any part of it) from any perspective, local or global.

Synergizing Concepts

In formulating the GSDM, the goal was to begin with fundamental, almost self-evident, concepts and arrange them in ways that accommodate the use of new technology while remaining consistent with established practice. Credit for development of the individual concepts brought together in the GSDM belongs to the many people who have written about various aspects of 3D geodesy and spatial data, including G. Bomford's (1971) "Cartesian Coordinates in Three Dimensions," Alfred Leick's (1990 and 1995) definition of the 3D geodetic model; Edward Mikhail's (1976) comprehensive discussion of functional and stochastic models; and, when discussing models, Helmut Moritz's (1978) com-

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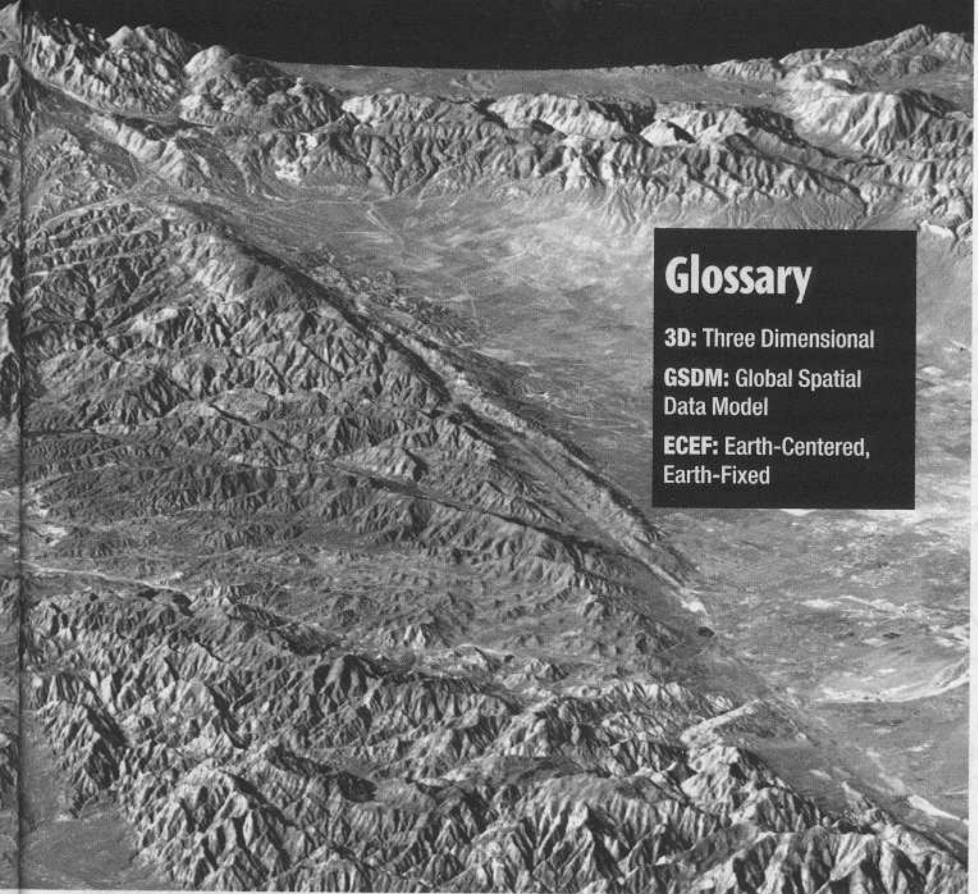


Glossary

3D: Three Dimensional

GSDM: Global Spatial Data Model

ECEF: Earth-Centered, Earth-Fixed

Image created from Shuttle Radar Topography Mission data by NASA/JPL.

ments about the simplicity of using the basic global rectangular X, Y, and Z system without an ellipsoid. When the aforementioned concepts are combined in a systematic way, with particular attention to the manner in which spatial data are used, the synergistic whole — the GSDM — appears to be greater than the sum of the parts.

The GSDM also borrows from the concept of a global three-dimensional polyhedron network, which was proposed by Heinrich Bruns in 1878, according to Gunter Seeber (1993). The difference now is that GPS and other modern technology have made a global network practical, and the polyhedron need not be limited to Earth-based points.

The Mathematics

So, what is the GSDM? Simply put, it is a collection of mathematical concepts and procedures that can be used to manage spatial data both locally and globally. The concept contains little or no new science; the solid geometry equations for manipulating spatial data are all in the public domain, and the GSDM fully supports continued derivative use of one- and two-dimensional spatial data. (Although the GSDM makes no attempt to accommodate non-Euclidean space or concepts, it does provide a simple, universal foundation for many disparate coordinate systems used in various parts of the world and offers advantages of standardization for spatial data users in disciplines such as those listed in Figure 1.)

In more specific terms, the GSDM consists of a functional model that describes the geometrical relationships as well as a stochastic model that describes the probabilistic characteristics (statistical qualities) of spatial data.

Functional Component. The functional model is specific with regard to geometric relationships, to absolute and relative spatial data, and to direct and indirect measurements. This portion of the GSDM includes equations of geometrical geodesy and rules of solid geometry as related to various coordinate systems, and it is intended to be consistent with the 3D geodetic model described by Leick (1990 and 1995) with the following exception: the GSDM, being strictly spatial, does not accommodate gravity measurements but presumes gravity affects are handled appropriately before data are entered into the spatial model.

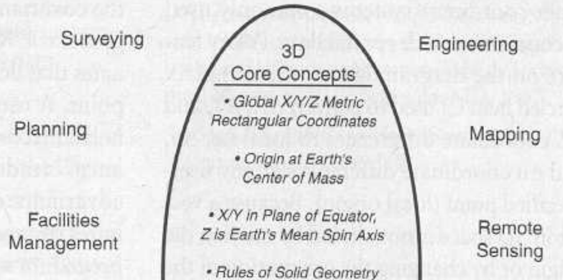


FIGURE 1 The GSDM provides a universal, 3D, mathematical foundation to support GIS database applications in several disciplines.

The GSDM functional model component is based on a three-dimensional right-handed rectangular Cartesian coordinate system with the origin located at the Earth's center of mass. The X–Y plane lies in the equatorial plane, with the X-axis at the 0-degree meridian. The Z-axis coincides with the mean spin axis of the Earth as defined by the International Earth Rotation Service Reference Pole. Widely used by many who work with GPS and related data, the U.S. Department of Defense (National Geospatial-Intelligence Agency 1997) calls this geocentric coordinate system the Earth-centered, Earth-fixed (ECEF) coordinate system. Rules of solid geometry and vector algebra are universally applicable when working with ECEF coordinates and coordinate differences.

As shown in Figure 2, the unique 3D position of any point on Earth or near space is equivalently defined by traditional latitude/longitude/ellipsoid height coordinates or by a triplet of X, Y, and Z coordinates expressed in meters. Because of the large distances involved, the X, Y, and Z coordinate values can be quite large. Fortunately, personal computers operating in double precision routinely handle 15 significant digits, and 12 significant digits will accommodate all ECEF coordinate values within

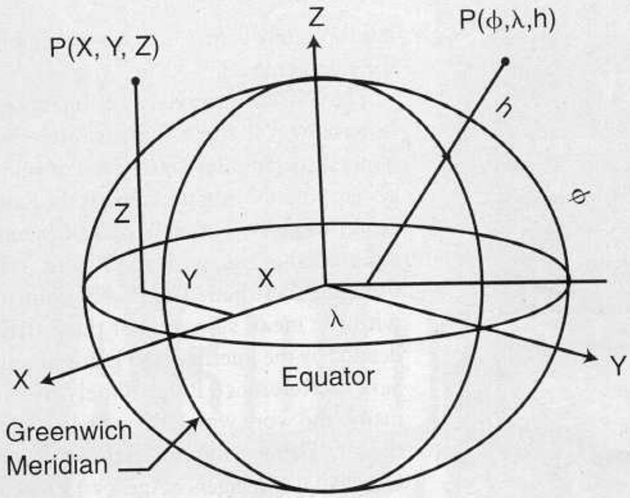
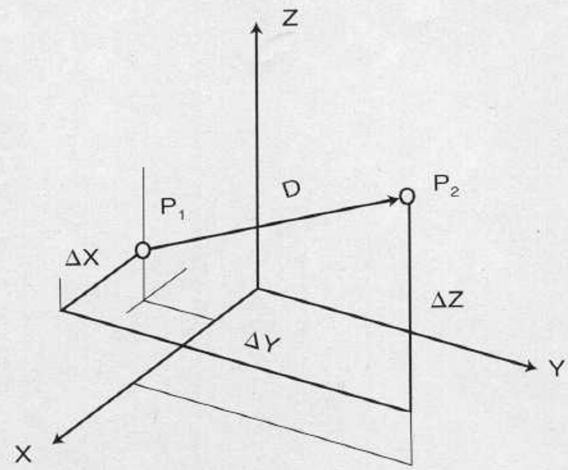


FIGURE 2 The 3D GSDM position of any point on Earth or near space is equivalently defined by traditional latitude, longitude, and ellipsoid height coordinates or by a triplet of X, Y, and Z coordinates expressed in meters.



$$X_2 = X_1 + \Delta X \qquad \Delta X = X_2 - X_1$$
$$Y_2 = Y_1 + \Delta Y \qquad \Delta Y = Y_2 - Y_1$$
$$Z_2 = Z_1 + \Delta Z \qquad \Delta Z = Z_2 - Z_1$$

FIGURE 3 The basic equations for 3D traverse and 3D inverse allow end-users to calculate coordinate differences employing smaller numbers and fewer digits.

the "birdcage" of GPS satellites down to 0.1 millimeter. Some users may object to working with such large coordinate values, but these objections will likely become inconsequential to the extent end-user applications are designed to use coordinate differences (much smaller numbers and fewer digits). For example, Figure 3 gives the 3D traverse equations on the left and 3D inverse equations on the right (For more details, see page 14 of Burkholder 1997).

Figure 4 illustrates relationships between the ECEF coordinate system and various other coordinate systems commonly used in connection with spatial data. A key feature on the diagram is a rotation matrix (circled item C) used to convert ΔX, ΔY, and ΔZ coordinate differences to local Δe, Δn, and Δu coordinate differences at any user-specified point (local origin). Because a vector in 3D space is not altered by moving the origin or by changing the orientation of the reference coordinate system, a vector defined by its geocentric ΔX, ΔY, and ΔZ components is equivalently defined by local components, and the rotation matrix is the mechanism that efficiently transforms a global perspective into a local one.

The transpose of the rotation matrix is used to transform local components of a space vector to corresponding geocentric components. Figure 4 also shows numbered boxes that represent various geometric elements and circled letters that represent mathematical transformations between the boxes. (A full explanation of all the numbered boxes and the circled letter processes, including the equations, is available in Burkholder 1997.)

Stochastic Model. The stochastic portion of the GSDM is an application of concepts described by Mikhail (1976). It provides unambiguous procedures for establishing, tracking, and using spatial data accuracy, and is devoted to answering the question, "Accuracy with respect to what?"

The stochastic model is based on storing the covariance matrix associated with the geocentric X, Y, and Z rectangular coordinates that define the location of each stored point. A user can compute the local east/north/up covariance matrix of any point on an as-needed basis using the standard covariance error propagation (this minimizes storage requirements). The same basic procedure is extended to other functional model computations and provides a statistically defensible method for tracking the influence of random errors to any derived quantity. In particular, the user can look at the standard deviation of a coordinate position (by individual component) in either the geocentric or local reference frame. The standard deviation of other derived quantities — such as distance, azimuth, slope, area, or volume — can be obtained using the same procedures and the appropriate functional model equations.

But, more importantly, the GSDM provides an efficient mathematical definition for datum accuracy, network accuracy, and local accuracy. As described in Burkholder (1999), a single point is described by datum accuracy. The inverse between a pair of points will reflect network accuracy if no statistical correlation exists between them. By using stored correlations, the GSDM also provides an efficient mechanism for computing the local accuracy between points stored in the database. An exhaustive study of spatial data accuracy and the GSDM is included in Burkholder (2004).

Choosing to Use GSDM

In all, the GSDM defines a model and computational environment that can be used to manage spatial data more efficiently. It takes advantage of and benefits modern spatial technologies by

- Accommodating all forms of 3D spatial data measurement
- Not distorting distances (as does a two-dimensional conformal map projection)
- Employing one set of standard solid geometry equations
- Handling horizontal and vertical data in the same 3D database
- Providing a way to obtain standard deviations for all spatial data
- Being appropriate for very precise applications

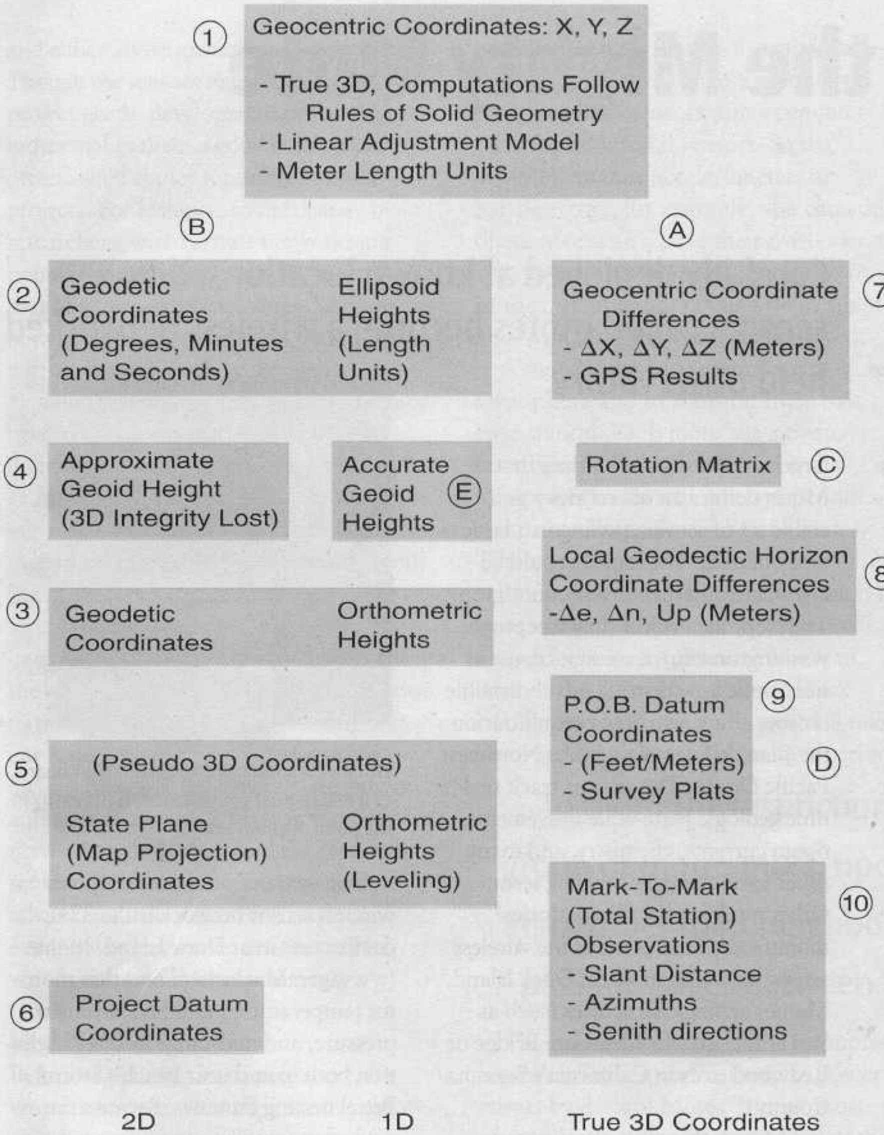


FIGURE 4 This schematic illustrates the relationships between the ECEF coordinate system and various other coordinate systems commonly used in connection with spatial data.

■ Permitting GIS and other computer intensive spatial data computations to be accomplished more efficiently in a dynamic environment.

Using the GSDM is primarily a matter of choosing to do so. To experiment with the GSDM, readers are encouraged to evaluate the prototype BURKORD software, a DOS-based and menu-driven program and database that combines horizontal and vertical geospatial data into a single 3D database and stores geocentric ECEF coordinates (metric only), as well as the geocentric covariance matrix associated with each point and the correlations between points. Gratis copies are available via return email from globalcogo@zianet.com.

A one-day workshop entitled "A Global Spatial Data Model (GSDM) for the Digital Revolution" will also be presented at the American Society for Photogrammetry and Remote Sensing (www.asprs.org) Annual Meeting in Denver, Colorado, May 23, 2004.

Acknowledgements

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