The NAVD 88 elevation of station "Reilly"
Based Upon the HARN Values of "Reilly" in 2005 with Geoid 03 and
The HARN Adjustment of 2011 with Geoid 12A

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This article comes under the "I thought you might like to know" category.

Station "Reilly" is a National Geodetic Survey (NGS) high accuracy reference network (HARN) ground-level brass tablet set in the top of a massive concrete vault in the middle of the NMSU "Horseshoe." NGS observed the HARN network in May 2000 with GPS and published the earth-centered earth-fixed (ECEF) X/Y/Z coordinates referenced to NAD 83(1992). Geoid03 was used to estimate the geoid height for the station and an approximate NAVD88 elevation for "Reilly" is listed on the 2005 NGS data sheet as 1,190.5 meters. The latitude, longitude and other data for "Reilly" are also listed on the data sheet.

The 2005 NGS data sheets for first-order benchmarks in the area included two recoverable monuments; A-245 and H-245 as shown in Figure 1.

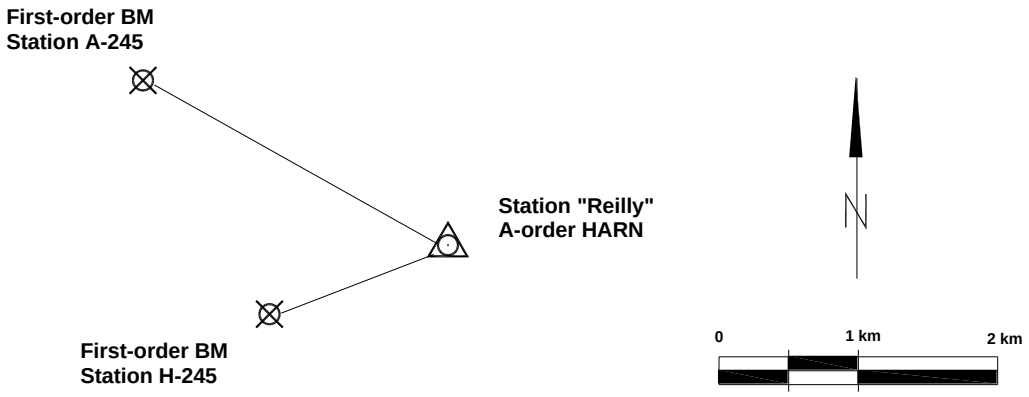

Figure 1, Location of GPS Points

As described in the paper posted at http://www.globalcogo.com/ReilElev.pdf, the NAVD 88 elevation of station "Reilly" was determined in 2005 using GPS vectors and geoid modeling. The process is documented in the posted article and summarized as:

- GPS vectors between the benchmarks and station "Reilly" were observed using the same brand and model GPS instruments with each instrument carefully oriented "north." This insured that the electrical center positions in the GPS units would cancel out during baseline computations. Sufficient sessions were observed to avoid trivial vectors and to insure adequate redundancy in the network.

- The baselines were processed to obtain the vector components (ΔX/ΔY/ΔZ) along with the covariance matrix for each baseline. A least squares adjustment was performed holding the published X/Y/Z values for station "Reilly."

- Geoid03 was used to compute the geoid height at each benchmark and at station "Reilly."

- The NAVD 88 elevation at station "Reilly" was computed from each benchmark as:

  - From A-245, the NAVD 88 elevation for "Reilly" is 1,190.495 m
  - From H-245, the NAVD 88 elevation for "Reilly" is 1,190.502 m

- **The mean NAVD 88 elevation of "Reilly" in 2005 is posted as 1,190.498 m +/- 0.0027 m**

All the information given so far is included in the posted article. But, since 2005, NGS has published subsequent geoid models (09, and12A) and the NAD 83(2011) position of station "Reilly" is different due to readjustment of the HARN network. There are no known seismic events in Las Cruces during the past 10 years and seemingly, there is no need to question the stability of any of the 3 points. With changes in the geoid model and re-adjustment of the HARN network, **"Is there a better NAVD 88 elevation for station "Reilly?"**

In order to address that question, the NAVD 88 elevation of station "Reilly" was recomputed holding its NAD 83(2011) ECEF coordinates, using the same GPS observed vectors (both the components and the covariance values), and using Geoid12A to compute geoid heights. The NAVD 88 elevations for stations A-245 and H-245 are the same on the 2015 NGS data sheet as listed on the 2005 NGS data sheet. The values used are summarized as:

Station "Reilly"	Published in 2005	Published in 2015	Diff. 2015 - 2005
X =	-1,556,177.615 m	-1,556,177.595 m	0.020 m
Y =	-5,169,235.319 m	-5,169,235.284 m	0.035 m
Z =	3,387,551.709 m	3,387,551.720 m	0.011 m

NAVD88 Elevation	Published in 2005	Published in 2015	Diff. 2015 - 2005
Benchmark "A 245"	1,186.626 m	1,186.626 m	0.000 m
Benchmark "H 245"	1,183.102 m	1,183.102 m	0.000 m

Geoid Height at:	Geoid03	Geoid12A	Diff. 2015 - 2005
Station "Reilly" =	-23.905 m	-23.943 m	-0.038 m
Station "A 245" =	-23.957 m	-23.999 m	-0.042 m
Station "H 245" =	-23.954 m	-23.993 m	-0.039 m

Results of Least Squares Adjustment of GPS Vectors

Several different software packages were used to compute a network adjustment of the observed GPS vectors. The adjustments all gave identical answers as summarized below.

Station "A 245"	Computed 2005	Computed 2015	Diff. 2015 - 2005
X =	-1,558,114.588 m +/- 0.0016 m	-1,558,114.568 m +/- 0.0016 m	0.020 m
Y =	-5,168,006.589 m +/- 0.0042 m	-5,168,006.554 m +/- 0.0042 m	0.035 m
Z =	3,388,522.031 m +/- 0.0027 m	3,388,522.042 m +/- 0.0027 m	0.011 m

Station "H 245"

X =	-1,557,508.610 +/- 0.0012 m	-1,557,508.590 m +/- 0.0012 m	0.020 m
Y =	-5,169,122.541 +/- 0.0029 m	-5,169,122.506 m +/- 0.0029 m	0.035 m
Z =	3,387,101.071 +/- 0.0020 m	3,387,101.082 m +/- 0.0020 m	0.011 m

The differences above are for ECEF coordinates only. Note that the ECEF coordinate differences are identical for all three points, Reilly, A 245, and H 245. The standard deviation of the X/Y/Z values at station "Reilly" were assumed to be zero, both in 2005 and in 2015. The standard deviations for A 245 and H 245 are the same in the 2015 adjustment and they were in the 2005 adjustment.

The 3-D coordinate geometry and error propagation software, BURKORD™ (free prototype download) was used to compute the NAD 83(2011) latitude/longitude/ellipsoid height at each point. Input includes the geocentric X/Y/Z coordinates and the covariance matrix of each point in the geocentric reference frame. BURKORD™ output includes local e/n/u standard deviations as well as the latitude/longitude/height at each point. The (derived) results are:

NAD 83(2011) Station "Reilly" (fixed):

Latitude	=	32º 16' 55."93001 N	(N) +/- 0.000 m
Longitude	=	106º 45' 15."16035 W	(E) +/- 0.000 m
Ellipsoid height	=	1,166.5429 m	(U) +/- 0.000 m

NAD 83(2011) Station "A 245"

Latitude	=	32º 17' 33."26573 N	(N) +/- 0.0012 m
Longitude	=	106º 46' 39."57079 W	(E) +/- 0.0009 m
Ellipsoid height	=	1,162.6198 m	(U) +/- 0.0050 m

NAD 83(2011) Station "H 245"

Latitude	=	32º 16' 38."78204 N	(N) +/- 0.0012 m
Longitude	=	106º 46' 05."09654 W	(E) +/- 0.0011 m
Ellipsoid height	=	1,159.0942 m	(U) +/- 0.0034 m

Computing the orthometric height of a point using GPS ellipsoid height differences and geoid height differences is discussed in the original 2005 document. That same process is applied to the 2015 adjusted ellipsoid height differences and the Geoid12A differences. With reference to Figure 2, the following computations were made.

Given: Known elevation at Point A = H_A.
GPS ellipsoid heights at Points A and B are h_A and h_B.
Geoid12A geoid heights at Points A and B are N_A and N_B.

Find: Elevation (orthometric height) at Point B.

Solution: $\Delta h = h_B - h_A$ (from GPS results)

$\Delta N = N_B - N_A$ (from Geoid12A)

$\Delta H = \Delta h - \Delta N$ and $H_B = H_A + \Delta H$

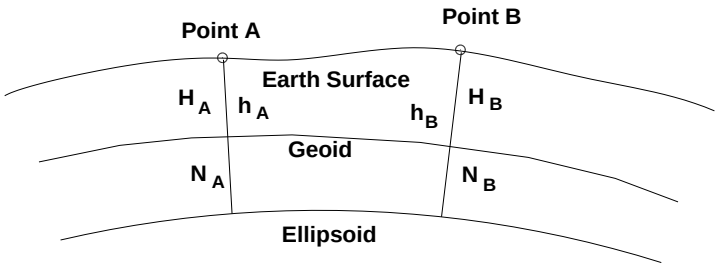

Figure 2, Computing Elevation Differences and the Elevation at Point B Using Ellipsoid Heights Differences and Geoid Height Differences

The recently computed 2015 values are shown below and the 2005 values from the posted paper are also shown for comparison.

Observed Orthometric Height Difference Between Published Benchmarks:

		2015	2005
$\Delta h = h_{StaH} - h_{Sta\ A}$	= 1,159.0942 m – 1,162.6198 m =	-3.5256 m	-3.5276 m
$\Delta N = N_{StaH} - N_{StaA}$	= -23.993 m – (-23.999 m) =	0.006 m	0.003 m
$\Delta H = \Delta h - \Delta N$	= -3.5256 m - 0.006 m =	-3.532 m	-3.531 m

Elevation at Station "Reilly" from Station "A 245"

$\Delta h = h_{Reilly} - h_{Sta\ A}$	= 1,166.5429 m – 1,162.6198 m =	3.9231 m	3.9210 m
$\Delta N = N_{Reilly} - N_A$	= -23.943 m – (-23.999 m) =	0.056 m	0.052 m
$\Delta H = \Delta h - \Delta N$	= 3.9231 m - 0.056 m =	3.867 m	3.869 m
Elevation at Station "Reilly" = 1,186.626 m + 3.867 m =		**1,190.493 m**	**1,190.495 m**

Elevation at Station "Reilly" from Station "H 245"

$\Delta h = h_{Reilly} - h_{StaH}$	= 1,166.5429 m - 1,159.0942 m =	7.4487 m	7.4486 m
$\Delta N = N_{Reilly} - N_H$	= -23.943 m – (-23.993) m =	0.050 m	0.049 m
$\Delta H = \Delta h - \Delta N$	= 7.449 m - 0.050 m =	7.399 m	7.400 m
Elevation at Station "Reilly" = 1,183.102 m + 7.399 m =		**1,190.501 m**	**1,190.502 m**

Average of the two 2015 values is: (1,190.493 m + 1,190.501 m)/2 = **1,190.497 m**

Average of the two 2005 values is: (1,190.495 m + 1,190.502 m)/2 = **1,190.498 m**

The difference between 2005 and 2015 values is 0.001 meters. Based upon that comparison, it is reasonable to conclude that the NAVD88 elevation of station "Reilly" has not changed over the intervening 10 years.

Additional observations, comments, and questions include:

1. The comparison would not be complete without also looking at the standard deviations computed for both the 2005 elevation and the 2015 elevation. The standard deviation part of the original paper is more subjective as assumptions (hopefully reasonable ones) were made as to the quality of ellipsoid height differences. Those assumptions were not changed in this update and the 2015 computed standard deviation for the NAVD88 elevation of "Reilly" rounded off to the same 0.003 meters as in the original paper.

2. Similar computations and comparisons could be made for each subsequent geoid model published by NGS – Geoid09. Those comparisons were made but the results were inconclusive, in part, because the changes in the geoid model were not simultaneous with the HARN re-adjustment. However, I believe that these two comparisons (using data published in 2005 and 2015) are sufficiently compatible as to be legitimate.

3. It appears that, elevation-wise, the change in X/Y/Z position of station "Reilly" was off-set by the difference in results obtained from the two geoid models.

4. But, a more important inference may be gotten from the following. It has been estimated that absolute geoid modeling in the past (i.e., Geoid03) could not be expected to be better than about 2 cm but that the accuracy of the newer geoid models (Geoid12A) is better – say about 1 cm.

5. Let's try it. The NAVD88 elevations computed and compared herein are based upon using geoid height **differences.** As described in the 2005 paper, using differences is better than using the modeled geoid height value at a single station. What would be the NAVD88 elevation of "Reilly" using the absolute Geoid03 model value and the Geoid12A value?

6. Those values are computed and compared to the average of 1,190.498 m as reported herein. If using the absolute geoid height value from the geoid model, the equation for the NAVD 88 elevation for "Reilly" is:

$$\text{NAVD 88 elevation } H_{Reilly} = h_{Reilly} - N_{Reilly}$$

	Absolute N	Pair-Wise	Comparison
2005 Data (Geoid03):			
H_{2005} = 1,166.570 m – (-23.905 m) =	1,190.475 m	1,190.497 m	0.023 m
2015 Data Geoid12A:			
H_{2015} = 1,166.543 m – (-23.943 m) =	1,190.486 m	1,190.497 m	0.011 m

If this comparison is legitimate, the evidence clearly shows that Geoid12A is significantly better than Geoid03, in particular when using only the absolute Geoid12A values at a station along with the ellipsoid height at the station obtained from high-quality GPS data.

Final question – what is meant by high-quality GPS data? The caveat is that high-quality GPS positions are obtained as a part of a formal rigorous least squares adjustment of a GPS network. It is left to the reader and others to determine if sufficiently high-quality GPS data can be obtained from a non-static GPS survey.