

Comparison of Arc and Chord on Ellipsoid
Earl F. Burkholder, PS, PE, F.ASCE - October 7, 2011

Use GRS 1980

$a =$ 6378137.000 m SPR = 206,264.806247096
 $e^2 =$ 0.006694380023 MtoFT = 3.2808333333333

$$R_m = \frac{a\sqrt{1-e^2}}{(1-e^2\sin^2\varphi)} \quad L = R\theta \quad \theta = 2\sin^{-1}\left(\frac{CH}{2R}\right)$$

Latitude at which to make the comparison = 30 deg. Geometrical mean radius = $R_m =$ 6,367,408.778 m

Chord Distance			Theta = subtended angle				Arc	Difference	One part in
			Radians	D	M	S			
1,000.0 m	0.621	mi.	0.0001570498	0	0	32.3938	1000.000001 m	1.02769E-06 m	973,053,584.00
2,000.0 m	1.243	mi.	0.0003140995	0	1	4.7877	2000.000008 m	8.22154E-06 m	243,263,362.36
5,000.0 m	3.107	mi.	0.0007852488	0	2	41.9692	5000.000128 m	0.000128462 m	38,922,135.92
10,000.0 m	6.21	mi.	0.0015704977	0	5	23.9384	10000.00103 m	0.001027693 m	9,730,531.98
20,000.0 m	12.43	mi.	0.0031409964	0	10	47.8770	20000.00822 m	0.008221551 m	2,432,630.97
50,000.0 m	31.07	mi.	0.0078525080	0	26	59.6960	50000.12846 m	0.128462485 m	389,218.69

Latitude at which to make the comparison = 35 deg. Geometrical mean radius = 6,370,783.223 m

Chord Distance			Theta - subtended angle				Arc	Difference	One part in
			Radians	D	M	S			
1,000.0 m	0.621	mi.	0.0001569666	0	0	32.3767	1000.000001 m	1.0266E-06 m	974,085,030.01
2,000.0 m	1.243	mi.	0.0003139331	0	1	4.7534	2000.000008 m	8.21284E-06 m	243,521,264.25
5,000.0 m	3.107	mi.	0.0007848329	0	2	41.8834	5000.000128 m	0.000128326 m	38,963,401.06
10,000.0 m	6.21	mi.	0.0015696659	0	5	23.7668	10000.00103 m	0.001026605 m	9,740,848.23
20,000.0 m	12.43	mi.	0.0031393327	0	10	47.5339	20000.00821 m	0.008212844 m	2,435,210.03
50,000.0 m	31.07	mi.	0.0078483487	0	26	58.8381	50000.12833 m	0.128326434 m	389,631.34

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$$R_m = \frac{a \sqrt{1 - e^2}}{(1 - e^2 \sin^2 \varphi)} \quad L = R\theta \quad \theta = 2 \sin^{-1} \left(\frac{CH}{2R} \right)$$

Latitude at which to make the comparison = 40 deg. Geometrical mean radius = 6,374,383.58 m

Chord Distance			Theta - subtended angle				Arc	Difference	One part in
			Radians	D	M	S			
1,000.0 m	0.621	mi.	0.0001568779	0	0	32.3584	1000.000001 m	1.02544E-06 m	975,186,556.47
2,000.0 m	1.243	mi.	0.0003137558	0	1	4.7168	2000.000008 m	8.20356E-06 m	243,796,591.82
5,000.0 m	3.107	mi.	0.0007843896	0	2	41.7920	5000.000128 m	0.000128181 m	39,007,452.71
10,000.0 m	6.21	mi.	0.0015687793	0	5	23.5840	10000.00103 m	0.001025445 m	9,751,861.15
20,000.0 m	12.43	mi.	0.0031375596	0	10	47.1681	20000.0082 m	0.008203569 m	2,437,963.26
50,000.0 m	31.07	mi.	0.0078439158	0	26	57.9238	50000.12818 m	0.128181512 m	390,071.85

Latitude at which to make the comparison = 45 deg. Geometrical mean radius = 6,378,101.03 m

Chord Distance			Theta - subtended angle				Arc	Difference	One part in
			Radians	D	M	S			
1,000.0 m	0.621	mi.	0.0001567865	0	0	32.3395	1000.000001 m	1.02425E-06 m	976,324,386.47
2,000.0 m	1.243	mi.	0.0003135730	0	1	4.6791	2000.000008 m	8.194E-06 m	244,081,042.44
5,000.0 m	3.107	mi.	0.0007839324	0	2	41.6977	5000.000128 m	0.000128031 m	39,052,963.11
10,000.0 m	6.21	mi.	0.0015678649	0	5	23.3954	10000.00102 m	0.00102425 m	9,763,238.77
20,000.0 m	12.43	mi.	0.0031357309	0	10	46.7909	20000.00819 m	0.008194009 m	2,440,807.67
50,000.0 m	31.07	mi.	0.0078393440	0	26	56.9808	50000.12803 m	0.128032134 m	390,526.96

