Example 3-D Computations Using the GSDM

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During recent travels, I've had the opportunity to photograph the highest point in Texas (Guadalupe Peak), about 90 miles east of El Paso, and some unusual rock formations in Texas Canyon (along Interstate 10 on our way to Tucson, Arizona). It is not a big deal, but each of those sites is interesting to explore further using Google Earth in conjunction with the 3-D global spatial data model (GSDM). I hope you might agree that this short article is both educational and fun. A press release on publication of the GSDM book and pictures of the two sites are included elsewhere in this issue of Benchmarks.

Interested readers are encouraged to visit http://google.earth.com for information on some excellent free software. With Google Earth installed on your computer and an internet connection, you can visit any part of the world and gain access to many kinds of valuable information. I've captured only a small portion of the data available for the two sites. In each case, I read the latitude, longitude, and elevation directly off of the screen. In case you wish to visit the same site, just "go to" the latitude and longitude values shown below. Once there, you can zoom in, rotate, tilt, and move the image. In addition, there are hot links posted on each site so that you can actually see pictures others have taken of prominent features. Given the high price of gas, traveling can still be enjoyed from home.

With the latitude/longitude/height values below read off of the Google Earth screen, the geocentric X/Y/Z coordinates for each point were computed using the GRS 1980 ellipsoid and the equations listed below.

Guadalupe Peak

φ =	31	53	29.12	N
λ =	104	51	37.81	W
λ =	255	8	22.19	E
h =			8763	ft
			2,671.0	m
N =	6,384,104.0959			m
X =	**-1,390,740.2732**			m
Y =	**-5,241,324.0923**			m
Z =	**3,351,626.2354**			m

Texas Canyon

φ =	32	3	34.73	N
λ =	110	4	58.71	W
λ =	249	55	1.29	E
h =			5,488	ft
			1,672.7	m
N =	6,384,160.5694			m
X =	**-1,858,364.9924**			m
Y =	**-5,082,900.4239**			m
Z =	**3,366,926.5738**			m

Geodesy Parameters and Equations

Seconds/radian =	206,264.806247096
GRS 1980 : a =	6,378,137.000 m
1/f =	298.25722210088
e^2 =	0.0066943800229

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$$

$$X = (N + h)\cos\varphi \cos\lambda$$
$$Y = (N + h)\cos\varphi \sin\lambda$$
$$Z = [N(1 - e^2) + h]\sin\phi$$
$$Dist = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}$$
where
$$\Delta X = X_2 - X$$
$$\Delta Y = Y_2 - Y_1$$
$$\Delta Z = Z_2 - Z_1$$

The point-to-point distance from Guadalupe Peak (in Texas) to Texan Canyon (in Arizona) is easily computed using the 3-D Pythagorean Theorem and the X/Y/Z differences.

ΔX =	-467,624.7193	m
ΔY =	158,423.6684	m
ΔZ =	15,300.3384	m

Mark-to-Mark Distance = **493,968.66** m = **306.94 mi.**
(slope distance point-to-point)

Comments:

1. The latitude/longitude/heights are all approximate and used as NAD 83 latitude/longitude/ellipsoid heights.
2. The GSDM allows the user to assign standard deviations on all input values. If no standard deviation values are input (as in this example), the values are used as exact.
3. If ellipsoid heights are input as zero (0.00 m), then the computed X/Y/Z coordinates are on the ellipsoid and the 3-D inverse distance (493,799.66 m) is the ellipsoid chord distance between the points.
4. The actual geodetic line arc distance on the ellipsoid between the same two points will be somewhat longer than the ellipsoid chord distance. Even when using the GSDM, computation of the geodetic arc distance between widely separated points involves quite a bit of work. But, it can be done - see equations 6.115 and 6.117 in the book. The geodetic arc distance between the points in this example is 493,922.84 meters.