Comparison of Geodetic, State Plane, and Geocentric Computational Models

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ABSTRACT: Surveyors make measurements and process those data to compute positions. Plane surveying uses two-dimensional (2D) coordinates based on assumptions of a flat earth. Plats, maps, and land boundary descriptions are prepared consistent with those assumptions and many "local practice" clients are well-served. With these descriptions, the client is well served, and, over the years, many "local practice" businesses have functioned successfully. But, as technology advances and as the scope of a project or service area gets larger, those flat earth assumptions become limiting, and plane surveyors are exposed to new challenges. Modern measurement systems, evolved during the digital revolution, now routinely collect 3D digital geospatial data. Similarly, computational processes now used in data reduction go well beyond the flat earth assumptions, and models for processing 3D digital geospatial data have evolved from flat earth models to various ellipsoidal models to a plethora of map projections to 3D models that support computations in 3D space worldwide, for example, the global spatial data model. This article includes a comparison of three models used to determine a 3D geodetic position based on a simple total-station side shot from a known station. The three methods are geodetic computation on the ellipsoid, state plane computation on a mapping grid, and geocentric computation in 3D space.

KEYWORDS: Spatial data, computational models, geometric geodesy, map projections, 3D positioning

Introduction

Station "Reilly" (PID AI5445) is geodetic control monument on the campus of New Mexico State University (NMSU) in Las Cruces, New Mexico. Used extensively by NMSU surveying engineering students, station Reilly was established as part of the New Mexico high-accuracy reference network (HARN), and its position was published by the National Geodetic Survey (NGS) on the North American Datum of 1983 (1992)—NAD 83 (1992). Although no longer referred to as an HARN station, NGS has subsequently published the position of Reilly on NAD 83 (2007) and NAD 83 (2011). As a learning exercise, students occupied Reilly with a total-station surveying instrument, backsighted a known azimuth mark, turned the horizontal angle, and observed the slope distance to the target (a retro-reflector placed on the desk of the NMSU Associate Dean of Engineering). The height of the instrument (HI), height of the target (HT), and zenith direction (Z) to the target were also observed. This article uses those data in three different models (geodetic, state plane, and geocentric) to compute an unmonumented 3D geodetic position on the associate dean's desk. It appears that, with no loss of integrity, the geocentric model has advantages of simplicity not shared by the other two models.

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Models and Objective

Models provide a conceptual connection between the abstract and human experience. Some models—for example, the flat earth model—are simple and easy to use, but spatial data models become more complex as needed to preserve the integrity of survey measurements and to account for geometrical relationships that extend beyond a local perspective. A general statement is that the "best" model is the simplest one that does not sacrifice geometrical integrity. Therefore, selection of the most appropriate spatial data model for a given application often involves a balance between simplicity and integrity. The objective of this article is to compare the ease of use and the complexity of the three computational procedures (models) that provide essentially identical answers for a geodetic position on the top of the NMSU Engineering Associate Dean's desk:

- Traditional geodetic computations on the ellipsoid.
- New Mexico central zone state plane coordinates.
- Geocentric *XYZ* values computed in 3D space.

Background, Control Values, and Measurements (Common to All Three Models)

The office of the Associate Dean of Engineering at NMSU is on the ground floor of Goddard Hall on the NMSU campus. Station Reilly is a ground-level brass tablet set in the top/middle of a massive concrete vault in an open area next to Goddard Hall. The 2015 NGS data sheet lists the geodetic latitude and longitude position, state plane values, and geocentric earth-centered earth-fixed (ECEF) coordinates for station Reilly. An approximate geoid height at Reilly is also listed on the data sheet. For this comparison, the North American Vertical Datum 1988 (NAVD 88) elevation for station Reilly was determined from local first-order benchmarks using Global Positioning System and geoid modeling (see http://www.globalcogo.com/ReilElev.pdf) and a finial on Skeen Hall about 240 m westerly of station Reilly was sighted for azimuth orientation. The azimuth to the finial was computed from four sets of Wild T-2 Polaris observations in 2001. A Laplace correction obtained using the NGS program "Deflect99" was used to compute a geodetic azimuth from the observed astronomic azimuth.

The following NAD 83 (2011) values were taken from the NGS data sheet:

	Geodetic	State Plane	Geocentric
Station Reilly	φ = 32°16′55.93001″ N	E = 452,506.490 m	X = –1,556,177.595 m
	λ = 106°45′15.16035″ W	N = 142,268.771 m	Y = –5,169,235.284 m
	= 253°14′44.83965″ E		Z = 3,387,551.720 m
	Ellipsoid height (h)	1,166.543 m	
	Geoid height (N) (Geoid12B)	–23.94 m	
	Grid scale factor	0.99992781	
	Convergence	–0° 16′ 09.5″	

Other values used in the computations include the following:

GRS80 ellipsoid parameters: a = 6,378,137.000 m and e^2 = 0.006694380023
Seconds per radian (spr) = 206,264.806247
NAVD 88 elevation of Reilly H = 1,190.497 m
Geodetic azimuth from Reilly to finial on Skeen Hall (α_{BS}) = 272°11′09″
Measurements
Electronic distance measurement slope distance (corrected for temperature & prism off-set) = 78.452 m
Angle right from finial to reflector on desk (mean of four sets direct/reverse) = 269°23′08″
Zenith direction to center of reflector (mean of two sets direct/reverse) = 090°54′08″
Height of Instrument at Reilly = 1.682 m
Height of Target on desk = 0.366 m

Equations and Computations

Geodesy Computations on the Ellipsoid

Equations for forward (also called "direct") geodetic computations on the ellipsoid are given in sources such as the works of Vincenty (1975, 1980) and Jank and Kivioja (1980). The equations used here are for one element of a geodetic line as described by Burkholder (2008) and based on the numerical integration method used by Jank and Kivioja (1980) who claim that millimeter accuracy of a computed position is maintained half way around the world when the individual line element used in the numerical integration is 200 m or less. Longer elements can be used on shorter lines while maintaining the same millimeter accuracy. Burkholder (2008) also describes a test for checking and assuring the accuracy of a geodetic forward computation.

$$\left.\begin{aligned} \varphi_{desk} &= \varphi_{Reilly} + \Delta\varphi \quad (1a)\\ \lambda_{desk} &= \lambda_{Reilly} + \Delta\lambda \quad (1b)\\ H_{desk} &= H_{Reilly} + \Delta H \quad (1c) \end{aligned}\right\}\ \text{See step-by-step equations below.}$$

$$\Delta\varphi'' = \frac{S\cos\alpha_{Geo}}{M}\,\text{spr} \quad (2)$$

$$\Delta\lambda'' = \frac{S \sin \alpha_{\mathrm{Geo}}}{N \cos \varphi} \mathrm{spr} \tag{3}$$

$$\Delta H = \mathrm{HI} + \mathrm{SD} \cos Z - \mathrm{HT} + (\text{curvature and refraction}) \tag{4}$$

$$S = \mathrm{SD} \times \sin Z \times \frac{R_{\mathrm{m}}}{R_{\mathrm{m}} + h} \tag{5}$$

$$\alpha_{\mathrm{Geo}} = \alpha_{\mathrm{BS}} + \text{angle right} \tag{6}$$

where α_{Geo} is geodetic azimuth, Z zenith direction to target (mean of two D/R sets), S ellipsoidal distance, SD slope distance, R_{m} Gaussian mean radius, h = ellipsoid height at station Reilly.

Radius of curvature in the meridian (M)

$$M = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \varphi^{1.5})} \tag{7}$$

Radius of curvature in the prime vertical (N)

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}} \tag{8}$$

$$R_m = \sqrt{M \times N} \tag{9}$$

Computations (use latitude and east longitude at station Reilly):

$$M = \frac{6,378,137.00(1 - 0.006694380023)}{\left[1 - 0.006694380023 \times \sin^2(32°16'55.93001'')^{1.5}\right]}$$
$$= 6,353,629.826 \text{ m}$$

$$N = \frac{6,378,137.000}{\sqrt{1 - 0.006694380023 \times \sin^2(32°16'55.93001'')}}$$
$$= 6,384,235.531 \text{ m}$$

$$R_m = \sqrt{6,353,629.826 \times 6,384,235.532}$$
$$= 6,368,914.294 \text{ m}$$

$$h = 1,166.543 \text{ m (from NGS data sheet)}$$

$$S = 78.452 \times \sin(90°54'08'') \times \frac{6,368,914.294}{6,370,080.837}$$
$$= 78.428 \text{ m}$$

$$\alpha_{\mathrm{Geo}} = 272°11'09'' + 269°23'08'' - 360° = 181°34'17''$$

$$\Delta\varphi' = \frac{78.428 \cos(181°34'17'')}{6,353,629.826} \times \mathrm{spr} = -2.54514''$$

$$\Delta\lambda'' = \frac{78.428 \sin(181°34'17'')}{6,384,235.531 \cos(32°16'55.93001'')} \times \mathrm{spr}(\mathrm{East}\ \Delta\lambda) = -0.08219''$$

$$\Delta H = 1.682 + 78.452 \cos(90°\ 54'08'') - 0.366 = 0.081 \text{ m}$$

In this case, curvature and refraction is <0.0005 m and is ignored, and the results are as follows:

$$\varphi_{\mathrm{desk}} = 32°16'55.93001'' - 2.54514'' = 32°16'53.38487'' \text{ N}$$

$$\lambda_{\mathrm{desk}} = 253°14'44.83965'' - 0.08219'' = 253°14'44.75746'' \text{ E} = 106°45'15.24254'' \text{ W}$$

$$H_{\mathrm{desk}} = 1,190.497 \text{ m} + 0.081 \text{ m} = 1,190.578 \text{ m}$$

State Plane Coordinate Computations: NM Central Zone

State plane coordinates are computed for the top of the desk and geodetic positions are computed from the state plane values. Although the equations are not listed herein, the geodetic latitude and longitude are computed from state plane coordinates using the algorithm given by Stem (1989). Elevation on the desk is computed the same way as that done in the geodetic method. Computation of state plane coordinates is as follows:

$$\varphi_{\mathrm{desk}} = \text{computed from NM central zone state plane coordinates} \tag{10a}$$

$$\lambda_{\mathrm{desk}} = \text{computed from NM central zone state plane coordinates} \tag{10b}$$

$$E_{\mathrm{desk}} = E_{\mathrm{Reilly}} + \mathrm{HD}_{\mathrm{Grid}} \times \sin(\mathrm{Az}_{\mathrm{Grid}}) \tag{11}$$

$$N_{\mathrm{desk}} = N_{\mathrm{Reilly}} + \mathrm{HD}_{\mathrm{Grid}} \times \cos(\mathrm{Az}_{\mathrm{Grid}}) \tag{12}$$

where:

$$\mathrm{HD}_{\mathrm{Grid}} = \mathrm{SD} \times \sin(Z) \times \text{combined factor} \tag{13}$$

$$\text{combined factor} = \text{grid scale factor} \times \text{elevation factor} \tag{14}$$

$$\text{elevation factor} = \frac{R_{\mathrm{m}}}{R_{\mathrm{m}} + h} \tag{15}$$

$$\mathrm{Az}_{\mathrm{Grid}} = \mathrm{Az}_{\mathrm{Grid\ to\ BS}} + \text{angle right} \tag{16}$$

$$\mathrm{Az}_{\mathrm{Grid\ to\ BS}} = \alpha_{\mathrm{Geo\ to\ BS}} - \text{convergence at Reilly} \tag{17}$$

Computations:

$$\mathrm{Az}_{\mathrm{Grid\ to\ BS}} = 272°11'09'' - (-00°16'09.''5) = 272°27'19''$$

$$\mathrm{Az}_{\mathrm{Grid\ to\ desk}} = 272°27'19'' + 269°23'08'' - 360° = 181°50'27''$$

$$\text{Combined factor} = 0.99992781\left(\frac{6,368,914.294}{6,370,080.837}\right)$$
$$= 0.999744695$$

$$HD_{Grid} = 78.452 \sin(90°54'08'')0.999744695$$
$$= 78.422 \text{ m}$$

$$\text{Easting}_{desk} = 452,506.490 \text{ m} + 78.422 \sin(181° \, 50'26.''6)$$
$$= 452,503.971 \text{ m}$$

$$\text{Northing}_{desk} = 142,268.771 \text{ m} + 78.422 \cos(181° \, 50'26.''6)$$
$$= 142,190.389 \text{ m}$$

$$H_{desk} = 1,190.497 \text{ m} + 0.081 \text{ m (same as before)}$$
$$= 1,190.578 \text{ m}$$

Geodetic latitude and longitude for the NM central zone NAD 83 state plane coordinates give:

Latitude on dean's desk	32°16′53.″38488 N
Longitude on dean's desk	106°45′15.″24253 W
Elevation on dean's desk	1,190.578 m

Geocentric ECEF XYZ Coordinates

Equations for computing ECEF geocentric coordinates are found in Chapter 1 of the book by Burkholder (2008), which describes the global spatial data model (GSDM). The equations and procedures can also be found in other geodesy texts. When using the GSDM for geodetic computations, the computations are performed in 3D space to obtain the geocentric *XYZ* coordinate values. For purposes of comparison with the other two methods, the geocentric *XYZ* coordinates need to be converted to geodetic latitude, longitude, and ellipsoid height. Geoid heights are required to determine orthometric heights (elevation) from ellipsoid heights. The NGS program, Geoid12B, was used to compute geoid heights, and those geoid heights were used to compute the NAVD 88 elevation on the desk.

$$\varphi_{desk} = \text{computed from } XYZ \text{ geocentric ECEF coordinate values} \tag{18a}$$

$$\lambda_{desk} = \text{computed from } XYZ \text{ geocentric ECEF coordinate values} \tag{18b}$$

$$H_{desk} = H_{Reilly} + \Delta H \text{ (Different than Equation 3)} \tag{18c}$$

$$X_{desk} = X_{Reilly} + \Delta X \tag{19}$$

$$Y_{desk} = Y_{Reilly} + \Delta Y \tag{20}$$

$$Z_{desk} = Z_{Reilly} + \Delta Z \tag{21}$$

$$\Delta X = -\Delta e \sin\lambda - \Delta n \sin\varphi \cos\lambda + \Delta u \cos\varphi \cos\lambda \tag{22}$$

$$\Delta Y = \Delta e \cos\lambda - \Delta n \sin\varphi \sin\lambda + \Delta u \cos\varphi \sin\lambda \tag{23}$$

$$\Delta Z = \Delta n \cos\varphi + \Delta u \sin\varphi \tag{24}$$

Note: Equations (22) to (24) use north latitude and east longitude at station Reilly.

$$\Delta e = \text{SD} \sin Z \sin\alpha_{Geo} \tag{25}$$

$$\Delta n = \text{SD} \sin Z \cos\alpha_{Geo} \tag{26}$$

$$\Delta u = \text{SD} \cos Z + \text{HI} - \text{HT} \tag{27}$$

Computations:

$$\alpha_{Geo} = 272° \, 11'09'' + 269°23'08'' - 360°$$
$$= 181°34'17''$$

$$\Delta e = 78.452 \sin(90°54'08) \sin(181°34'17'')$$
$$= -2.151 \text{ m}$$

$$\Delta n = 78.452 \sin(90°54'08) \cos(181°34'17'')$$
$$= -78.413 \text{ m}$$

$$\Delta u = 78.452 \cos(90°54'08'') + 1.682 - 0.366$$
$$= 0.081 \text{ m}$$

$$\Delta X = -(-2.151)\sin(253°14'44.''83965) - (-78.413) \sin(32°16'55.''93001)\cos(253°14'44.''83965) + 0.081 \cos(32°16'55.''93001)\cos(253°14'44.''83965)$$
$$= -14.152 \text{ m}$$

$$\Delta Y = (-2.151)\cos(253°14'44.''83965) - (-78.413) \sin(32°16'55.''93001)\sin(253°14'44.''83965) + 0.081 \cos(32°16'55.''93001)\sin(253°14'44.''83965)$$
$$= -39.547 \text{ m}$$

$$\Delta Z = -78.413\cos(32°16'55.''93001) + 0.081\sin(32°16'55.''93001) = -66.249 \text{ m}$$

$$X_{desk} = -1,556,177.595 \text{ m} + (-14.152 \text{ m})$$
$$= -1,556,191.747 \text{ m}$$

$$Y_{desk} = -5,169,235.284 \text{ m} + (-39.547 \text{ m})$$
$$= -5,169,274.831 \text{ m}$$

$$Z_{desk} = 3,387,551.720 \text{ m} + (-66.249 \text{ m})$$
$$= 3,387,485.471 \text{ m}$$

These geocentric *XYZ* values need to be converted to latitude, longitude, and ellipsoid height (and to elevation) for a comparison to be made. The longitude computation is very straight forward but the latitude and ellipsoid height computations are more challenging. Techniques such as one of those described by Meyer (2010) can be used with excellent results. But, the iteration method used here provides fully rigorous results with fewer mathematical gymnastics.

Since the geocentric X and Y values are both negative, the east longitude lies in the third quadrant of the equator and is computed (with due regard to radian units) as

$$\lambda = 180° + \text{atan}\left(\frac{Y}{X}\right) = \text{East longitude} \quad (28)$$

$$\text{Longitude} = 180° + \text{atan}\left(\frac{-5,169,274.831}{-1,556,191.747}\right)$$
$$= 253°14'44.''75745\ \text{E}$$
$$= 106°45'15.''24255\ \text{W}$$

Closed form equations for computing geocentric XYZ coordinates from latitude, longitude, and ellipsoid height are called a BK1 transformation by Burkholder (2008) and given as

$$X = (N + h)\cos\varphi\cos\lambda \quad (29)$$

$$Y = (N + h)\cos\varphi\sin\lambda \quad (30)$$

$$Z = \left[N\left(1 - e^2\right) + h\right]\sin\varphi \quad (31)$$

A mathematical inversion of Equations (35) to (37) can be used to compute the latitude and ellipsoid height from the XYZ coordinates. That inversion is also closed form but must be solved using iteration. Solving those inverted equations is referred to as a BK2 transformation and given in Chapter 6 of Burkholder (2008)

$$P = \sqrt{X^2 + Y^2}, \text{ an intermediate value} \quad (32)$$

$$\varphi_0 = \text{arc tan}\left(\frac{Z}{P(1 - e^2)}\right), \text{"seed" value for subsequent use} \quad (33)$$

$$N_0 = \frac{a}{\sqrt{1 - e^2\sin^2\varphi_0}}, \text{ needed in next step} \quad (34)$$

$$\varphi_i = \text{arc tan}\left[\frac{Z}{P}\left(1 + \frac{e^2 N_{i-1}\sin\varphi_{i-1}}{Z}\right)\right], \text{ second and subsequent values} \quad (35)$$

$$N_i = \frac{a}{\sqrt{1 - e^2 sin^2\varphi_i}}, \text{ second and subsequent values} \quad (36)$$

Once the solution for latitude has converged sufficiently, the ellipsoid height is computed using the latest values of φ and N:

$$h = \frac{P}{\cos\varphi} - N \quad (37)$$

Using a spreadsheet, the computation of the latitude for the top of the dean's desk is shown in Table 1.

The latitude is computed by converting radians to degrees-minutes-seconds using the conversion spr = 206,264.806247096 sec/rad.

φ = 0.563418389267(206,264.806247)
= 116,213.''384899 = 32°16′53.''38490

Twelve decimal places of latitude or longitude in radians translate to six decimal places of seconds when expressed as degrees, minutes, and seconds. More decimal places are included in the latitude tabulation that can be justified. This is done to show where differences begin to occur. It is safer to use more iterations than needed to stop the iteration prematurely. Good judgment is essential in reporting and interpreting results. In this case, the comparison between models is made at five decimal places of seconds for latitude and longitude (0.''00001 ≅ 0.0003 m) but, because of original observations being limited to the millimeter, the computed position can only be justified at five decimal places of seconds.

To compute the elevation (orthometric height) of the top of the dean's desk, the ellipsoid height must be converted to elevation. Milbert (1991) states that modeled geoid height differences are more accurate than a modeled geoid height at a single point. Therefore, two alternates are included for computing the NAVD 88 elevation of the top of the desk:

1. Apply the modeled geoid height at the desk as obtained from the NGS Geoid12B model.

Iteration	Latitude (radian)	Difference (radian)	Normal (meter)	Difference (meter)
0	0.563418945242	–	6,384,570.81481	–
1	0.563418550755	–0.000000394487	6,384,235.29594	–335.51888
2	0.563418390041	–0.000000160715	6,384,235.29283	–0.00311
3	0.563418389270	–0.000000000770	6,384,235.29281	–0.00002
4	0.563418389267	–0.000000000004	6,384,235.29281	0.00000

Table 1. Computation of latitude for top of the dean's desk.

Value	Geodetic	State Plane	3D Geocentric
Latitude	32°16′53.″38487 N	32°16′53.″38488 N	32°16′53.″38490 N
Longitude	106°45′15.″24254 W	106°45′15.″24253 W	106°45′15.″24255 W
Elevation (meter)	1,190.578	1,190.578	1,190.568 (alternative 1) 1,190.579 (alternative 2)

Table 2. A comparison of geodetic positions and elevations obtained for top of the dean's desk.

This method relies on the absolute value of modeled geoid height.

$$H_{\text{desk}} = h_{\text{desk}} - \text{geoid height}_{\text{desk}} \tag{38}$$

2. Determine the geoid height at both station Reilly and on the desk using the NGS Geoid12B model. The difference in geoid height combined with the difference in ellipsoid height should provide a stronger solution than using only a single modeled geoid height.

$$H_{\text{desk}} = H_{\text{Reilly}} + \left(h_{\text{desk}} - h_{\text{Reilly}}\right) - \left(\text{geoid height}_{\text{desk}} - \text{geoid height}_{\text{Reilly}}\right) \tag{39}$$

Using NGS interactive software, Geoid12B computations of geoid heights at station Reilly and on the dean's desk are as follows:

Geoid height at station Reilly	–23.943 m
Geoid height on dean's desk	–23.944 m

Elevation on dean's desk using the alternative 1:

$$H_{\text{desk}} = 1{,}166.624\ \text{m} - (-23.944\ \text{m}) = 1{,}190.568\ \text{m}$$

Elevation on dean's desk using alternative 2:

$$H_{\text{desk}} = 1{,}190.497\,\text{m} + (1{,}166.624\,\text{m} - 1{,}166.543\,\text{m}) - (-23.944\,\text{m} - (-)23.943\,\text{m}) = 1{,}190.579\,\text{m}$$

Summary of Results

A comparison of geodetic position and elevation of the top of the dean's desk for all the three methods is shown in Table 2.

Conclusions and Comments

1. All three methods yielded latitude/longitude values within 0.00003 sec of arc, that is, agreement within about 0.001 m, consistent with the quality of the observations.
2. Elevations derived from the geodetic and state plane methods are identical. There are two geocentric solution elevations. The first alternative uses the modeled absolute geoid model value for the top of the desk, and the result agrees with other methods within 0.011 m. The second alternative uses ellipsoid height difference along with modeled geoid height difference, and the result agrees with the first two methods within 1 mm. This illustrates the importance of using geoid modeled differences (relative) as opposed to using absolute geoid heights.
3. The geodetic model uses differential geometry equations on the ellipsoid. Although those geodesy equations are straight forward, they can be intimidating to persons not familiar with the same. But, all data and equations are listed herein.
4. Except maybe for needing to use grid azimuth and grid distance, Equations (11) and (12) in the state plane model are quite familiar to plane surveyors. Equations (13) to (17) deal with concepts of grid scale factors, elevation factors, combined factors, and convergence. Although not needed or used in the 3D model, they are required when using the state plane model.
5. The process of computing latitude and longitude from state plane coordinates is rather complicated, and the equations are not listed. However, those computations have become ingrained in modern practice, and software is readily available for making those conversions. Stem's (1989) work is an excellent source for equations and algorithms for NAD 83 state plane coordinate conversions.
6. The geocentric computations are performed in 3D space using rules of solid geometry. The equations for geocentric computations are readily available in Burkholder (2008) and other sources. Additional on-line resources are available as follows:

www.globalcogo.com/GM008.pdf	Gives equations for $\varphi/\lambda/h$ to X/Y/Z (BK1)
www.globalcogo.com/GM009.pdf	Gives equations for X/Y/Z to $\varphi/\lambda/h$ (BK2)
www.globalcogo.com/GM010.pdf	Diagrams illustrating BK1 & BK2 computation
www.globalcogo.com/DD-BK2.xlsx	Excel file for dean's desk BK2 computation
www.globalcogo.com/DD-BK2.pdf	PDF file for dean's desk BK2 computation

7. The BK2 computation is the most difficult part of the GSDM geometrical computations. The BK1 transformations from latitude/longitude/height to geocentric X/Y/Z coordinates are fairly straight forward but the reverse process (BK2) inverts those equations. The solution is also closed form, but these equations must be iterated for a solution. Iteration is used in the above spreadsheet file for BK2 computations.
8. Other alternatives to the iteration procedure used in the BK2 spreadsheet include the following:
   - Vincenty (1980) devised a noniterative algorithm as used in the following link: www.globalcogo.com/GM009.pdf. Comparisons have been made using these equations with excellent results, even out to satellite heights.
   - Equations (28) to (35) given by Meyer (2010) can be used for a reliable noniterative computational alternative.
9. The author concludes that the 3D GSDM can be used to perform most 3D spatial data computations in 3D space with greater ease and efficiency than performing similar computations on the ellipsoid or using a map projection such as state plane or Universal Transverse Mercator coordinates. Furthermore, the GSDM preserves geometrical integrity in spatial data computations.

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