From 3-D GPS Data to a 2-D Plat -
A Direct "No Distortion" Solution

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Introduction

I was privileged during the 2006 fall semester to teach SUR 292, U.S. Public Land Survey System Boundaries. Of the various challenges in teaching this sophomore level class, one goal was to incorporate GPS equipment, measurements, and data into a class project devoted to teaching principles of the U.S. Public Land Survey System. The class collected GPS data, individual students computed vectors and misclosures, the instructor computed a least squares adjustment of the network, and each student used those adjusted results in the global spatial data model (GSDM) to determine the local easting/northing coordinates of each surveyed point with respect to the chosen P.O.B. – see www.globalcogo.com/psgsdm.pdf From there, the section breakdown computations proceeded as a local tangent plane survey referenced to the true meridian through the P.O.B. Each student was asked to write a description for and to compute the area of a separate forty acre aliquot part of Section 31, T23S-R1E, NMPM.

Data Collection

Although other options such as RTK were available, we focused on keeping it simple. We used four Topcon dual frequency GPS receivers to collect static data. In the field, we set each GPS unit up over a found monument, turned it on, pressed the start button, measured the height of instrument (HI), and wrote that information on a station data sheet. At the end of the session, pressing one button terminated the survey and the data were saved in a file having a default name. The field notes consisted of the data file in the GPS unit and the session data sheet showing the station name, date, persons on the crew, the serial number of the GPS instrument, the start/stop times, and the antenna height measurement.

Yes, the learning process includes "hitches" - both anticipated and unanticipated. Mission planning, fully charged batteries, cables, field books, drive-to instructions, and coordination are all important. Of course we encountered some problems but, with careful planning and five separate observing sessions, on three different days, we ended up with 15 vectors connecting two HARN control points with four section corners and four quarter corners. Figure 1 shows three vector diagrams – one diagram for each day of observations. The diagrams show the two HARN control points (Reilly and Crucesair) and Section 31, T23S-R1E, New Mexico Principal Meridian (four section corners and four quarter corners). Section 31 is the same section in all three diagrams.

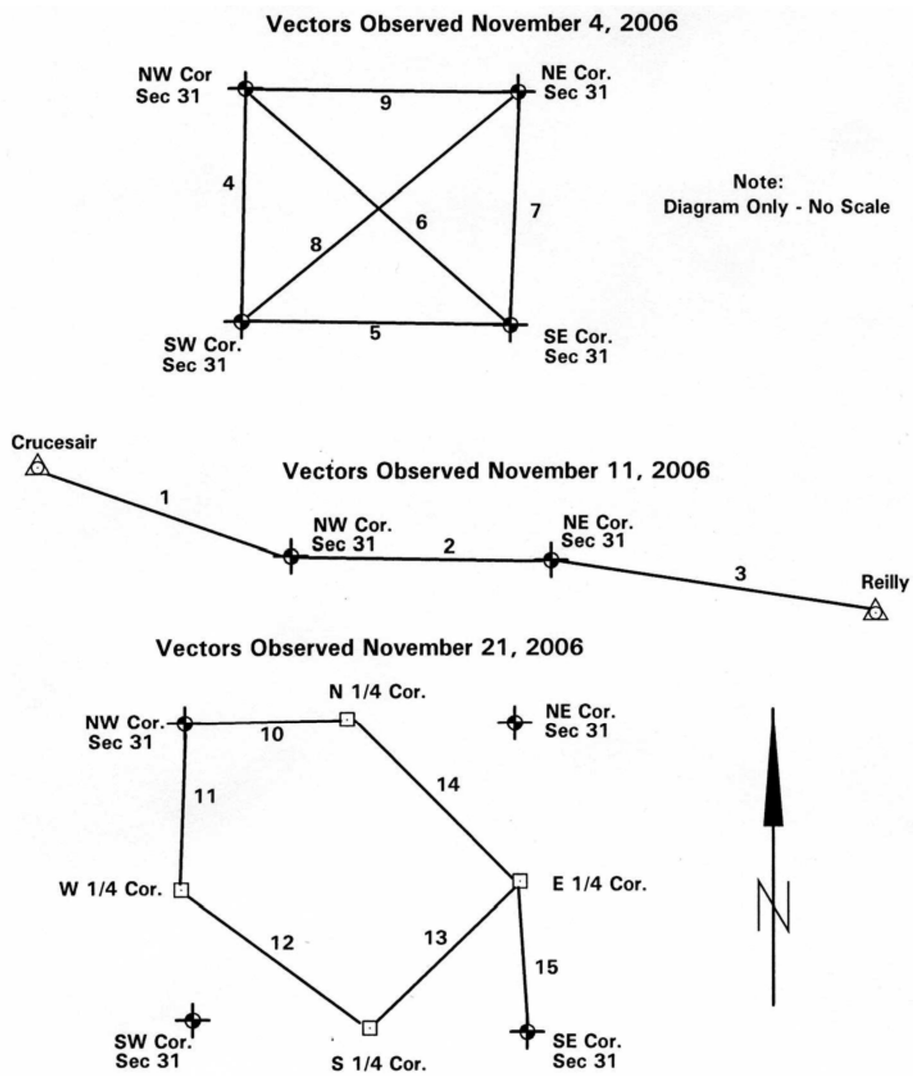

Figure 1 Diagrams of Vector Observations

We avoided trivial vectors. If two static GPS receivers collect data simultaneously, the computed vector between the two receivers is non-trivial. If a third receiver is added, two non-trivial baselines can be obtained. The third (closing) vector is trivial. We used four GPS receivers collecting data simultaneously and were thus able to obtain three non-trivial vectors during each session. We observed 15 vectors in surveying Section 31 and connecting it to the two HARN control points during the following 5 sessions.

November 11, 2006 Session A	Vectors	1, 2, and 3
November 4, 2006 Session A	Vectors	4, 5, and 7
November 4, 2006 Session B	Vectors	6, 8, and 9
November 21, 2006 Session A	Vectors	10, 11, and 12
November 21, 2006 Session B	Vectors	13, 14, and 15

Baseline Processing

Back at the office we downloaded the data files, replaced the default file names with more appropriate names, and entered the antenna height for each set-up. Each student went through the whole process for at least one baseline. They downloaded and renamed the two files for a given baseline, they entered the HI for each set-up, and they processed a baseline. The baseline results were saved and printed as geocentric coordinate differences ($\Delta X/\Delta Y/\Delta Z$ in meters), standard deviations, and correlations as shown below. Note that each baseline is directional. If the baseline is to be used in the opposite direction, the sign of each component is changed.

```
Crucesair to NW Cor 31                      Baseline 1
2983.7038,-1500.6159,-1030.1033,
0.00126368,0.00282992,0.00165760,
0.2600,-0.3096,-0.6778

NE Cor 31 to NW Cor 31                      Baseline 2
-1540.1416,465.5256,1.5518,
0.00062900,0.00177295,0.00087746,
0.2582,-0.2844,-0.7360

NE Cor 31 to Reilly                         Baseline 3
10729.2000,-2486.8700,980.1667,
0.00264627,0.00705109,0.00373377,
0.2333,-0.2095,-0.7225

NW Cor 31 to SW Cor 31                      Baseline 4
-251.1226,-824.1955,-1359.0102,
0.00060790,0.00169368,0.00085682,
0.3555,-0.3421,-0.7518

SE Cor 31 to SW Cor 31                      Baseline 5
-1540.5953,464.0666,2.8006,
0.00052881,0.00150311,0.00072976,
0.2729,-0.2712,-0.7387

NW Cor 31 to SE Cor 31                      Baseline 6
1289.4774,-1288.2622,-1361.8141,
0.00082659,0.00174433,0.00125784,
0.4457,-0.2963,-0.6632

NE Cor 31 to SE Cor 31                      Baseline 7
-250.6673,-822.7391,-1360.2552,
0.00059782,0.00180027,0.00087708,
0.1581,-0.1015,-0.7533

NE Cor 31 to SW Cor 31                      Baseline 8
-1791.2591,-358.6591,-1357.4656,
0.00084431,0.00182336,0.00137961,
0.4267,-0.2777,-0.6726

NE Cor 31 to NW Cor 31                      Baseline 9
-1540.1356,465.5338,1.5500,
0.00076400,0.00159682,0.00117659,
0.4058,-0.2765,-0.6558
```

```
N QTR to NW Cor 31                          Baseline 10
-764.5292,227.8853,-5.0120,
0.00048475,0.00086581,0.00077709,
0.6254,-0.3901,-0.6649

NW Cor 31 to W QTR                          Baseline 11
-125.8134,-412.0716,-679.5711,
0.00045859,0.00089418,0.00077360,
0.5998,-0.4204,-0.6770

S QTR to W QTR                              Baseline 12
-644.7279,644.5742,680.2069,
0.00055464,0.00108800,0.00088786,
0.5573,-0.3727,-0.6258

E QTR to S QTR                              Baseline 13
-895.3968,-178.7033,-678.5322,
0.00091636,0.00132630,0.00074513,
0.7750,-0.5652,-0.6381

E QTR to N QTR                              Baseline 14
-649.7821,650.0585,686.2564,
0.00103544,0.00145094,0.00077741,
0.8131,-0.6219,-0.6772

E QTR to SE Cor 31                          Baseline 15
-124.8328,-410.3149,-680.5715,
0.00086527,0.00115583,0.00061119,
0.8314,-0.6163,-0.6666
```

Computing Misclosures – Blunder Checks

In order to verify the absence of blunders in the baselines, we computed misclosures using all of the observed (independent) components as follows:

Traverse between fixed points Crucesair and Reilly using baselines 1, 2, and 3:

	X	Y	Z
Station "Crucesair"	-1,571,430.672 m	-5,164,782.312 m	3,387,603.188 m
Baseline 1 deltas	2,983.704 m	-1,500.616 m	-1,030.103 m
Baseline 2 deltas	1,540.142 m	-465.526 m	-1.552 m
Baseline 3 deltas	10,729.200 m	-2,486.870 m	980.167 m
Station "Reilly" observed	-1,556,177.626 m	-5,169,235.324 m	3,387,551.700 m
Station "Reilly" actual	-1,556,177.615 m	-5,169,235.319 m	3,387,551.709 m
Misclosures (observed-actual)	**-0.011 m**	**-0.005 m**	**-0.009 m**

Loop including baselines 4, 5 and 6 (being careful to preserve sign convention):

Baseline 4	-251.123 m	-824.196 m	-1,359.010 m
Baseline 5	1,540.595 m	-464.067 m	-2.801 m
Baseline 6	-1,289.477 m	1,288.262 m	1,361.814 m
Misclosure (sum to zero)	**-0.005 m**	**-0.001 m**	**0.003 m**

Loop including baselines 6, 7 and 8 (being careful to preserve sign convention):

Baseline 5	1,540.595 m	-464.067 m	-2.801 m
Baseline 7	250.667 m	822.739 m	1,360.255 m
Baseline 8	-1,791.259 m	-358.659 m	-1,357.466 m
Misclosure (sum to zero)	**0.003 m**	**0.013 m**	**-0.012 m**

Loop including baselines 2 and 9 (being careful to preserve sign convention):

Baseline 2	1,540.142 m	-465.526 m	-1.552 m
Baseline 9	-1,540.136 m	465.534 m	1.550 m
Misclosure (sum to zero)	**0.006 m**	**0.008 m**	**-0.002 m**

Loop including baselines 10, 11, 12, 13, and 14 (being careful to preserve sign convention):

Baseline 10	-764.529 m	227.885 m	-5.012 m
Baseline 11	-125.813 m	-412.072 m	-679.571 m
Baseline 12	644.728 m	-644.574 m	-680.207 m
Baseline 13	895.397 m	178.703 m	678.532 m
Baseline 14	-649.782 m	650.058 m	686.256 m
Misclosure (sum to zero)	**0.001 m**	**0.000 m**	**-0.002 m**

Loop including baselines 10, 14, 15, and 6 (being careful to preserve sign convention):

Baseline 10	764.529 m	-227.885 m	5.012 m
Baseline 14	649.782 m	-650.058 m	-686.256 m
Baseline 15	-124.833 m	-410.315 m	-680.572 m
Baseline 6	-1,289.477 m	1,288.262 m	1,361.814 m
Misclosure (sum to zero)	**0.001 m**	**0.004 m**	**-0.002 m**

All 15 baselines have been included in the checks and all misclosures are acceptable (in fact they are quite impressive). Therefore, it is legitimate to perform a least squares adjustment of the 15 baselines to determine the "best" adjusted position for the four Section Corners and four Quarter Corners of Section 31, T23S-R1E, NM Principal Meridian. The adjustment will provide statistics for the adjusted positions allowing us to assign and quote standard deviations for the adjusted coordinate values.

Least Squares Adjustment

With the misclosures being so small, one could argue that no adjustment is required. But, in addition to an adjustment providing one set of "best" numbers for the occupied points, the least squares adjustment also provides positional tolerance statistics for each point. The 15 baselines were adjusted simultaneously using option 2 procedures as documented in the article "Magical Least Squares" as printed in the January 2006 issue of Benchmarks and as posted at www.globalcogo.com/nmsunet1.pdf. The same two HARN stations, "Crucesair" and "Reilly," were used as fixed control. The fixed and adjusted stations (along with standard deviations of each component at the 1 sigma level) are shown in the ECEF reference frames. The geodetic coordinates and local standard deviations, also shown below, were computed using Windows BURKORD™ (WBK). See www.globalcogo.com/WBK3D.html.

Crucesair:	**X = -1,571,430.6720 m**	**fixed**	**Lat.**	**32° 16' 54."63123 N**	**fixed**
(HARN pt)	**Y = -5,164,782.3120 m**	**fixed**	**Long.**	**106° 55' 22."24784 W**	**fixed**
	Z = 3,387,603.1880 m	**fixed**	**El Hgt**	**h = 1,326.250 m**	**fixed**
Reilly:	**X = -1,556,177.6150 m**	**fixed**	**Lat.**	**32° 16' 55.92906 N**	**fixed**
(HARN pt)	**Y = -5,169,235.3190 m**	**fixed**	**Long.**	**106° 45' 15.16070 W**	**fixed**
	Z = 3,387,551.7090 m	**fixed**	**El Hgt**	**h = 1,166.570 m**	**fixed**

		ECEF Frame			**Local Frame**
NW Cor 31:	**X = -1,568,446.9652 m**	**+/- 0.0033 m**	**Lat.**	**32° 16' 16.51587 N**	**+/- 0.0054 m**
	Y = -5,166,282.9266 m	**+/- 0.0077 m**	**Long.**	**106° 53' 16.50858 W**	**+/- 0.0039 m**
	Z = 3,386,573.0861 m	**+/- 0.0044 m**	**El Hgt**	**h = 1,256.511 m**	**+/- 0.0067 m**

NE Cor 31:	**X = -1,566,906.8273 m +/- 0.0034 m**	**Lat.**	**32° 16' 16.50308 N +/- 0.0057 m**
	Y = -5,166,748.4577 m +/- 0.0080 m	**Long.**	**106° 52' 15.04095 W +/- 0.0040 m**
	Z = 3,386,571.5363 m +/- 0.0046 m	**El Hgt**	**h = 1,254.233 m +/- 0.0070 m**
SW Cor 31:	**X = -1,568,698.0864 m +/- 0.0035 m**	**Lat.**	**32° 15' 24.28753 N +/- 0.0058 m**
	Y = -5,167,107.1198 m +/- 0.0083 m	**Long.**	**106° 53' 16.54155 W +/- 0.0041 m**
	Z = 3,385,214.0743 m +/- 0.0047 m	**El Hgt**	**h = 1,259.609 m +/- 0.0072 m**
SE Cor 31:	**X = -1,567,157.4899 m +/- 0.0035 m**	**Lat.**	**32° 15' 24.26696 N +/- 0.0058 m**
	Y = -5,167,571.1861 m +/- 0.0081 m	**Long.**	**106° 52' 15.08320 W +/- 0.0041 m**
	Z = 3,385,211.2732 m +/- 0.0047 m	**El Hgt**	**h = 1,255.365 m +/- 0.0071 m**
N ¼ Cor 31:	**X = -1,567,682.4363 m +/- 0.0036 m**	**Lat.**	**32° 16' 16.72241 N +/- 0.0058 m**
	Y = -5,166,510.8119 m +/- 0.0080 m	**Long.**	**106° 52'46.03144 W +/- 0.0042 m**
	Z = 3,386,578.0990 m +/- 0.0048 m	**El Hgt**	**h = 1,255.823 m +/- 0.0070 m**
W ¼ Cor 31:	**X = -1,568,572.7788 m +/- 0.0035 m**	**Lat.**	**32° 15' 50.39908 N +/- 0.0058 m**
	Y = -5,166,694.9985 m +/- 0.0080 m	**Long.**	**106° 53' 16.53459 W +/- 0.0041 m**
	Z = 3,385,893.5160 m +/- 0.0048 m	**El Hgt**	**h = 1,258.017 m +/- 0.0070 m**
S ¼ Cor 31:	**X = -1,567,928.0513 m +/- 0.0038 m**	**Lat.**	**32° 15' 24.28747 N +/- 0.0060 m**
	Y = -5,167,339.5732 m +/- 0.0083 m	**Long.**	**106° 52' 45.81734 W +/- 0.0044 m**
	Z = 3,385,213.3104 m +/- 0.0050 m	**El Hgt**	**h = 1,258.181 m +/- 0.0073 m**
E ¼ Cor 31:	**X = -1,567,032.6554 m +/- 0.0038 m**	**Lat.**	**32° 15' 50.37719 N +/- 0.0059 m**
	Y = -5,167,160.8706 m +/- 0.0082 m	**Long.**	**106° 52' 15.06861 W +/- 0.0043 m**
	Z = 3,385,891.8435 m +/- 0.0048 m	**El Hgt**	**h = 1,255.952 m +/- 0.0072 m**

From 3-D GPS Data to a 2-D Plat

For seasoned GPS users, performing a least squares adjustment of a GPS network is fairly routine. One difference in this case may be that a commercial network package does the least squares adjustment in the geodetic reference system. Done that way, questions remain about the third dimension and how vertical is handled. Many vendor packages do a competent adjustment in the geodetic reference system but users need to exercise caution with respect to vertical. This least squares adjustment was done in the 3-D ECEF environment according to strict rules of solid geometry. The mathematical operations were handled efficiently by matrices, see www.globalcogo.com/nmsunet1.pdf, and no approximations were made during the adjustment regarding elevation, geoid height, or map projections.

This section is the crux of this article – the global spatial data model (GSDM) defines the procedures for obtaining 2-D plat data (local tangent plane bearings, distances, and standard deviations) from the 3-D GPS data. The BURKORD™ Diagram in Figure 2 shows a box for the geocentric X/Y/Z coordinate values at the top. The local tangent plane latitude/departure differences are shown in a box on the right. The user specifies the P.O.B. origin (by choosing an existing point) and a rotation matrix is used to transform the geocentric differences between the user selected origin and all other points to local (plane surveying) differences.

Equations for performing the various computations as shown on the 3-D Diagram are readily available and, among others, can be found at www.globalcogo.com/psgsdm.pdf or www.globalcogo.com/ionpaper.pdf. Equation numbers in the ION paper are used below.

3-D Diagram	In ION paper see
BK1 equations - Use φ/λ/h to find X/Y/Z	equations (11), (12), and (13).
BK2 equations - Use X/Y/Z to find φ/λ/h	equations (14), (15), and (16).
BK3 equations - Use X/Y/Z's to find ΔX/ΔY/ΔZ	equations (1), (3), and (5).
BK4 equations - Use Pt_1 & ΔX/ΔY/ΔZ to find Pt_2	equations (2), (4), and (6).
BK8 equations - Use ΔX/ΔY/ΔZ to find Δe/Δn/Δu	matrix equation (7).
BK9 equations - Use Δe/Δn/Δu to find ΔX/ΔY/ΔZ	matrix equation (8).

Specifically, the local tangent plane latitudes and departures (northing and easing coordinates with respect to the P.O.B.) are obtained using the rotation matrix as shown in Box 7 of Figure 2.

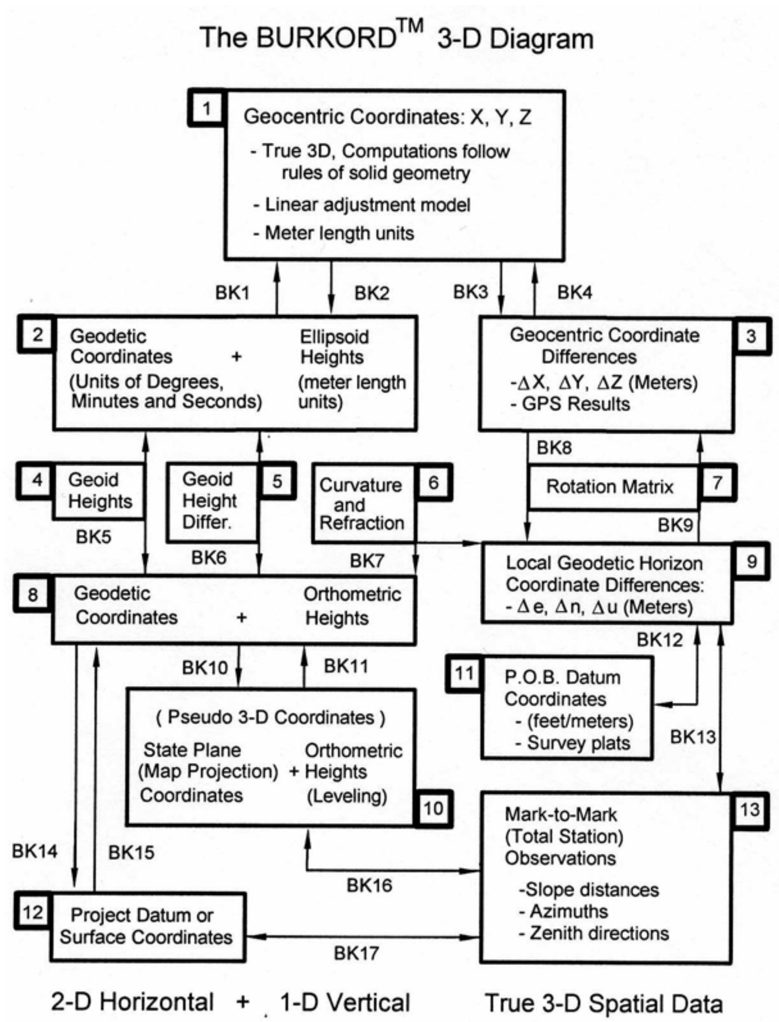

Figure 2 – Schematic Showing Relationship of Geometrical Elements

For the SUR 292 GPS survey of Section 31, T23S-R1E. NMPM, we decided to place the P.O.B origin on the SW Corner. Having selected that origin, the local easting and northing distances to the other points in Section 31 are shown below. We did not use the "up" component data.

Point	Description	P.O.B. East (meters)	P.O.B. North (meters)	P.O.B. Up (meters)
1001	Crucesair	-3,290.100	2,783.992	65.183
1002	Reilly	12,598.767	2,831.217	-106.099
1013	NW Cor Sec 31	0.863	1,609.117	-3.302
1014	NE Cor Sec 31	1,609.819	1,608.850	-5.783
1015	SW Cor Sec 31	0.000	0.000	0.000
1016	SE Cor Sec 31	1,608.970	-0.506	-4.447
1017	N Qtr Cor Sec 31	798.622	1,615.512	-4.041
1018	W Qtr Cor Sec 31	0.182	804.478	-1.643
1019	S Qtr Cor Sec 31	804.355	0.030	-1.479
1020	E Qtr Cor Sec 31	1,609.224	803.931	-3.910

Using the P.O.B. latitudes/departures as local tangent plane coordinates, we computed directions and distances around Section 31 as shown below and on the plat in Figure 3. It is redundant, but each line is computed both ways. Each distance is the P.O.B. tangent plane distance (no grid to ground correction is needed) and each direction is with respect to the true meridian through the P.O.B. The standard deviation of each computed direction and distance is also provided by the GSDM – for computational details, see www.globalcogo.com/accuracy.pdf.

From Pt	To Point	Azimuth D M S	St. dev. Seconds	Distance	St .dev
SW Cor	W Qtr Cor	0 00 46.7	+/- 1.5	804.478 m	+/- 0.0082 m
W Qtr Cor	NW Cor	0 02 54.5	+/- 1.4	804.640 m	+/- 0.0079 m
NW Cor	N Qtr Cor	89 32 26.7	+/- 2.1	797.785 m	+/- 0.0057 m
N Qtr Cor	NE Cor	90 28 13.7	+/- 2.1	811.224 m	+/- 0.0058 m
NE Cor	E Qtr Cor	180 02 32.6	+/- 1.5	804.920 m	+/- 0.0082 m
E Qtr Cor	SE Cor	180 01 05.2	+/- 1.5	804.437 m	+/- 0.0082 m
SE Cor Sec	S Qtr Cor	270 02 17.4	+/- 2.1	804.615 m	+/- 0.0060 m
S Qtr Cor	SW Cor	269 59 52.3	+/- 2.1	804.355 m	+/- 0.0060 m
SW Cor	S Qtr Cor	89 59 52.3	+/- 2.1	804.355 m	+/- 0.0060 m
S Qtr Cor	SE Cor	90 02 17.4	+/- 2.1	804.615 m	+/- 0.0060 m
SE Cor	E Qtr Cor	0 01 05.2	+/- 1.5	804.437 m	+/- 0.0082 m
E Qtr Cor	NE Cor	0 02 32.6	+/- 1.5	804.920 m	+/- 0.0082 m
NE Cor	N Qtr Cor	270 28 13.7	+/- 2.1	811.224 m	+/- 0.0058 m
N Qtr Cor	NW Cor	269 32 26.7	+/- 2.1	797.785 m	+/- 0.0057 m
NW Cor	W Qtr Cor	180 02 54.5	+/- 1.4	804.640 m	+/- 0.0079 m
W Qtr Cor	SW Cor	180 00 46.7	+/- 1.5	804.478 m	+/- 0.0082 m

The North/South and East/West Quarter lines were also computed as:

S Qtr Cor	N Qtr Cor	359 47 48.0	+/- 0.8	1,615.492 m	+/- 0.0084 m
W Qtr Cor	E Qtr Cor	90 01 10.1	+/- 1.1	1,609.042 m	+/- 0.0060 m
N Qtr Cor	S Qtr Cor	179 47 48.0	+/- 0.8	1,615.492 m	+/- 0.0084 m
E Qtr Cor	W Qtr Cor	270 01 10.1	+/- 1.1	1,609.042 m	+/- 0.0060 m

The user may choose any length units when drafting a plat using P.O.B. datum coordinates. For example, the following inverses replicate those in meters above except that U.S. Survey Foot units are displayed. All GSDM computations are performed in meters, but other units (even chains) may be displayed and/or printed by the user. A point is that coordinate differences are usually small – when compared to state plane coordinate values. An implication of that is that U.S. Survey Feet and International Feet distances will be identical (to a very small tolerance).

From Pt	To Point	Azimuth D M S	St. dev. Seconds	Distance	St .dev
SW Cor	W Qtr Cor	0 00 46.7	+/- 1.5	2,639.357 ft	+/- 0.027 ft
W Qtr Cor	NW Cor	0 02 54.5	+/- 1.4	2,639.889 ft	+/- 0.026 ft
NW Cor	N Qtr Cor	89 32 26.7	+/- 2.1	2,617.399 ft	+/- 0.019 ft
N Qtr Cor	NE Cor	90 28 13.7	+/- 2.1	2,661.492 ft	+/- 0.019 ft
NE Cor	E Qtr Cor	180 02 32.6	+/- 1.5	2,640.808 ft	+/- 0.027 ft
E Qtr Cor	SE Cor	180 01 05.2	+/- 1.5	2,639.222 ft	+/- 0.027 ft
SE Cor	S Qtr Cor	270 02 17.4	+/- 2.1	2,639.807 ft	+/- 0.020 ft
S Qtr Cor	SW Cor	269 59 52.3	+/- 2.1	2,638.955 ft	+/- 0.020 ft

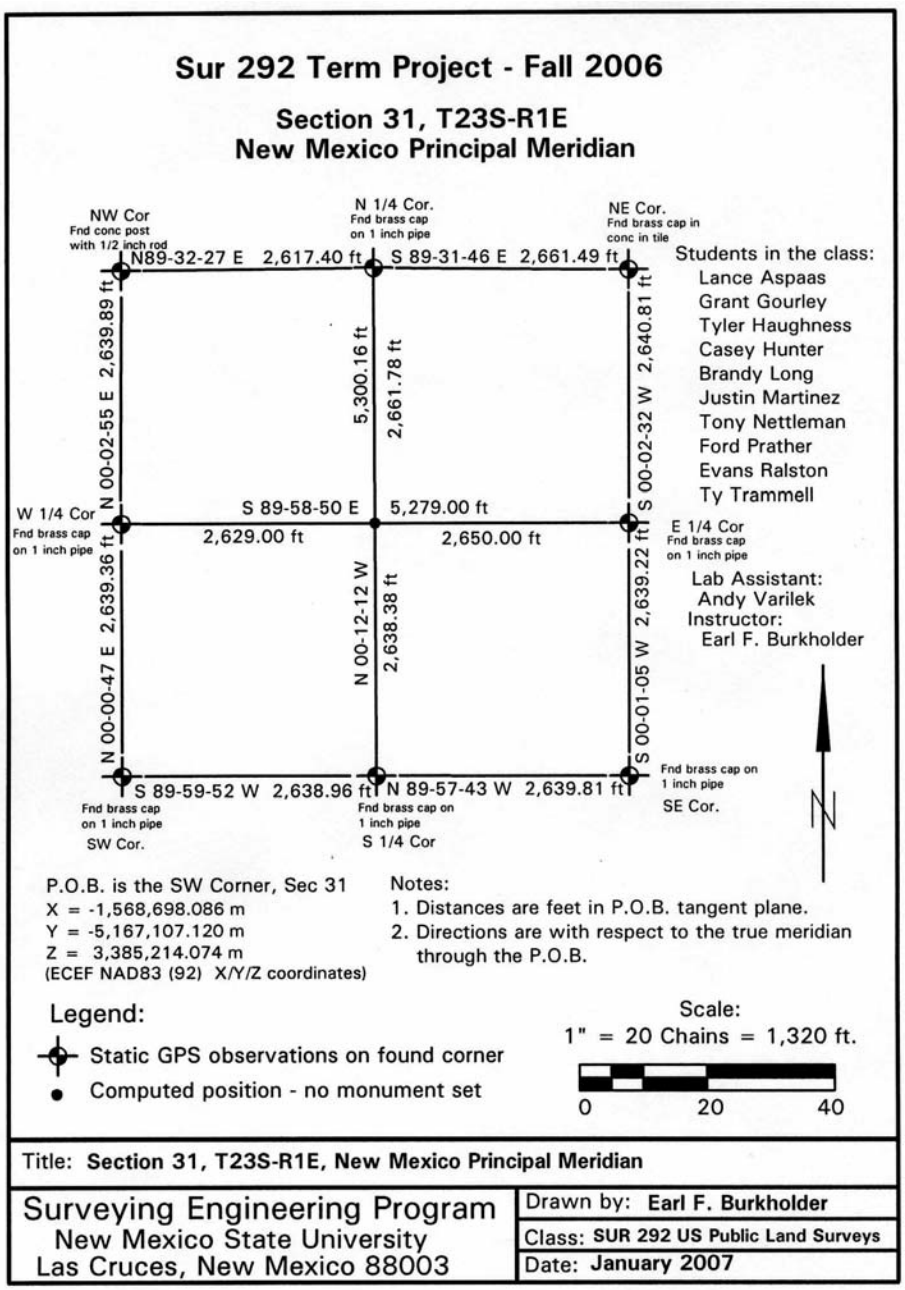

Figure 3 Plat of Survey – Section 31, T23S-R1E, New Mexico Principal Meridian

Conclusions

Many conclusions can be drawn from this example. A few of the most obvious ones are:

1. There is a direct process by which static GPS data can be depicted on a 2-D plat.
2. All the equations are already in place and being used by many spatial data users.
3. Local ground level bearings and distances are readily available without using state plane coordinates, scale factors, elevation factors, map projections, or geoid modeling.
4. The advantages of working on a standard globally recognized referenced system are preserved. Latitude/longitude/ellipsoid height are readily computed for each point.
5. A local tangent plane grid is established by deciding which point to use as the P.O.B.
6. Others:

Software

Software for performing 3-D coordinate geometry and error propagation computations based upon the GSDM has been developed by the author and offered to spatial data users by Global COGO, Inc. under the BURKORD™ trademark. A DOS-based, menu-driven BURKORD program was written in FORTRAN and has been available since 1997. BURKORD8 is an excellent proof-of-concept tool but it is not very user-friendly. A Windows-based version of BURKORD™ has been developed and is called WBK. A free 50 point version, WBK-Basic, was released in November 2006 and is available at www.globalcogo.com/WBK3D.html.