Concepts of Spatial Data Accuracy Need Our Attention

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Abstract

The digital revolution has created disruptive innovation in many sectors of society — spatial data accuracy being the focus of this paper. Surveyors have used geometry and spatial data for many years to compute control traverses and level loops for many applications. With the advent of computers and least squares adjustments, the traditional ratio of precision for judging the quality of a traverse has been replaced by positional tolerance and error ellipses. Not surprisingly, those concepts have entered the main stream of surveying practice and are included in the 2016 version of the Minimum Standards for ALTA/NSPS Land Title Surveys. The problem is that the method described in the ALTA/NSPS standard does not provide the stipulated "relative positional precision." This paper provides some background, identifies the issues, discusses models, and shows how "relative positional precision" (also called local accuracy) can be determined with statistical reliability.

Introduction

Members of the American Land Title Association need reliable surveys to support guarantees of title to clients who purchase those insurance policies. Accuracy standards are a part of a reliable survey and are included in the "Minimum Standard Detail Requirements for ALTA/NSPS Land Title Surveys" as Relative Positional Precision (RPP) measurement standards (ALTA/NSPS 2016). The relative position of a corner monument to any other property survey corner is to be reported at the 95% (2 sigma) confidence level. The standards also state that the RPP is to be estimated as the semimajor axis of the error ellipse obtained from a properly weighted least squares adjustment. Therein lies the problem. The error ellipse as determined from a least squares adjustment provides an error estimate with respect to the survey control as chosen by the user — not an adjacent (or any other) point in the survey. The intent is stated rather well and a properly weighted least squares adjustment is the "gold standard" for survey adjustments. But, as noted by Bill Hazelton (2016), the least squares adjustment would need to be run repeatedly — successively holding each point in the survey "fixed" to determine the relative position of that point with respect to all other points in the survey. As currently stated in the ALTA/NSPS standards, there is a mismatch between the prescribed computational procedure and the desired result. This paper shows how the local accuracy of any point in the survey with respect to any other point in the survey (or project) can be determined from one run of a least squares adjustment.

Possible Solution

Possible solutions include:

1. Changing the wording in the Minimum Standards to reflect current practice and use of error ellipses. While less than ideal, replacing "relative positional precision" with "positional tolerance" may be appropriate. Such a change of wording could eliminate the conflict within the standard but it would not provide the intuitive understanding implied by "relative positional precision."

2. Stipulating that "local accuracy" as computed using the correlation information in the adjustment covariance matrix be used as the basis of comparing the relative position of one point in the survey with respect to any other point in the survey.
3. Some process that is adequate, that is comprehensible, and that preserves mathematical and geometrical integrity.

Background

Spatial data are three-dimensional (3-D), four-dimensional if time is also counted. The current ALTA/NSPS standards and associated error ellipses are based on assumptions of a 2-D survey. The least squares adjustment is configured accordingly — no fault is attributed to the least squares adjustment. And, it is presumed in any case that the least squares adjustment is properly weighted. The difference is that the desired correlation information (rarely used) is contained in the covariance matrix associated with the adjustment. Various kinds of accuracies can be computed following a least squares adjustment depending on choices made by the user (Burkholder 1999). Those choices and accuracies can be understood better following a short discussion of models.

Models

Simplicity and integrity are deciding factors when selecting an appropriate model for spatial data computations. The best model is the simplest one that is adequate for the intended application. Flat-Earth rectangular 2-D or 3-D equations are used extensively in many disciplines – often without detrimental consequences. But the Earth is not flat and more complex models are needed when curvature of the Earth makes a difference. Geometrical integrity is preserved (at the expense of simplicity) by performing geodetic computations on the mathematical ellipsoid. Alternatively, cartographers invented map projections to recover some of the simplicity for the end user without sacrificing integrity. Both models are legitimate and used extensively. But both models assume separate origins for horizontal and vertical (3-D data) and both models require measurements of spatial data to be reduced to a computational surface — the mathematical ellipsoid for geodetic computations and to a projection surface for mapping.

The 3-D global spatial data model (GSDM), (Burkholder 1997a), is an improvement over traditional models in that spatial data computations are performed in 3-D space. The GSDM embodies a single origin (center of mass of the Earth) for both horizontal and vertical data and the GSDM eliminates the need for many "reductions."

Among others, advantages of the GSDM include:

1. The model does not distort spatial data components.
2. The simplicity of "flat-Earth" uses is fully supported.
3. Backward compatibility supports derivative uses of traditional 2-D and 1-D models.
4. The same data base supports both global and local perspectives with no loss of integrity.
5. The same equations are applicable worldwide for all spatial data disciplines.
6. Computations are less complicated and data storage is more efficient.
7. Spatial data accuracy is well defined for both global and local perspectives.

Example

The 2nd Edition of *The 3-D Global Spatial Data Model* (Burkholder 2017) is scheduled for release in July 2017. Chapter 14 in that book contains a comprehensive example of computing network accuracy and local accuracy. Rather than repeat that lengthy example, a more recent data set obtained on the newly installed and measured calibration baseline at New Mexico State University (NMSU) is used to illustrate computation of local accuracy. While the book example shows computation of network and local accuracies, this example applies and extends those concepts to the solution of an existing dilemma.

NMSU EDMI Calibration Baseline

Due to unfortunate circumstances, the previous EDMI calibration baseline at NMSU became un-usable. Eventually, new baseline monuments were established on the NMSU Leyendecker Research Farm and the National Geodetic Survey (NGS 2016) measured the baseline. NGS published the baseline distances in July 2016 and quoted a standard error of 0.2 mm for each baseline distance.

In April 2017, Robert Green, local Trimble dealer from Vectors, Inc. in Albuquerque, NM, collected static GPS data on the new baseline as part of a test of the new Trimble SX10 total station/scanner. He used a Trimble R8 receiver and several R8s GNSS receivers programmed for 5-second logging intervals. The tribrachs had been recently calibrated and the data were processed using both the Ultra Rapid Orbit and several weeks later using the Precise orbit. The survey was tied to two CORS stations – one at NMSU, the other at Hatch, New Mexico, and trivial baselines were studiously avoided. Those static GPS data are used as the basis for the discussion of local accuracy included in this paper.

Control Data and Observations

The data obtained from the NGS data sheets for the CORS are:

NMSU (PID DM3997) NAD 83(2011)		HATCHARP (PID P026) NAD 83(2011)	
X =	-1,555,457.539 m	X =	-1,589,272.343 m
Y =	-5,169,962.408 m	Y =	-5,135,739.911 m
Z =	3,386,819.086 m	Z =	3,422,845.167 m

Figure 1 is a diagram (not to scale) showing the two control points and the four baseline monuments. The observed GPS vectors are also shown along with an arrow indicating the direction (From-To) for the computation of each baseline.

Baseline 1 — NMSU to 950 observed 04/25/17 (use subscript BL1):

			Sxx	Syy	Szz
ΔX_{BL1} =	-714.312 m	Sxx	1.33766E-05		
ΔY_{BL1} =	-4,811.351 m	Syy	3.31024E-06	1.08730E-04	
ΔZ_{BL1} =	-7,705.140 m	Szz	-2.03729E-05	-6.50663E-05	4.39296E-05

Baseline 2 — 950 to ZERO observed 04/25/17 (use subscript BL2):

			Sxx	Syy	Szz
ΔX_{BL2} =	612.228 m	Sxx	1.76440E-06		
ΔY_{BL2} =	-517.924 m	Syy	1.06300E-06	6.22450E-06	
ΔZ_{BL2} =	-509.040 m	Szz	2.05000E-08	-3.58500E-06	5.38040E-06

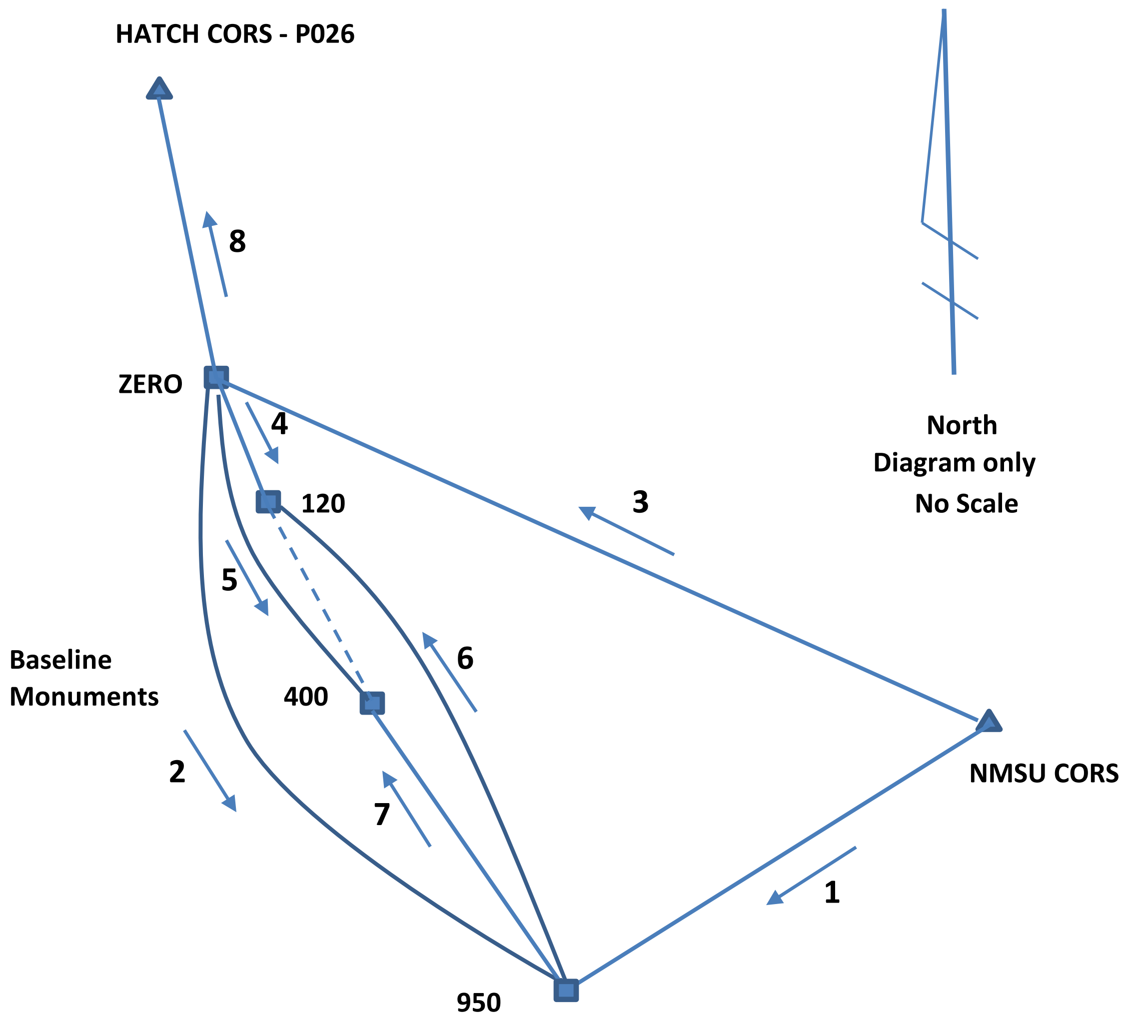

Figure 1 **Control Points and Calibration Baseline Monuments**

<u>Baseline 3 — NMSU to ZERO observed 04/25/17 (use subscript BL3):</u>

			Sxx	Syy	Szz
ΔX_{BL3} =	-1,326.538 m	Sxx	7.08100E-06		
ΔY_{BL3} =	-4,293.433 m	Syy	1.46015E-05	6.74901E-05	
ΔZ_{BL3} =	-7,196.095 m	Szz	-8.41390E-06	-4.55390E-05	3.79188E-05

<u>Baseline 4 — ZERO to 120 observed 04/25/17 (use subscript BL4):</u>

			Sxx	Syy	Szz
ΔX_{BL4} =	77.477 m	Sxx	8.15900E-07		
ΔY_{BL4} =	-64.949 m	Syy	5.13000E-07	2.73460E-05	
ΔZ_{BL4} =	-64.547 m	Szz	-3.00000E-09	-1.04660E-06	1.34850E-06

Baseline 5 — ZERO to 400 observed 04/25/17 (use subscript BL5):

			Sxx	Syy	Szz
ΔX_{BL5} =	257.936 m	Sxx	1.02820E-06		
ΔY_{BL5} =	-217.583 m	Syy	4.51400E-07	3.13680E-06	
ΔZ_{BL5} =	-214.615 m	Szz	2.63200E-07	-1.57500E-06	2.501200E-06

Baseline 6 — 950 to 120 observed 04/25/17 (use subscript BL6):

			Sxx	Syy	Szz
ΔX_{BL6} =	-534.749 m	Sxx	2.11930E-06		
ΔY_{BL6} =	452.971 m	Syy	3.11220E-06	1.18796E-05	
ΔZ_{BL6} =	444.495 m	Szz	-1.54030E-06	-4.82990E-06	3.32680E-06

Baseline 7 — 950 to 400 observed 04/25/17 (use subscript BL7):

			Sxx	Syy	Szz
ΔX_{BL7} =	-354.294 m	Sxx	1.36820E-06		
ΔY_{BL7} =	300.343 m	Syy	1.70440E-06	7.114200E-06	
ΔZ_{BL7} =	294.423 m	Szz	-5.90700E-07	-2.525400E-06	1.97780E-06

Baseline 8 — ZERO to P026 observed 04/25/17 (use subscript BL8):

			Sxx	Syy	Szz
ΔX_{BL8} =	-32,488.275 m	Sxx	5.70620E-06		
ΔY_{BL8} =	38,515.929 m	Syy	1.03618E-05	5.031901E-05	
ΔZ_{BL8} =	43,222.178 m	Szz	-7.95760E-06	-3.376260E-05	3.112740E-06

Blunder Checks

There is no method of adjustment of survey data proven better than a least squares adjustment. But, before performing an adjustment, a "blunder check" is used to confirm acceptable observations by adding up vector components to determine misclosures. All vectors in the adjustment are to be included in the blunder check. Some vectors may be used more than once. If a vector leads to a side shot with no redundant checks, that vector cannot be included in the least squares adjustment. A large misclosure typically means that the direction of the observed vector is "backward," that the sign of a baseline component has been switched, or that the observed data cannot be accepted as reliable.

It is also a cardinal rule in computing a least squares adjustment of GNSS vectors that trivial baselines are not to be used. Trivial vectors are discussed in Burkholder (2017) and other sources.

The following sequences of vectors is used to determine acceptable misclosures in each of the three components — other sequences could also be used. If a computational direction opposes the observation direction, the sign of vector each component is reversed as noted (-) in the following tabulations.

Loop: NMSU to 950 to ZERO to NMSU

Station/Baseline	ΔX (m)	ΔY (m)	ΔZ (m)
BL1	-714.312	-4,811.351	-7,705.140
BL2 (-)	-612.228	517.924	509.040
BL3 (-)	1,326.538	4,293.433	7,196.095
Loop misclosure	- 0.002	0.006	- 0.005

Loop: ZERO to 0120 to 0950 to 0400 to ZERO

Station/Baseline	ΔX (m)	ΔY (m)	ΔZ (m)
BL4	77.477	-64.949	-64.547
BL6 (-)	534.749	-452.971	-444.495
BL7	-354.294	300.343	294.423
BL5 (-)	-257.936	217.583	214.615
Loop misclosure	- 0.004	0.006	- 0.004

PT to PT – NMSU to ZERO to P026

Station/Baseline	X (m)	Y (m)	Z (m)
NMSU	-1,555,457.539	-5,169,962.408	3,386,819.086
BL3	-1,326.538	-4,293.433	-7,196.095
BL8	-32,488.275	38,515.929	43,222.178
P026 (cmptd)	-1,589,272.352	-5,135,739.912	3,422,845.169
P026 (pub)	-1,589,272.343	-5,135,739.911	3,422,845.167
Misclosure (cmptd-pub)	- 0.009	- 0.001	0.002

These small misclosures provide assurance that a least squares adjustment is the appropriate next step.

Least Squares Adjustment

This example could be considered a "Cliff Notes" version of a more complete procedure. Among others, the reader is advised to rely on standard least squares sources such as Mikhail (1976), Mikhail/Gracie (1980), and Ghilani (2010). The least squares adjustment used on this project is a linear adjustment patterned after the example given in Chapter 14 of Burkholder (2017). The adjustment consists of manipulating three matrices input by the user. They are the ***f*** vector (observations and constants), the ***B*** matrix of coefficients, and ***Q***, the covariance matrix of the observations. The weight matrix (***W***) is computed as the inverse of the covariance matrix and the solution (***Δ***) is the vector of computed parameters. The matrix formulation is given as:

$$\boldsymbol{\Delta} = \left(\boldsymbol{B}^{T}\,\boldsymbol{W}\,\boldsymbol{B}\right)^{-1}\boldsymbol{B}^{T}\,\boldsymbol{W}\,\boldsymbol{f} \qquad (1)$$

In many cases, the normal equations are listed as $\boldsymbol{N} = \left(\boldsymbol{B}^{T}\,\boldsymbol{W}\,\boldsymbol{B}\right)$ in which case, the solution is:

$$\boldsymbol{\Delta} = (\boldsymbol{N})^{-1}\,\boldsymbol{B}^{T}\,\boldsymbol{W}\,\boldsymbol{f} \qquad (2)$$

Standard least squares references continue the derivation to show that the residuals (*v*) for each observation are computed as:

$$\boldsymbol{v} = \boldsymbol{f} - \boldsymbol{B}\,\boldsymbol{\Delta} \qquad (3)$$

A side note is that a successful adjustment will have small residuals. Small, in this case, is related to the quality of the observations - if a residual is 20 mm and the standard deviation of the observations is estimated to be in the 5-mm range, the residual should not be considered "small." Given that a residual is a mathematical correction to the observation, the likelihood that a legitimate correction is larger than 3 standard deviations of an observation is very small. Large residuals are evidence of problems to be addressed before re-running an adjustment – for example, finding and eliminating blunders.

The *a posterior* reference variance is computed following a successful least squares adjustment as:

$$\hat{\sigma}_0^2 = \frac{\boldsymbol{v^T W v}}{(n-u)} \quad \text{and} \quad \Sigma_{\Delta\Delta} = \hat{\sigma}_0^2 \boldsymbol{N^{-1}} \qquad (4) \;\&\; (5)$$

Equation 4 is very important because the covariance matrix of the computed answers ($\Sigma_{\Delta\Delta}$) is obtained as the product of the *a posterior* reference variance (a scaler) and $\boldsymbol{N}^{-1}$ (a matrix). The resulting matrix is the basis for computing both network accuracy and local accuracy (Equation 9, Burkholder 1999).

Formulation of the matrices used in a least squares solution is very specific and can be quite tedious. There is no substitute for writing observations equations to be consistent with the problem being solved. In this Cliff Notes version, formulation of the matrices is given as a rote rule-driven process. Applicable rules include:

1. This is a linear observations-only least squares solution. No iterations are needed or included.
2. The control points are taken as unknowns with standard deviations chosen by the user.
    a. A very small standard deviation (in this case, 0.001 m) will essentially "fix" the control point.
    b. But, if the control is "imperfect" a larger standard deviation could/should be assigned.
3. Consequently, the ***f*** vector is composed of control station X/Y/Z coordinates and observed baseline components, ΔX/ΔY/ΔZ. In the theoretical formulation, the constants part of each equation is moved to the other side of the equals (=) sign. That means that the sign of each element in the ***f*** vector is changed – contrary to what one would intuitively expect.
4. The ***B*** matrix is composed of 0s and 1s (zeros and ones) because it is formulated as a linear problem.
    a. The ***B*** matrix is all zeros (0s) except in the 3x3 sub-matrix for each station or baseline.
    b. For a control station, the diagonal element for each X/Y/Z coefficient value is -1.
    c. For baselines, the diagonal elements for each station are assigned:
        i). Plus 1 for each "from" station.
        ii). Minus 1 for each "to" station.
5. The user has great latitude in choosing order of data input – both control stations and vectors. But, once an order is selected, it must be followed exactly.
    a. The order chosen for input into the ***f*** vector determines the order of rows in the ***B*** matrix.
    b. The order chosen for columns in the ***B*** matrix determines the order of both rows and columns in the ***Q*** matrix.
6. The ***Q*** matrix consists of block-diagonal 3x3 submatrices obtained from baselines and stations:
    a. Covariance matrix of the associated baseline.
    b. The station covariance matrix for each control point.
7. In this problem, there are 30 observations and 18 unknowns to be computed. Redundancy is the difference between the number of observations and the number of unknowns. In this case, n – u = 12 redundancies. The 6 control point values are "unknown," but have small standard deviations.

The ***f*** vector consists of the X/Y/Z coordinate values of each control station and the ΔX/ΔY/ΔZ components of each vector. For this problem, the order chosen is — start at NMSU CORS, use each baseline in numerical order, and close on the Hatch CORS (Station P026). That makes the ***f*** vector a 30 x 1 vector as follows:

$\boldsymbol{f}_{30,1}$

1,555,457.539		1,326.538	
5,169,962.408	NMSU CORS	4,293.433	BL 3
-3,386,918.086		7,196.095	
714.312		-77.477	
4,811.351	BL 1	64.949	BL 4
7,705.140		64.547	
-612.228		-257.936	
517.924	BL 2	217.583	BL 5
509.040		214.615	
534.749		32,488.275	
-452.971	BL 6	-38,515.929	BL 8
-444.495		-43,222.178	
354.294		1,589,272.849 .344	2/29/2024
-300.343	BL 7	5,135,739.911	Hatch CORS (P026)
-294.423		-3,422.845.167	

$\boldsymbol{B}_{30,18}$

NMSU	950	400	120	ZERO	P026	
-1 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	
0 -1 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	NMSU
0 0 -1	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	
1 0 0	-1 0 0	0 0 0	0 0 0	0 0 0	0 0 0	
0 1 0	0 -1 0	0 0 0	0 0 0	0 0 0	0 0 0	BL 1 (from NMSU to 950)
0 0 1	0 0 -1	0 0 0	0 0 0	0 0 0	0 0 0	
0 0 0	-1 0 0	0 0 0	0 0 0	1 0 0	0 0 0	
0 0 0	0 -1 0	0 0 0	0 0 0	0 1 0	0 0 0	BL 2 (from ZERO to 950)
0 0 0	0 0 -1	0 0 0	0 0 0	0 0 1	0 0 0	
1 0 0	0 0 0	0 0 0	0 0 0	-1 0 0	0 0 0	
0 1 0	0 0 0	0 0 0	0 0 0	0 -1 0	0 0 0	BL 3 (from NMSU to ZERO)
0 0 1	0 0 0	0 0 0	0 0 0	0 0 -1	0 0 0	
0 0 0	0 0 0	0 0 0	-1 0 0	1 0 0	0 0 0	
0 0 0	0 0 0	0 0 0	0 -1 0	0 1 0	0 0 0	BL 4 (from ZERO to 120)
0 0 0	0 0 0	0 0 0	0 0 -1	0 0 1	0 0 0	

0 0 0 0 0 0 -1 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 -1 0 0 0 0 0 1 0 0 0 0 BL 5 (from ZERO to 400)
0 0 0 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0

0 0 0 1 0 0 0 0 0 -1 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 -1 0 0 0 0 0 0 0 BL 6 (from 950 to 120)
0 0 0 0 0 1 0 0 0 0 0 -1 0 0 0 0 0 0

0 0 0 1 0 0 -1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 -1 0 0 0 0 0 0 0 0 0 0 BL 7 (from 950 to 400)
0 0 0 0 0 1 0 0 -1 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 -1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 -1 0 BL 8 (from ZERO to P026)
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 -1

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 Hatch CORS (P026)
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1

The $\boldsymbol{Q}_{30,30}$ matrix is built from the covariance assumptions made for the control stations and from the covariance matrix of each vector determined during baseline processing. The ***Q*** matrix is symmetrical and the order of 3x3 submatrices is determined by the order of rows selected for the ***B*** matrix. The standard deviation for each X/Y/Z component at stations NMSU and P026 is assumed to 0.001 m.

NMSU
0.000001 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0.000001 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0.000001 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

BL 1
0 0 0 0.0000133766 0.0000033102 -0.0000203729 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0.0000033102 0.000108730 -0.0000650663 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 -0.0000203729 -0.0000650663 0.0000439296 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

BL 2
0 0 0 0 0 0 0.0000017644 0.000001063 0.0000000205 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0.0000010630 0.0000062245 -0.0000035850 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0.0000001063 -0.0000035850 0.0000053804 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

BL 3
0 0 0 0 0 0 0 0 0 0.0000070810 0.0000146015 -0.0000084139 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0.0000146015 0.0000674901 -0.0000455390 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 -0.0000084139 -0.0000455390 0.0000379188 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

BL 4
0 0 0 0 0 0 0 0 0 0 0 0 0.0000008159 0.0000005130 -0.0000000030 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0.0000005130 0.0000273460 -0.0000010466 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 -0.0000000030 -0.0000010466 0.0000013485 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

BL 5
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0000010282 0.0000004514 0.0000002632 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0000004514 0.0000031368 -0.0000015750 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0000002632 -0.0000015750 0.0000025501 0 0 0 0 0 0 0 0 0 0 0 0

BL 6
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0000021193 0.0000031122 -0.0000015403 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0000031122 0.0000118796 -0.0000048299 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.0000015403 -0.0000048299 0.0000033268 0 0 0 0 0 0 0 0 0

BL 7
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0000013682 0.0000017044 -0.0000005907 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0000017044 0.0000071142 -0.0000025254 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.0000005907 -0.0000025254 0.0000019778 0 0 0 0 0 0

BL 8
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0000057062 0.0000103618 -0.0000079576 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0000103618 0.0000503190 -0.0000337626 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.0000079576 -0.0000337626 0.0000031127 0 0 0

P026
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.000001 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.000001 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.000001

With these three matrices in hand, the next step is to manipulate the matrices to obtain a solution. The following manipulations can be accomplished using any reputable matrix manipulation software.

With the ***f***, ***B***, and ***Q*** matrices in hand, the following matrix operations were performed:

Transpose matrix $\boldsymbol{B}$	to get	$\boldsymbol{B}^T$
Invert the $\boldsymbol{Q}$ matrix	to get	$\boldsymbol{W}$
Multiply $\boldsymbol{B}^T$ by $\boldsymbol{W}$	to get	$\boldsymbol{B}^T\boldsymbol{W}$
Multiply again by $\boldsymbol{B}$	to get	$\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{B}$ (Rename to $\boldsymbol{N} = \boldsymbol{B}^T\boldsymbol{W}\boldsymbol{B}$)
Multiply $\boldsymbol{B}^T\boldsymbol{W}$ by $\boldsymbol{f}$	to get	$\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{f}$
Invert $\boldsymbol{N}$ (Normal Equations)	to get	$\boldsymbol{N}^{-1}$
Multiply $\boldsymbol{N}^{-1}$ by $\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{f}$	to get	$\boldsymbol{\Delta}$ (these are the coordinate values)
Multiply $\boldsymbol{B}$ by $\boldsymbol{\Delta}$	to get	$\boldsymbol{B}\boldsymbol{\Delta}$
Subtract $\boldsymbol{f}$ minus $\boldsymbol{B}\boldsymbol{\Delta}$	to get	$\boldsymbol{v}$ = residuals
Transpose vector $\boldsymbol{v}$	to get	$\boldsymbol{v}^T$
Multiply $\boldsymbol{v}^T$ by W	to get	$\boldsymbol{v}^T\boldsymbol{W}$
Multiply $\boldsymbol{v}^T\boldsymbol{W}$ by $\boldsymbol{v}$	to get	$\boldsymbol{v}^T\boldsymbol{W}\boldsymbol{v}$
Divide $\boldsymbol{v}^T\boldsymbol{W}\boldsymbol{v}$ by $(n - u)$	to get	*a posteriori* reference variance
Multiply reference variance by $\boldsymbol{N}^{-1}$	to get	$\Sigma_{\Delta\Delta}$ - covariance matrix of parameters

Matrix software was used to perform the computations listed above. A full print-out of the least squares adjustment is available at www.globalcogo.com/SAGES-LS-2017.pdf. A summary of the results is:

Computed coordinates of all stations – including the "fixed" control points.

Δ – X/Y/Z values in meters

-1,555,457.53761		-1,556,526.13912		-1,556,784.07362	
-5,169,962.4083	NMSU	-5,174,473.4184	400	-5,174,255.83654	ZERO
3,386,819.08608		3,379,408.37065		3,379,622.98756	
-1,556,171.84634		-1,556,706.59602		-1,589,272.34439	
-5,174,773.76021	950	-5,174,320.78641	120	-5,135,739.9107	P026
3,379,113.94685		3,379,558.44106		3,422,845.16692	

Residuals in meters for each control station and for each baseline are:

0.00139		0.0006		0.00423	
-0.00030	NMSU	-0.00087	BL4	-0.00316	BL8
0.00008		0.0005		0.00137	
0.00327		-0.0015		-0.00139	
-0.00091	BL1	0.00114	BL5	0.0003	P026
0.00078		-0.00191		-0.00008	
-0.00072		-0.00068		0.00	
0.00083	BL2	0.00281	BL6	-0.00	Place holder
-0.00021		-0.00079		0.00	
0.00199		0.00122		-0.00	
0.00476	BL3	-0.00118	BL7	0.00	Place holder
-0.00352		0.0008		-0.00	

The A posterior reference variance is = 2.14046665 meters squared.

And, the 18 x 18 covariance matrix ($\Sigma_{\Delta\Delta}$) of the computed values is:

NMSU

1.8576173E-6 5.12532542E-8 -2.02934436E-8 1.28673285E-6 -3.9617315E-7 1.79163501E-7 1.24590099E-6 -3.99151732E-7 1.71768055E-7 1.23741018E-6 -3.92769104E-7 1.62092936E-7 1.20428127E-6 -3.84601843E-7 1.31616668E-7 2.82849356E-7 -5.12532542E-8 2.02934436E-8

5.12532542E-8 2.04836973E-6 -9.01929598E-8 -1.06482445E-7 1.19834816E-6 2.08307079E-7 -1.0720551E-7 1.162551E-6 2.07423928E-7 -1.0614196E-7 1.16067293E-6 2.03865965E-7 -1.07144141E-7 1.15010657E-6 1.96086514E-7 -5.12532542E-8 9.20969261E-8 9.01929598E-8

-2.02934436E-8 -9.01929598E-8 1.99114245E-6 -8.57195078E-8 -1.70347204E-7 1.65346411E-6 -9.93622393E-8 -2.02186362E-7 1.63707188E-6 -1.0068467E-7 -1.97034306E-7 1.62263454E-6 -1.1760895E-7 -2.02675103E-7 1.59076237E-6 2.02934436E-8 9.01929598E-8 1.493242E-7

Mon 950

1.28673285E-6 -1.06482445E-7 -8.57195078E-8 7.28756266E-6 1.26033623E-5 -8.00489209E-6 6.61720621E-6 1.1698004E-5 -7.63374416E-6 6.48892762E-6 1.15728254E-5 -7.55510627E-6 6.14653788E-6 1.14166887E-5 -7.63485141E-6 8.53733804E-7 1.06482445E-7 8.57195078E-8

-3.9617315E-7 1.19834816E-6 -1.70347204E-7 1.26033623E-5 5.16412508E-5 -3.13419677E-5 1.15204892E-5 4.8161514E-5 -3.01039091E-5 1.13578169E-5 4.76038595E-5 -2.9743056E-5 1.10633049E-5 4.67023757E-5 -2.94881446E-5 3.9617315E-7 9.42118496E-7 1.70347204E-7

1.79163501E-7 2.08307079E-7 1.65346411E-6 -8.00489209E-6 -3.13419677E-5 2.59519237E-5 -7.40274561E-6 -2.97754887E-5 2.50049871E-5 -7.29095238E-6 -2.93964181E-5 2.4612945E-5 -7.23531401E-6 -2.89890603E-5 2.41048256E-5 -1.79163501E-7 -2.08307079E-7 4.87002544E-7

Mon 400
1.24590099E-6 -1.0720551E-7 -9.93622393E-8 6.61720621E-6 1.15204892E-5 -7.40274561E-6 7.63501912E-6
1.23316966E-5 -7.56147296E-6 6.43473827E-6 1.14175986E-5 -7.4391896E-6 6.31929334E-6 1.14162732E-5 -
7.54588206E-6 8.94565661E-7 1.0720551E-7 9.93622393E-8
-3.99151732E-7 1.162551E-6 -2.02186362E-7 1.1698004E-5 4.8161514E-5 -2.97754887E-5 1.23316966E-5
5.22090008E-5 -3.14795977E-5 1.13570001E-5 4.7597109E-5 -2.96205427E-5 1.12008396E-5 4.74953444E-5 -
2.96975428E-5 3.99151732E-7 9.77915654E-7 2.02186362E-7
1.71768055E-7 2.07423928E-7 1.63707188E-6 -7.63374416E-6 -3.01039091E-5 2.50049871E-5 -7.56147296E-6
-3.14795977E-5 2.68713183E-5 -7.36599128E-6 -2.9532046E-5 2.46454097E-5 -7.30701041E-6 -2.94205518E-5
2.45427977E-5 -1.71768055E-7 -2.07423928E-7 5.03394771E-7
Mon 120
1.23741018E-6 -1.0614196E-7 -1.0068467E-7 6.48892762E-6 1.13578169E-5 -7.29095238E-6 6.43473827E-6
1.13570001E-5 -7.36599128E-6 7.58494167E-6 1.23913366E-5 -7.73490466E-6 6.35469073E-6 1.14050242E-5
-7.53505404E-6 9.03056469E-7 1.0614196E-7 1.0068467E-7
-3.92769104E-7 1.16067293E-6 -1.97034306E-7 1.15728254E-5 4.76038595E-5 -2.93964181E-5 1.14175986E-5
4.7597109E-5 -2.9532046E-5 1.23913366E-5 5.23873811E-5 -3.14520342E-5 1.12184946E-5 4.76995364E-5 -
2.9875683E-5 3.92769104E-7 9.79793722E-7 1.97034306E-7
1.62092936E-7 2.03865965E-7 1.62263454E-6 -7.55510627E-6 -2.9743056E-5 2.4612945E-5 -7.4391896E-6 -
2.96205427E-5 2.46454097E-5 -7.73490466E-6 -3.14520342E-5 2.65296027E-5 -7.32014679E-6 -2.96251508E-5
2.4809515E-5 -1.62092936E-7 -2.03865965E-7 5.17832108E-7
Mon ZERO
1.20428127E-6 -1.07144141E-7 -1.1760895E-7 6.14653788E-6 1.10633049E-5 -7.23531401E-6 6.31929334E-6
1.12008396E-5 -7.30701041E-6 6.35469073E-6 1.12184946E-5 -7.32014679E-6 6.45256756E-6 1.12283226E-5
-7.28878375E-6 9.36185377E-7 1.07144141E-7 1.1760895E-7
-3.84601843E-7 1.15010657E-6 -2.02675103E-7 1.14166887E-5 4.67023757E-5 -2.89890603E-5 1.14162732E-5
4.74953444E-5 -2.94205518E-5 1.14050242E-5 4.76995364E-5 -2.96251508E-5 1.12283226E-5 4.79667171E-5
-2.99862149E-5 3.84601843E-7 9.90360083E-7 2.02675103E-7
1.31616668E-7 1.96086514E-7 1.59076237E-6 -7.63485141E-6 -2.94881446E-5 2.41048256E-5 -7.54588206E-6
-2.96975428E-5 2.45427977E-5 -7.53505404E-6 -2.9875683E-5 2.4809515E-5 -7.28878375E-6 -2.99862149E-5
2.53283128E-5 -1.31616668E-7 -1.96086514E-7 5.49704285E-7
P026
2.82849356E-7 -5.12532542E-8 2.02934436E-8 8.53733804E-7 3.9617315E-7 -1.79163501E-7 8.94565661E-7
3.99151732E-7 -1.71768055E-7 9.03056469E-7 3.92769104E-7 -1.62092936E-7 9.36185377E-7 3.84601843E-7
-1.31616668E-7 1.8576173E-6 5.12532542E-8 -2.02934436E-8
-5.12532542E-8 9.20969261E-8 9.01929598E-8 1.06482445E-7 9.42118496E-7 -2.08307079E-7 1.0720551E-7
9.77915654E-7 -2.07423928E-7 1.0614196E-7 9.79793722E-7 -2.03865965E-7 1.07144141E-7 9.90360083E-7 -
1.96086514E-7 5.12532542E-8 2.04836973E-6 -9.01929598E-8
2.02934436E-8 9.01929598E-8 1.493242E-7 8.57195078E-8 1.70347204E-7 4.87002544E-7 9.93622393E-8
2.02186362E-7 5.03394771E-7 1.0068467E-7 1.97034306E-7 5.17832108E-7 1.1760895E-7 2.02675103E-7
5.49704285E-7 -2.02934436E-8 -9.01929598E-8 1.99114245E-6

The diagonals of each station 3x3 submatrix are underlined to aid in isolating desired values.

In many cases, this is where the adjustment is complete. But, the real purpose here is obtaining the covariance matrix ($\boldsymbol{\Sigma}_{\Delta\Delta}$) to be used as the basis for computing network and local accuracies of the inverse distances between the stations.

The BURKORD™ database file and computation of Accuracies

The BURKORD™ data base file is populated with the station coordinates, station names, point numbers, the covariance matrix for each station, and the correlations between stations. The station coordinates are in the $\boldsymbol{\Delta}$ vector and the covariance values are in the $\boldsymbol{\Sigma}_{\Delta\Delta}$ above. Those values were extracting and the BURKORD™ data base was populated using a separate program. The format for the BURKORD™ database file is given at www.globalcogo.com/burkord.html. The BURKORD™ file for this project and a summary of both network and local accuracies is included in the following computer printout.

```
Program name/version:  Local Accuracy - version 2013-B

The user is:              Earl F. Burkholder
Program used on:          June 20, 2017

The input file for program is:       BurkordDataFile.xyz
The name of this output file is:     LocalAccuracy.txt                    File renamed: BURKORD-File.docx

p 101 -1555457.5376 -5169962.4083 3386819.0861 1.8576173E-6 2.04836973E-6 1.99114245E-6 5.12532542E-8 -2.02934436E-8 -9.01929598E-8  NMSU
p 100 -1556784.0736 -5174255.8365 3379622.9876 6.45256756E-6 4.79667171E-5 2.53283128E-5 1.12283226E-5 -7.28878375E-6 -2.99862149E-5  Zero
p 120 -1556706.596 -5174320.7864 3379558.4411 7.58494167E-6 5.23873811E-5 2.65296027E-5 1.23913366E-5 -7.73490466E-6 -3.14520342E-5  120
p 400 -1556526.1391 -5174473.4184 3379408.3706 7.63501912E-6 5.22090008E-5 2.68713183E-5 1.23316966E-5 -7.56147296E-6 -3.14795977E-5  400
p 950 -1556171.8463 -5174773.7602 3379113.9468 7.28756266E-6 5.16412508E-5 2.59519237E-5 1.26033623E-5 -8.00489209E-6 -3.13419677E-5  950
p 102 -1589272.3444 -5135739.9107 3422845.1669 1.8576173E-6 2.04836973E-6 1.99114245E-6 5.12532542E-8 -2.02934436E-8 -9.01929598E-8  P026

c 101 100 1.20428127E-6 -3.84601843E-7 1.31616668E-7 -1.07144141E-7 1.15010657E-6 1.96086514E-7 -1.1760895E-7 -2.02675103E-7 1.59076237E-6
c 101 120 1.23741018E-6 -3.92769104E-7 1.62092936E-7 -1.0614196E-7 1.16067293E-6 2.03865965E-7 -1.0068467E-7 -1.97034306E-7 1.62263454E-6
c 101 400 1.24590099E-6 -3.99151732E-7 1.71768055E-7 -1.0720551E-7 1.162551E-6 2.07423928E-7 -9.93622393E-8 -2.02186362E-7 1.63707188E-6
c 101 950 1.28673285E-6 -3.9617315E-7 1.79163501E-7 -1.06482445E-7 1.19834816E-6 2.08307079E-7 -8.57195078E-8 -1.70347204E-7 1.65346411E-6
c 101 102 2.82849356E-7 -5.12532542E-8 2.02934436E-8 -5.12532542E-8 9.20969261E-8 9.01929598E-8 2.02934436E-8 9.01929598E-8 1.493242E-7
c 100 101 1.20428127E-6 -1.07144141E-7 -1.1760895E-7 -3.84601843E-7 1.15010657E-6 -2.02675103E-7 1.31616668E-7 1.96086514E-7 1.59076237E-6
c 100 120 6.35469073E-6 1.12184946E-5 -7.32014679E-6 1.14050241E-5 4.76995364E-5 -2.96251508E-5 -7.53505404E-6 -2.9875683E-5 2.4809515E-5
c 100 400 6.31929334E-6 1.12008396E-5 -7.30701041E-6 1.14162732E-5 4.74953444E-5 -2.94205518E-5 -7.54588206E-6 -2.96975428E-5 2.45427977E-5
c 100 950 6.14653788E-6 1.10633049E-5 -7.23531401E-6 1.14166887E-5 4.67023757E-5 -2.89890603E-5 -7.63485141E-6 -2.94881446E-5 2.41048256E-5
c 100 102 9.36185377E-7 1.07144141E-7 1.1760895E-7 3.84601843E-7 9.90360083E-7 2.02675103E-7 -1.31616668E-7 -1.96086514E-7 5.49704285E-7
c 120 101 1.23741018E-6 -1.0614196E-7 -1.0068467E-7 -3.92769104E-7 1.16067293E-6 -1.97034306E-7 1.62092936E-7 2.03865965E-7 1.62263454E-6
c 120 100 6.35469073E-6 1.14050241E-5 -7.53505404E-6 1.12184946E-5 4.76995364E-5 -2.9875683E-5 -7.32014679E-6 -2.96251508E-5 2.4809515E-5
c 120 400 6.43473827E-6 1.13570001E-5 -7.36599128E-6 1.14175986E-5 4.7597109E-5 -2.9532046E-5 -7.4391896E-6 -2.96205427E-5 2.46454097E-5
c 120 950 6.48892762E-6 1.13578169E-5 -7.29095238E-6 1.15728254E-5 4.76038595E-5 -2.93964181E-5 -7.55510627E-6 -2.9743056E-5 2.4612945E-5
c 120 102 9.03056469E-7 1.0614196E-7 1.0068467E-7 3.92769104E-7 9.79793722E-7 1.97034306E-7 -1.62092936E-7 -2.03865965E-7 5.17832108E-7
c 400 101 1.24590099E-6 -1.0720551E-7 -9.93622393E-8 -3.99151732E-7 1.162551E-6 -2.02186362E-7 1.71768055E-7 2.07423928E-7 1.63707188E-6
c 400 100 6.31929334E-6 1.14162732E-5 -7.54588206E-6 1.12008396E-5 4.74953444E-5 -2.96975428E-5 -7.30701041E-6 -2.94205518E-5 2.45427977E-5
c 400 120 6.43473827E-6 1.14175986E-5 -7.4391896E-6 1.13570001E-5 4.7597109E-5 -2.96205427E-5 -7.36599128E-6 -2.9532046E-5 2.46454097E-5
c 400 950 6.61720621E-6 1.15204892E-5 -7.40274561E-6 1.1698004E-5 4.8161514E-5 -2.97754887E-5 -7.63374416E-6 -3.01039091E-5 2.50049871E-5
c 400 102 8.94565661E-7 1.0720551E-7 9.93622393E-8 3.99151732E-7 9.77915654E-7 2.02186362E-7 -1.71768055E-7 -2.07423928E-7 5.03394771E-7
c 950 101 1.28673285E-6 -1.06482445E-7 -8.57195078E-8 -3.9617315E-7 1.19834816E-6 -1.70347204E-7 1.79163501E-7 2.08307079E-7 1.65346411E-6
```

```
c 950 100 6.14653788E-6 1.14166887E-5 -7.63485141E-6 1.10633049E-5 4.67023757E-5 -2.94881446E-5 -7.23531401E-6 -2.89890603E-5 2.41048256E-5
c 950 120 6.48892762E-6 1.15728254E-5 -7.55510627E-6 1.13578169E-5 4.76038595E-5 -2.9743056E-5 -7.29095238E-6 -2.93964181E-5 2.4612945E-5
c 950 400 6.61720621E-6 1.1698004E-5 -7.63374416E-6 1.15204892E-5 4.8161514E-5 -3.01039091E-5 -7.40274561E-6 -2.97754887E-5 2.50049871E-5
c 950 102 8.53733804E-7 1.06482445E-7 8.57195078E-8 3.9617315E-7 9.42118496E-7 1.70347204E-7 -1.79163501E-7 -2.08307079E-7 4.87002544E-7
c 102 101 2.82849356E-7 -5.12532542E-8 2.02934436E-8 -5.12532542E-8 9.20969261E-8 9.01929598E-8 2.02934436E-8 9.01929598E-8 1.493242E-7
c 102 100 9.36185377E-7 3.84601843E-7 -1.31616668E-7 1.07144141E-7 9.90360083E-7 -1.96086514E-7 1.1760895E-7 2.02675103E-7 5.49704285E-7
c 102 120 9.03056469E-7 3.92769104E-7 -1.62092936E-7 1.0614196E-7 9.79793722E-7 -2.03865965E-7 1.0068467E-7 1.97034306E-7 5.17832108E-7
c 102 400 8.94565661E-7 3.99151732E-7 -1.71768055E-7 1.0720551E-7 9.77915654E-7 -2.07423928E-7 9.93622393E-8 2.02186362E-7 5.03394771E-7
c 102 950 8.53733804E-7 3.9617315E-7 -1.79163501E-7 1.06482445E-7 9.42118496E-7 -2.08307079E-7 8.57195078E-8 1.70347204E-7 4.87002544E-7
```

The points in the project file are:

PT	X/Y/Z (meters)			Station
101	-1,555,457.5376	-5,169,962.4083	3,386,819.0861	NMSU
100	-1,556,784.0736	-5,174,255.8365	3,379,622.9876	Zero
120	-1,556,706.5960	-5,174,320.7864	3,379,558.4411	120
400	-1,556,526.1391	-5,174,473.4184	3,379,408.3706	400
950	-1,556,171.8463	-5,174,773.7602	3,379,113.9468	950
102	-1,589,272.3444	-5,135,739.9107	3,422,845.1669	P026

List of stored correlations:

PT to PT		
101 100	120 101	950 101
101 120	120 100	950 100
101 400	120 400	950 120
101 950	120 950	950 400
101 102	120 102	950 102
100 101	400 101	102 101
100 120	400 100	102 100
100 400	400 120	102 120
100 950	400 950	102 400
100 102	400 102	102 950

Note: The correlation 3x3 submatrix “there” to “here” is the transpose of the submatrix “here” to “there” meaning that twice as many correlations as minimally needed are stored and listed. The same applies to the inverses listed in the following section.

Output of the Local Accuracy program is tabulated as:

Distances and standard deviations in this tabulation were computed using the equations in Burkholder (1999). Both given in meters. These accuracies are based on assuming 0.001 m uncertainty in each component at both “control” stations, NMSU and P026. The number of digits shown in each standard deviation cannot be justified except to show where differences begin to occur.

PT to PT	Accuracies based on - - -			3-D Slope Distance	GSDM Hor. Dist.
NMSU to ZERO	Hor. Dist. =	8,483.819	network	0.0025871	0.0025832
	3-D Dist. =	8,483.929	local	0.0019585	0.0019547
NMSU to 120	Hor.Dist. =	8,559.827	network	0.0027418	0.0027399
	3-D Dist. =	8,559.940	local	0.0021403	0.0021392
NMSU to 400	Hor.Dist. =	8,741.158	network	0.0027754	0.0027733
	3-D Dist. =	8,741.271	local	0.0021766	0.0021751
NMSU to 950	Hor.Dist. =	9,111.888	network	0.0026307	0.0026292
	3-D Dist. =	9,111.998	local	0.0019666	0.0019657
NMSU to P026	Hor.Dist. =	60,103.612	network	0.0019425	0.0019425
	3-D Dist. =	60,104.068	local	0.0018072	0.0018072
Zero to NMSU	Hor.Dist. =	8,483.869	network	0.0025871	0.0025841
	3-D Dist. =	8,483.929	local	0.0019585	0.0019557
ZERO to 120	Hor.Dist. =	119.946	network	0.0029009	0.0029045
	3-D Dist. =	119.948	local	0.0009729	0.0009762
ZERO to 400	Hor.Dist. =	399.915	network	0.0028981	0.0028997
	3-D Dist. =	399.916	local	0.0009999	0.001001
ZERO to 950	Hor.Dist. =	949.836	network	0.0028326	0.0028332
	3-D Dist. =	949.837	local	0.0010018	0.0010023
ZERO to P026	Hor.Dist. =	66,385.645	network	0.0024365	0.0024359
	3-D Dist. =	66,386.151	local	0.0021496	0.0021493
120 to NMSU	Hor.Dist. =	8,559.879	network	0.0027418	0.0027403
	3-D Dist. =	8,559.940	local	0.0021403	0.0021394
120 to ZERO	Hor.Dist. =	119.946	network	0.0029009	0.0029045
	3-D Dist. =	119.948	local	0.0009729	0.0009761
120 to 400	Hor.Dist. =	279.968	network	0.0030279	0.0030288
	3-D Dist. =	279.969	local	0.0013089	0.0013097
120 to 950	Hor.Dist. =	829.890	network	0.0029649	0.0029651
	3-D Dist. =	829.890	local	0.0011734	0.0011735

PT to PT	Accuracies based on - - -		3-D Slope Distance	GSDM Hor. Dist.
120 to P026	Hor.Dist. = 66,503.271	network	0.0025925	0.0025909
	3-D Dist. = 66,503.779	local	0.0023322	0.0023307
400 to NMSU	Hor.Dist. = 8,741.211	network	0.0027754	0.0027738
	3-D Dist. = 8,741.271	local	0.0021766	0.0021754
400 to ZERO	Hor.Dist. = 399.915	network	0.0028981	0.0028997
	3-D Dist. = 399.916	local	0.0009999	0.0010010
400 to 120	Hor.Dist. = 279.968	network	0.0030279	0.0030287
	3-D Dist. = 279.969	local	0.0013089	0.0013097
400 to 950	Hor.Dist. = 549.922	network	0.0029605	0.0029603
	3-D Dist. = 549.922	local	0.0010905	0.0010903
400 to P026	Hor.Dist. = 66,777.878	network	0.0025921	0.0025907
	3-D Dist. = 66,778.394	local	0.0023347	0.0023334
950 to NMSU	Hor.Dist. = 9,111.943	network	0.0026307	0.0026296
	3-D Dist. = 9,111.998	local	0.0019666	0.0019658
950 to ZERO	Hor.Dist. = 949.836	network	0.0028326	0.0028331
	3-D Dist. = 949.837	local	0.0010018	0.0010022
950 to 120	Hor.Dist. = 829.890	network	0.0029649	0.002965
	3-D Dist. = 829.890	local	0.0011734	0.0011734
950 to 400	Hor.Dist. = 549.922	network	0.0029605	0.0029602
	3-D Dist. = 549.922	local	0.0010905	0.0010902
950 to P026	Hor.Dist. = 67,317.398	network	0.0025072	0.0025058
	3-D Dist. = 67,317.932	local	0.0022548	0.0022536
P026 to NMSU	Hor.Dist. = 60,103.146	network	0.0019425	0.0019425
	3-D Dist. = 60,104.068	local	0.0018072	0.0018072
P026 to ZERO	Hor.Dist. = 66,384.739	network	0.0024365	0.0024384
	3-D Dist. = 66,386.151	local	0.0021496	0.0021512
P026 to 120	Hor.Dist. = 66,502.357	network	0.0025925	0.0025962
	3-D Dist. = 66,503.779	local	0.0023322	0.0023357
P026 to 400	Hor.Dist. = 66,776.957	network	0.0025921	0.0025955
	3-D Dist. = 66,778.394	local	0.0023347	0.0023379
P026 to 950	Hor.Dist. = 67,316.471	network	0.0025072	0.0025103
	3-D Dist. = 67,317.932	local	0.0022548	0.0022578

Notes and conclusions

1. Both network and local (relative positional precision) accuracy are listed for each line.
2. HD(1) is the horizontal distance "here" to "there" in each case - see (Burkholder 1991).
3. Due to being in different horizontal planes, the distance "here" to "there" is not the same as "there" to "here" – see Part II of Burkholder (1997) http://www.globalcogo.com/psgsdm.pdf.
4. The 3-D distance is the same both ways. It is also known as the slope or mark-to-mark distance.
5. Network and local accuracy for longer lines are quite similar. Local accuracy for "short" lines is significantly smaller that network accuracy on the same line.
6. The line segment between 120 and 400 was not directly measured. The local accuracy for that indirectly measured line is slightly larger than other local accuracies but significantly smaller than the network accuracy for same line.
7. In all cases, the printout provides a defensible answer, "what is the accuracy of this point with respect to that point?"

Although additional conclusions will emerge from discussions of the concepts presented, several of the more obvious conclusions include:

1. It is possible to compute "relative positional precision" with statistical reliability.
2. Revising the wording in the existing 2016 standard may be preferred to stipulating computation of local accuracy as described herein.
3. It may serve professionals and clients alike to use "datum accuracy" to describe the quality of position of a point – see Burkholder (1999).

Bonus Comparison with NGS Distances

In addition to documenting computation of network and local accuracies, this exercise also demonstrates the ability to duplicate baseline distances with static GPS observations. It shows that local accuracies compare very favorably with NGS numbers and statistics. Note, NGS accuracy is given as 0.2 mm without specifying whether that applies to mark-to-mark or to horizontal distances. Relevant comparisons are:

Baseline	NGS Published Distances (m)			Observed GNSS Distances (m)			
Segment	Mark-to-Mark	Horizontal,	σ	Mark-to-Mark,	σ	Horizontal,	σ
ZERO to 120	119.9474	119.9456	0.0002	119.948	0.0010	119.946	0.0010
ZERO to 400	399.9147	399.9136	0.0002	399.916	0.0010	399.915	0.0010
ZERO to 950	949.8348	949.8346	0.0002	949.837	0.0010	949.836	0.0010
120 to 400	279.9682	279.9681	0.0002	279.969	0.0013	279.968	0.0013
120 to 950	829.8889	829.8890	0.0002	829.890	0.0012	829.890	0.0012
400 to 950	549.9209	549.9209	0.0002	549.922	0.0011	549.922	0.0011

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