A METRIC MAP PROJECTION FOR THE STATE OF MICHIGAN

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ABSTRACT

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The existing Michigan State Plane Coordinate System is based on the North American Datum of 1927 and the Michigan Spheroid. The coordinates are expressed in U.S. ~~American~~ Survey Feet. The National Geodetic Survey, NOS, NOAA, U.S. Department of Commerce, is preparing a readjustment of the horizontal geodetic control network throughout the United States and will publish their results in metric units on the North American Datum of 1983. In order to accommodate the change in the datum and the units of the point coordinates, a revision to the existing Michigan State Plane Coordinate System Law will have to be enacted by the Michigan Legislature.

This thesis presents a brief explanation of the state plane coordinate systems in general and of the Michigan State Plane Coordinate System (MSPCS) in particular. A review of the existing applications, obstacles, and benefits in using the MSPCS is presented in order to determine how the system might be improved rather than just changed. Finally, several possible map projections are considered and a recommendation for a set of three Lambert conic conformal projections is presented. A model law designed to replace the existing law is also included.

CHAPTER I. INTRODUCTION

Many people have considered the problem of the best way to represent a sphere-like earth on a flat map. The problem has been illustrated by using an orange peel to show the distortion, stretching, and tearing which occurs when one attempts to lay the peeling out flat on a plane surface. If one considers only a very small part of the orange peel the distortion may not be discernible. Even so, the distortion still exists and must not be ignored when one relates points on a curved surface to the corresponding ones on a plane.

Map projections provide ways to establish a one-to-one correspondence between a point on the surface of the earth and a point on a two dimensional map. The parallels of latitude and the meridians of longitude on the ellipsoidal surface of the earth are portrayed systematically on a flat surface. A plane rectangular grid is then superimposed upon the graticule of parallel and meridian lines. The systematic portrayal of the graticule can be achieved using any one of a large number of map projections. Thus a map projection can be thought of as a consistent set of rules and formulas which establishes a unique pair of plane coordinates for a given latitude and longitude. The transformation must also be reversible.

Of the various types of map projections, the conformal projection is of particular interest to surveyors and engineers because the angle formed by the intersection of two lines on the ellipsoid transforms to a plane angle without being distorted. That is, the shape of a small feature on the surface of the earth is accurately represented on a map. However, as illustrated by the orange peel, something must give when a curved surface is flattened out. With a conformal map projection a distance on the map is generally longer or shorter than its corresponding distance on the ellipsoid. The mathematical condition of conformality (page 52 of [12]) ". . . is that the distortion or scale at any point must be the same in all directions: the scale may change from point to point, but at each point it will be independent of azimuth." The conformal state plane projections presently in use were designed with the goal of keeping the scale factor distortion less than 1/10,000 within the boundaries of each zone.

The existing state plane coordinate systems were defined on the Clarke Spheroid of 1866 and provide a method of determining X & Y plane coordinates for each point whose latitude and longitude is known. The plane coordinates are expressed in feet and enable control surveys on the ellipsoid to be computed using plane Euclidean geometry and trigonometry rather than the more complicated formulas for latitude and longitude using spherical trigonometry and units of angular measure; degrees, minutes and seconds.

Mitchell (page 21 of [12]) defines a geodetic datum as a mathematical model of the earth consisting of five quantities: "the latitude and longitude of an initial point, the azimuth of a line from the point, and two constants necessary to define the terrestrial spheroid."[1] The ellipsoid is obtained by rotating the ellipse defined by the two constants about its minor axis. Given a specific geodetic datum, the location of any point on the surface of the Earth is defined by its latitude and longitude on the ellipsoid and its height above (or below) the ellipsoid. State plane coordinates can then be computed from the latitude and longitude of the point.

The sea-level surface of the Earth, the geoid, being a geopotential surface of the physical Earth, will not coincide exactly with the mathematical model of the Earth, the ellipsoid. In the past, parameters of the ellipsoid and datum have been selected which minimize some measure of the difference between the geoid and the ellipsoid in a given area. The North American Datum of 1927 (page 22 of [12]) is defined by the latitude and longitude of triangulation station MEADES RANCH in Kansas; the azimuth of the line from MEADES RANCH to station WALDO; and the parameters of the Clarke Spheroid of 1866. The latter was chosen in 1880 because it fit the geoid in North America more closely than other available ellipsoids. The geoid and ellipsoid were taken to coincide at station MEADES RANCH and the geoid separation, N, at all other stations in North America is related to $N_0 = 0$ at station MEADES RANCH. The magnitude of the separation in North America rarely exceeds 30 meters [24]. However, if one were to extend the North American Datum of 1927 into South America, the

[1] The term "spheroid" has been used in the past to refer to a particular mathematical model of the Earth. The more recent convention, which will be used in this thesis, is to use the term "ellipsoid" for a mathematical model of the Earth and reserve the use of the term "spheroid" to describe any nearly ellipsoidal surface.

geoid separation becomes large, cumbersome to work with, and intolerable for mapping purposes. To eliminate the problem a separate datum has been defined which fits the geoid better in South America [9]. Similarly, numerous geodetic datums have been defined and used in different areas of the world.

In 1927 the horizontal geodetic control network in the United States, consisting of approximately 20,000 control points, was adjusted and the geodetic position of each point was computed on the North American Datum of 1927. Since then approximately 100,000 additional control points have been added to the network [16]. Areas have been covered with geodetic control which were not previously covered and connections between previously adjusted points have been made which should improve the quality of the network. Although many of the more recent observations are more accurate than those used in the 1927 adjustment, additions to the network have been adjusted to fit into the network on a localized basis. On a national scale the magnitude of the effort required for patching and local readjustments contributed to the decision to readjust the national network [16]. The National Geodetic Survey (NGS) is currently cooperating with the Danish and Canadian governments in the readjustment of the North American Datum of 1983.[1]

Space age technology consisting of satellite triangulation, Doppler positioning, and mass data processing added to traditional geodetic surveying practice [20] has made it feasible to determine a best fitting

[1]National Geodetic Survey, NOS,NOAA, "Notice of New North American Datum of 1983, Reference for the National Network of Horizontal Geodetic Control," Federal Register, Vol. 44, No. 127, page 37969, Friday, June 29, 1979, "The new datum shall be known as the 'North American Datum of 1983' and may be referred to as 'NAD of 1983,' '1983 NAD,' or 'NAD 83.'"

ellipsoid for the entire Earth instead of treating each continent or country separately. Therefore, the reference ellipsoid for the North American Datum of 1983 will be Earth-centered (page 23 of [16]). Instead of being defined by the 5 quantities given by Mitchell, the geodetic datum will be the Geodetic Reference System 1980 as adopted by the International Union of Geodesy and Geophysics at their meeting in Canberra, Australia in December, 1979 [5]. Although other geodetic parameters are involved, the parameters of the ellipsoid for the Geodetic Reference System 1980 are

$$a = 6{,}378{,}137 \text{ meters and } 1/f = 298.257.$$

These values will be imposed as constraints in the adjustment of the North American Datum of 1983 [5] which in turn will be used to define coordinates for the existing horizontal control points. Hence, the state plane coordinates of each control point in the network will change for two reasons:

1. The geodetic datum will be changed.
2. The horizontal control network will be readjusted.

Because of these changes, the various state plane coordinate systems will have to be revised. There are two areas in which all existing state plane coordinate systems will have to be changed. First, the defining parameters will have to be those of the North American Datum of 1983. Secondly, it is the policy [17] of the National Geodetic Survey to publish all state plane coordinates based on the NAD 1983 in meters. Additionally, in those states where a state plane coordinate system has been legally adopted by legislative action a revision to the enabling act will have to be prepared and adopted by the various state legislatures. The scope of this thesis is to redefine the Michigan State Plane Coordinate System according to the parameters of the North American Datum of 1983 and to propose revisions to the Michigan Coordinate System Law necessary to facilitate an orderly transition to the use of Michigan Metric Coordinates 1983.[1]

[1] It is proposed by the author that the Michigan State Plane Coordinate System derived by projection of the 1983 NAD and expressed in metric units be known as "Michigan Metric Coordinates 1983" or "MMC 83."

CHAPTER II. THE STATE PLANE COORDINATE SYSTEMS

Development in the United States

Although map projections were commonly in use before the development of the state plane coordinate systems, it was not until the early 1930's that the first conformal projection was designed to cover an entire state. During a cooperative triangulation project between the United States Coast & Geodetic Survey (USC&GS, now the National Geodetic Survey, NGS) and the North Carolina State Highway Commission, the Senior Highway Engineer, Mr. George F. Syme, and the Public Works Commission requested the USC&GS to ". . . consider setting up a system or systems of coordinates for the state." The assignment was given to Dr. Oscar S. Adams, Senior Mathematician for the USC&GS, who designed a two-standard-parallel Lambert conic conformal projection for the State of North Carolina in 1933 (page 34 of [2]). Recognizing the potential value of a map projection for each state, the USC&GS expanded the project and designed a state plane coordinate system for each of the (at that time) 48 states.

There were three important goals to be considered in designing the state plane coordinate systems:

1. The angular relationship of intersecting lines on the ellipsoid must be preserved on the projection.

2. The distortion of geodetic distance from the ellipsoid to the grid distance on the projection plane should be minimized.

3. Each projection should cover a large area; an entire state if possible.

The first criterion was met by using a conformal map projection; however, the last two goals are contradictory and require a trade off. When the maximum allowable distortion is restricted, so also is the useful area of the projection. On the other hand, if the projection is designed to cover an entire state, the distance distortion may become considerable.

There are two conformal map projections which were used in the original design of the state plane coordinate systems, the Lambert conic conformal projection and the transverse Mercator projection. The construction of a Lambert projection is illustrated as a cone which intersects the ellipsoid at two "standard" parallels as shown in Figure 1. First, the meridians and parallels are projected onto the cone. The cone is then cut along a convenient meridian and developed onto a plane. Finally, a rectangular coordinate grid is superimposed upon it.

The transverse Mercator projection is illustrated by wrapping a cylinder about the ellipsoid with the axis of the cylinder parallel with the plane of the equator as shown in Figure 2. Here too, the cylinder is cut along the back, rolled out flat to form the projection surface and a rectangular coordinate grid superimposed upon it. In each case the transformation is defined mathematically and is not a true geometrical projection due to imposing the condition of conformality.

The distance distortion from the ellipsoid to the projection plane is given by the deviation of the scale factor from unity. By selecting a scale factor of 0.9999+ on the axis of a state plane coordinate system the distance distortion is restricted to be less than 1/10,000 at the center of the zone. Moving parallel to the axis of a zone the scale factor is constant; however, the scale factor increases from a minimum on

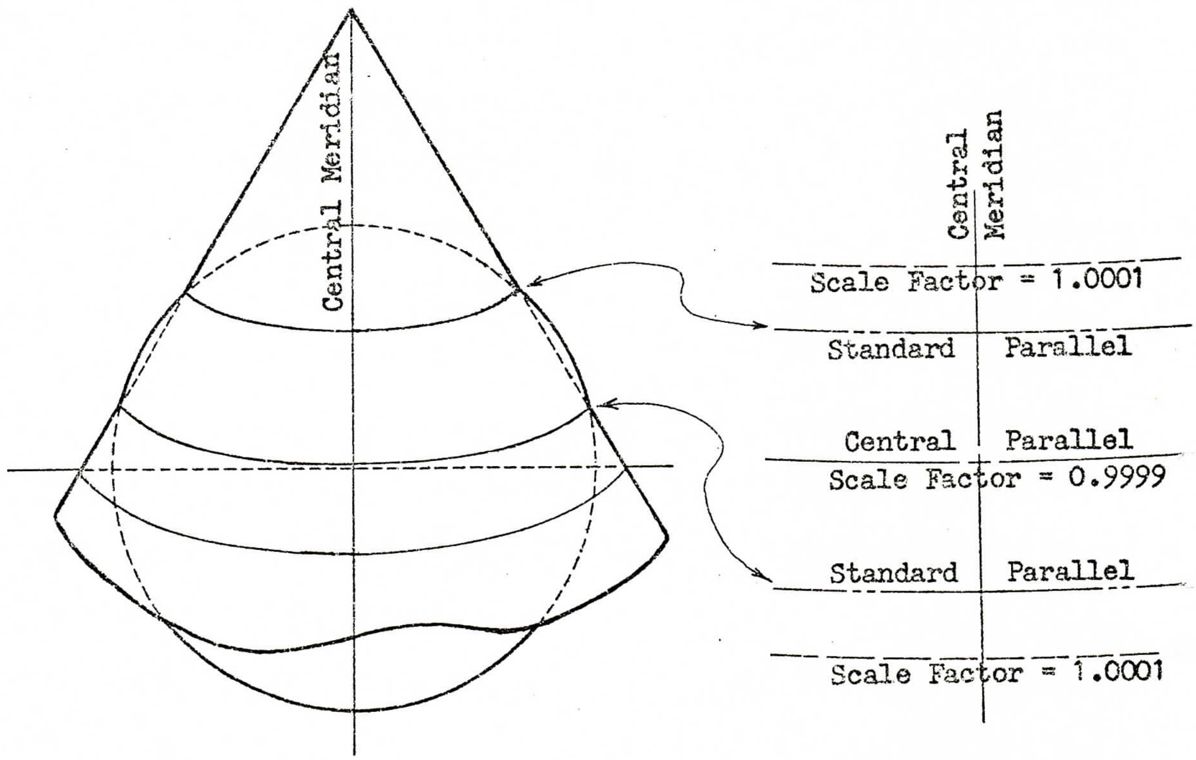

Figure No. 1 Lambert Conic Conformal Projection

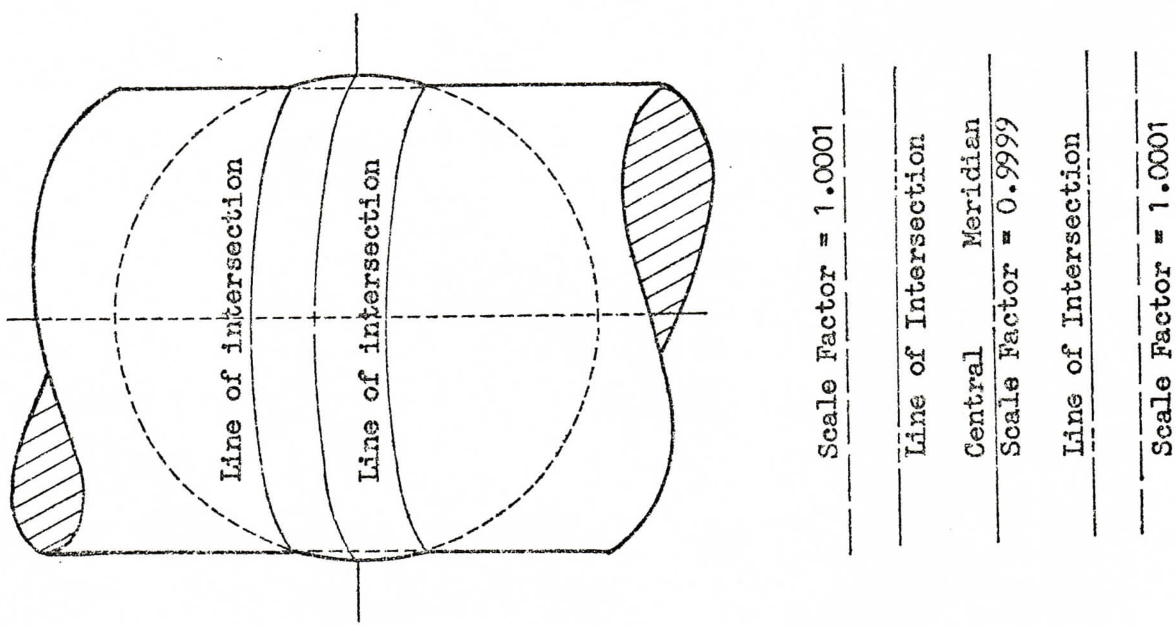

Figure No. 2 Transverse Mercator Projection the axis of a zone to a maximum at the edge of a zone (See Figure No. 3). At the standard parallel on a Lambert projection the scale factor is unity and an elemental grid distance is the same as the corresponding elemental geodetic distance. The scale factor then increases to a certain limit at the edge of the zone.

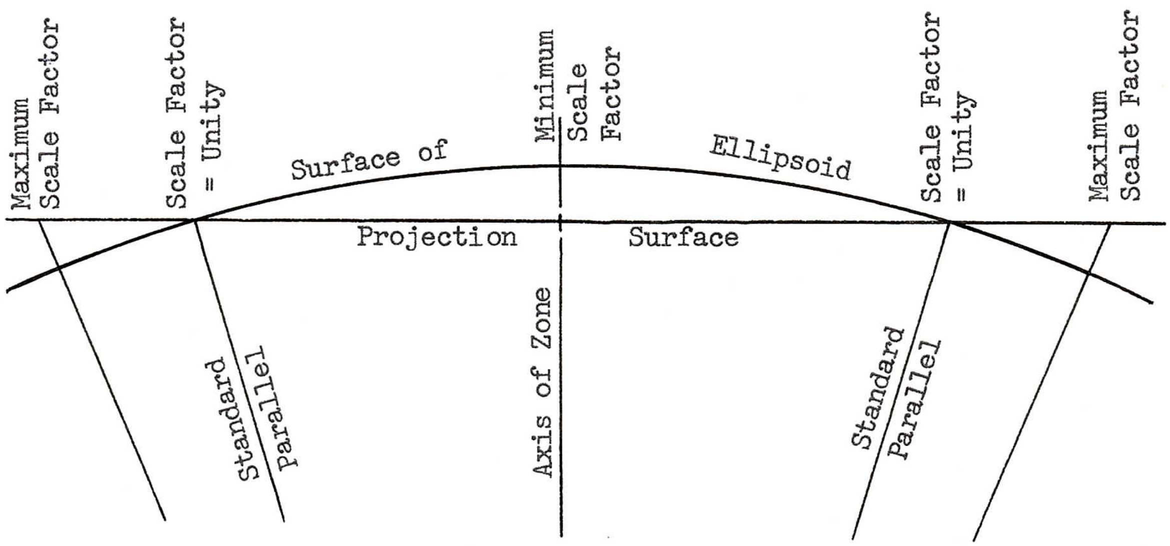

Figure No. 3 Variation of Scale Factor

The maximum width of either a Lambert zone or a transverse Mercator zone is determined by the selection of a scale factor on the axis of the zone and specifying the maximum scale factor allowable at the outer edge of the zone. If the scale factor on the axis of the zone is chosen to be 0.9999 (scale distortion = -1/10,000) and the scale factor limit at the edge of the zone is 1.0001 (scale distortion = +1/10,000) the total width of the zone is approximately 158 miles. If it is desirable to cover a wider strip, either a second zone is required or the scale distortion criterion must be relaxed.

For those states having a large east-west extent (e.g. North Carolina or Tennessee) a Lambert projection is applicable because it extends east and west indefinitely. The north-south limit of a Lambert zone is determined by scale factor considerations. For those states having a predominately north-south extent (e.g. Indiana or Illinois) a transverse Mercator projection is applicable because it extends north and south. The east-west limit of a transverse Mercator projection is again determined by scale factor considerations. There are also several states having a configuration which lends itself to the use of a combination of projections. For example, the State of Florida uses one Lambert and two transverse Mercator projections while the State of New York uses one Lambert projection for Long Island and three transverse Mercator projections for the remainder of the state. California, on the other hand, uses seven Lambert projections and no transverse Mercator projections.

Although not included in the original 48 states, map projections have also been designed for various United States territories and protectorates. Of particular interest is the tangent plane projection for Guam which approximates an azimuthal equidistant projection and the combination of map projections used for the (now) State of Alaska. Due to the size and configuration of the State of Alaska, there are eight transverse Mercator projections, one Lambert projection for the Aleutian Islands, and one oblique Mercator projection for the southeastern panhandle. There is quite a variety of conformal map projections in use throughout the United States for the state plane coordinate systems.

In addition to those map projections used in the United States the military has adopted a series of Universal Transverse Mercator (UTM) projection zones to cover the entire world from 80° south latitude to 80°

north latitude. The central meridian of each zone is 6° from the central meridian of the two adjacent zones and the scale factor on the central meridian is 0.9996. UTM coordinates are expressed in meters and the UTM system in the United States is based on the Clarke Spheroid of 1866 and the North American Datum of 1927.

After each projection was designed, the USC&GS published the formulas to be used to compute the transformations between geodetic positions (latitude and longitude) and state plane coordinates. As most of the projections were designed in the 1930's, the formulas were arranged to use logarithms or mechanical desk-top calculators. People were employed to compute projection tables containing a tabulation of the more difficult factors for each state and zone. They also computed intersection tables of the transformation at each 2½ minute intersection of latitude and longitude throughout the United States as well as the state plane coordinates for each of the horizontal control points for which the latitude and longitude were known.

As the latitude and longitude of the grid intersections are perfectly known, the accuracy of the derived coordinates is determined by the number of terms and by the significant figures carried in the various terms of the transformation. Since the latitude and longitude of the triangulation stations were originally expressed only to three decimal places of seconds, the derived state plane coordinates were accurate only to the nearest 0.1 foot. Thus the coordinate inverse computation between nearby points was inaccurate. This problem was solved by expressing the geodetic position to five decimal places of a second before making the transformation to state plane coordinates. Claire (page 2 of [7]) summarizes by saying,

> Owing to the limited number of terms used in converting geographic positions to plane coordinates, the values of the plane coordinates are not defined as precisely as they might be in an absolute sense. But no significant harm is done since the effect of the omitted terms is virtually the same on all coordinates in a limited area; thus in a relative sense, the plane coordinates are well defined.

Therefore, as long as one uses the same formulas as were used in the original computations, the results will be consistent even though we now use computers with more significant-figure-capacity than was obtainable from mechanical desktop calculators.

Although the state plane coordinate systems were designed in 1933 and projection tables were available to practicing engineers and surveyors shortly afterwards, the system enjoyed no legal status. It became the responsibility of each individual state to enact legislation adopting the system of plane coordinates for the state. In some cases the actual use preceded legal adoption and some states, even now, have not enacted a state plane coordinate law.

The first state to enact a state plane coordinate system law was the State of New Jersey in 1935, followed by Pennsylvania in 1937, and six other states before 1945. In 1945, the Council of State Governments (page 48 of [13]) included a model law for the state plane coordinate systems in its General Report on Suggested State War and Postwar Legislation for 1945. Enactment of the model law was intended:

a. to establish the legal status of the state systems,

b. to insure uniformity and definiteness in terms used, and

c. to impose reasonable standards in the use of the systems when the state coordinates are to become part of the public records.

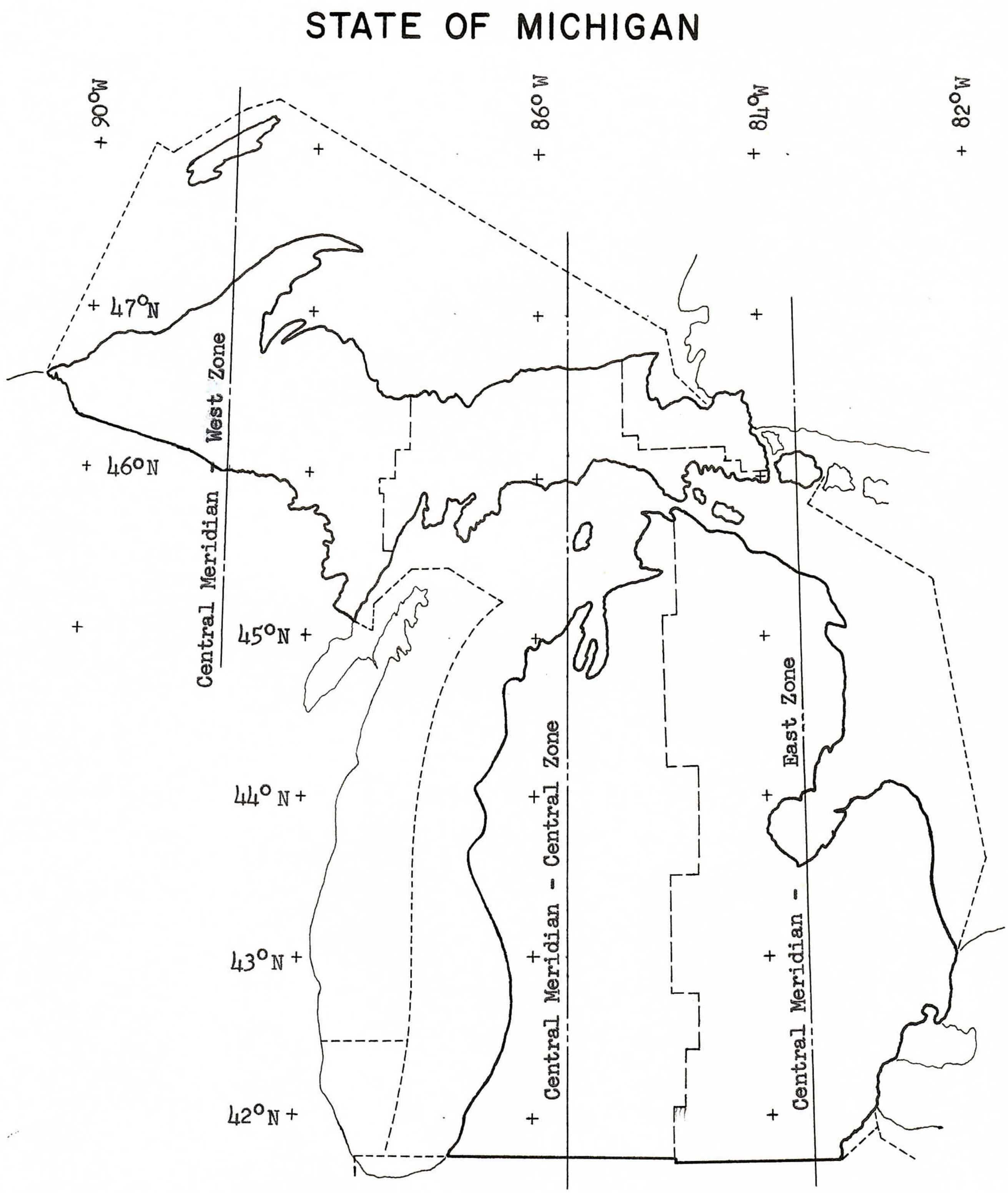

Figure No. 4 Transverse Mercator Projection Zones - 1933

STATE OF MICHIGAN

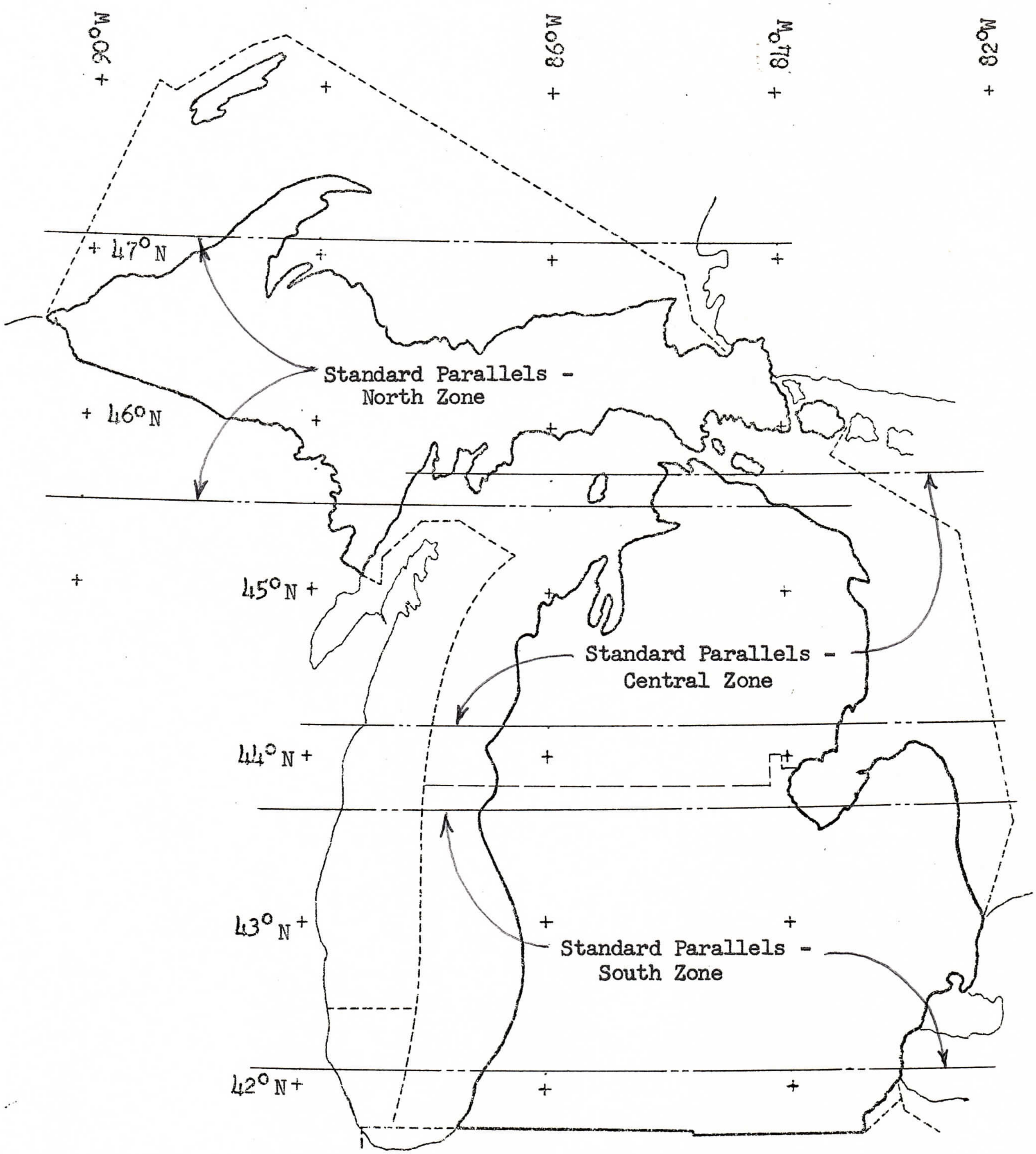

Figure No. 5 Lambert Conic Conformal Projection Zones - 1964 reduction in Appendix C. "The Michigan Scale Factor.") The result is that the grid factor, which is the ratio of the grid distance over the horizontal ground distance, is closely approximated by the scale factor at a given point.

When the USC&GS designed the various state plane coordinate systems, an attempt was made to keep the scale factor from varying from unity by more than 1/10,000. However, rather than design two zones for the North Carolina State Plane Coordinate System the scale factor on the central parallel was chosen to be 0.99987255. By stretching the 1/10,000 criterion a little, the entire state could be covered with one zone.

The minimum scale factor is a convenient way to measure the distance distortion caused by the projection, but it is only one component of the total distance distortion from the ground level to the projection plane. The total distance distortion is obtained from the grid factor which is the product of the scale factor and the sea level factor. Since it is really the grid factor which is of interest to most surveyors, engineers, lawyers, planners and others, the elevation of the ground at each computation point should be considered. In North Carolina the elevation along the central parallel, $\phi = 35^\circ\ 15'$, ranges from sea level at the coast to well over 4000 feet in the Smoky Mountains in the western part of the state. When one includes the elevation component of the grid factor at the 4000 foot elevation, the grid factor on the central parallel becomes:

$$
\begin{aligned}
\text{Grid factor} &= \text{scale factor} * \text{sea level factor} \\
&= 0.99987255 * \frac{20{,}902{,}018}{20{,}902{,}018 + 4000} \\
&= 0.999681
\end{aligned}
$$

Where 20,902,018 = radius of curvature of ellipsoid in feet at $\phi = 35^\circ\ 15'$.
4000 = elevation in feet.

Thus the grid factor on the central parallel at the 4000 foot elevation varies from unity by 1/3135. Although the stated goal for the scale factor was nearly met, the problem of an excessive grid factor was not really addressed by the USC&GS.

The elevated reference surface designed by Berry for the State of Michigan accommodates the effect of elevation, yielding a <u>grid</u> factor which varies from unity less than 1/10,000 for all but a very small part of the state. Since little of the land area in Michigan is over 1000 (or under 600) foot elevation the grid factor can be taken as the scale factor for most land surveys. However if one is conducting a precise control survey or working in an area having an elevation over 1000 feet, the effect of the elevation should be considered as explained in Appendix C.

Another difference in the MSPCS is the transformation formulas used for converting geographic positions (GP) to state plane coordinates (SPC) and vice versa. Instead of using the logarithmic form of the formulas, Berry [3] programmed a set of closed-form formulas for the direct transformation (GP to SPC) for an electronic computer. The inverse transformation is not in closed-form; but sufficient terms were carried in the series to yield results consistent to the nearest 0.001 foot.

The final distinguishing characteristic of the MSPCS is that the Central Meridian for the south and central zones was chosen to coincide quite closely with the Michigan Principal Meridian of the U.S. Public Land Survey System ($\lambda \approx 84^\circ\ 22'$W). Hence, lands lying east of the Michigan Principal Meridian generally have state plane coordinate "X" (Easting) values greater than 2,000,000 feet, and those lying west of the Michigan Principal Meridian have "X" values less than 2,000,000 feet.

Uses and Users

The State of Michigan enjoys the benefits of a unique map projection for its state plane coordinate system. However with publication and use of the 1983 NAD, a break with the past will be made. Not that the value of the work done in the past will be diminished - but there will be new coordinates for the same points. Besides, the 1927 datum coordinates are in feet while the 1983 datum coordinates will be in meters. Since a definite and specific break with the past is being made - are there any other changes which should be made to the map projection which will increase the effectiveness of the Michigan State Plane Coordinate System? To answer this question, an attempt was made during the past year to contact all users of the MSPCS, to inquire what features of the present system (if any) were obstacles to its use, and to determine what changes (if any) should be made to improve upon the existing system.

In August, 1979 a questionnaire was sent to all 692 members of the Michigan Society of Registered Land Surveyors. (See Appendix A, "Tabulation of Questionnaire Responses from Members of the Michigan Society of Registered Land Surveyors.") Following that, 32 letters of inquiry were sent to various state agencies, colleges & universities, utility companies, and related professionals who might be users of the MSPCS. (See Appendix B, "Tabulation of Contacts and Summary of Responses from State Agencies, Colleges and Universities, Utility Companies, and Related Professionals.") In short, the results of the MSRLS inquiries were disappointing and rather inconclusive. Of the 692 inquiries to MSRLS members, only 50 bothered to reply; and of those who did - more than half stated that they had never used the MSPCS. On the other hand, 24 responses were received to the 32 letters of inquiry to state agencies etc. Some of these replies were excellent and very helpful; but taken as a whole, these replies also established little except the overwhelming need for continuing education in the effective use of the MSPCS.

<u>As a Data Base for Engineering Projects</u>. As established from personal experience and the results of the inquiries, the most prominent use of the MSPCS is as a coordinate data base for various engineering projects. The Surveying Section of the Design Division of the Michigan Department of Transportation (MDOT) has unofficially used the MSPCS for quite a few years [11]. Although there was no adopted policy within the MDOT on use of the MSPCS, occasionally the necessary control monuments were to be found in the area of a MDOT project and the system was used by the surveyors as a convenience to their own operations. However, as a result of the need for a consistent data base for automated mapping and design activities, the MDOT has adopted a policy on the use of the MSPCS as a data base for development of proposed improvements to existing and proposed state trunk lines. In the past five years the MDOT has used the MSPCS quite extensively and has even established second-order geodetic control on a cooperative arrangement with the National Geodetic Survey as a part of several projects.

The two major utility companies within the state, the Detroit Edison Company and Consumers Power Company, have also made use of the MSPCS as a data base for engineering projects. Both companies have used the MSPCS as part of the site control at power plant sites and on major transmission lines within their respective service areas. Although both companies do use the MSPCS, neither company provided a positive policy statement on the use of the MSPCS.

Other organizations within the State of Michigan also use the MSPCS as a data base for engineering projects. Some use of the system exists on a county or city wide basis and certain engineering firms use the MSPCS as a coordinate data base for construction projects, hydrographic surveys, subdivisions, photogrammetric mapping, and other applications.

As Monument to the U.S. Public Land Survey System. Another significant use of the MSPCS is as a data base for referencing the location of section corners of the U.S. Public Land Survey System. The locations of the section corners were established years ago when the state was being settled. Many of the original section corner monuments have been obliterated or lost over the years and it has become quite costly to re-establish the location of a missing section corner. Added to that is the problem created by the presence of multiple monuments in the same general area, each purporting to mark the same section corner. Resolving such a discrepancy can be even more costly than determining the correct location of a missing corner. When the state plane coordinates are reliably determined for a section corner whose position is known, the location of the corner will never be lost again. If the monument is destroyed (obliterated) the position can be re-established by setting a monument to mark the location of the recorded coordinates.

Of the many people and organizations within the state who are vitally concerned with preserving the location of the section corner, it is the land surveyors who are responsible for locating them. Thus, it is not surprising that it was a group of land surveyors who spearheaded the Ingham County Remonumentation Project in Michigan. The project was begun in 1977 and has two goals:

1. to establish and monument the correct location of each section, quarter section, and center of section corners within the county; and

2. to establish accurately the Michigan State Plane Coordinates of each corner.

When the project is completed more than 2000 corners in Ingham County will be monumented and referenced to the Michigan State Plane Coordinate System.

The Michigan Department of Transportation has also established MSPCS values for many section corners lying within or adjacent to the numerous state trunk line highways throughout the state. Although the MDOT has adopted a positive policy on the use of MSPCS for engineering design, much more could be done by the MDOT in recording MSPCS values for the section corners on and along state trunk line highways.

Although other utility companies may have little use for the MSPCS as a data base for section corners, both the Detroit Edison Company and the Consumers Power Company have used state plane coordinates on the property and section corners on the right-of-way of some of their transmission lines. The author was personally involved in a power line project for the Detroit Edison Company on which more than 500 section corners were defined using the MSPCS. However, due to lack of positive policy on the part of the Detroit Edison Company, the state plane coordinate values were not made a part of the public record of the surveys of the land parcels involved. The Consumers Power Company has also used the MSPCS on power line right-of-way surveys (See reply in Appendix B). However, they too lack a positive policy on use of the MSPCS and use it only as dictated by permit requirements or when the control points are immediately accessible.

There are other firms and organizations within the State of Michigan who have benefically used the MSPCS to reference the location of section corners. However, there is no organization or agency within the state which has been given responsibility for coordinating the use of the statewide system. If such responsibility were designated and funded, much more use and benefit could be achieved.

As a Regional Data Base. Although the Michigan State Plane Coordinate System has three separate zones, it covers the entire state and can be used as a regional data base for numerous purposes. Any information which is of a demographic nature can be analyzed more readily if it is catalogued by its relative location within its particular population. When populations of different events or objects are defined on a common data base and given an absolute location, correlation of one data file with another becomes possible. The MSPCS provides such a data base.

According to the responses received from the MDOT and several colleges and universities, the MSPCS is being used as a regional data base quite successfully. The MDOT is using the MSPCS as a data base for their county mapping program. They are also providing land use data to the Great Lakes Coastal Unit of the Michigan Department of Natural Resources on the MSPCS. Charles E. Olsen Jr., Professor of Natural Resources at the University of Michigan, writes that although they do not use the MSPCS, they do use the UTM system for land use mapping and would like to have a metric system which would cover the entire state with no seams or zone boundaries. Several geography professors responded that the use of the state plane coordinates on the USGS quad maps is being taught in their courses and Professor Hodler of Western Michigan University tells of using the state plane coordinate system as a data base for the inventory of utility equipment for a small utility company.

In the past several years there has been a national movement toward the use of the state plane coordinate system as a "Land parcel identifier" [14]. Much work has been done in this area and many benefits could be achieved once the state plane coordinate system is fully implemented. However, little activity was found of anyone in Michigan using the MSPCS for land resource management, tax mapping, or land parcel identification. Could these uses help justify implementation of the new system of metric coordinates based on the 1983 North American Datum?

In summary, three categories of use of the MSPCS have been identified:

1. as a coordinate data base for engineering projects,
2. as a reference system for property corners and land boundaries, and
3. as a regional data base for mapping, planning, and analysis of geographic, demographic, and environmental data.

These uses have many overlaps and enjoy many benefits as a result of their similarities. However, in extreme cases, there are two conflicting criteria to be met:

1. The ideal for engineers and surveyors is for the grid factor to be very close to unity. This criterion restricts the effective area of a given zone. Hence, it may take more than one zone to cover the state.
2. The ideal for planners and data analysts is to have one zone cover the entire State of Michigan, eliminating coordinate discontinuities at zone boundaries. However, with only one zone, the scale factor distortion must be greater than 1/10,000.

Fortunately, the choice of one method or the other does not eliminate the usefulness of the MSPCS for other applications. These will be discussed later.

Obstacles to Effective Use

Given that the MSPCS has been defined, legally adopted and offers certain benefits to several categories of users - why hasn't the use or implementation of the MSPCS proceeded more rapidly than it has? Is it because the law establishes "permissive use" rather than mandatory use? Is it because the law provides no funds for implementation? Is the MSPCS, for some reason, unacceptable due to its very nature? Is it so exotic that no one can afford it? Is it too complicated to be understood? Proponents of the system argue that none of the explanations given above constitutes a legitimate obstacle to the use of the system. The law establishes the legal basis of the system. The benefits derived through using the system justify the cost involved in using the MSPCS. And, the mathematical formulation is straightforward and well within the understanding of a reasonably competent surveyor.

Perhaps a better understanding of the reluctance of the surveying profession to use the MSPCS can be obtained by studying the replies to the MSRLS questionnaire listed in Appendix A. Of those who responded to the questionnaire the largest number of responses in the "obstacle" category (34 out of 50) was, "Lack of request or acceptance by client or employer." However, of the 34 who listed acceptance as an obstacle, 19 have never used the MSPCS and 9 more have used it only once or twice.

The second largest response in the obstacle category was the "Inadequate number of control points in the area." Note that it was listed as an obstacle by users of the MSPCS as well as non-users. The lack of adequate control is an obstacle which can be quantified for a given area and one which is tied closely to the availability of funds for control surveys.

The third major obstacle listed in the responses was the "Lack of familiarity in the use of the system." These responses came primarily from those who have seldom or never used the MSPCS. It is interesting to note here that the items involving the sea level reduction, the scale factor computation, and crossing zone boundaries were considered by very few to be obstacles to use of the MSPCS, yet many cited lack of familiarity as an obstacle. This suggests that few Michigan surveyors are qualified to judge what features of the MSPCS may, or may not, be an obstacle to the use of the MSPCS.

Another factor considered by many to be an obstacle to the use of the MSPCS is that of cost. Mr. Hooth of the MDOT pointed to the failure of the present Michigan Coordinate System Act to provide funding for implementation of the MSPCS. The statutory charge of the MDOT is to build and maintain transportation facilities, not to spend time and money on geodetic control surveys. Thus, the MDOT is reluctant to commit funds or resources to "extra" surveying required to use the MSPCS.

Although cost was not included as a specific item on the MSRLS questionnaire, a number of comments in the "obstacle" section stated that it is not feasible to charge a client for tying a survey into the MSPCS. Generally the comments considered only the first-time cost and didn't acknowledge that points, once established, could be reused on other surveys. Admittedly, cost and profitability are just as important in the operation of a business as efficient and responsible expenditures of tax monies are for public agencies. However, it seems that short sighted arguments are presented with little or no consideration given to coordinating surveying activities, eliminating duplicate efforts, or maximizing long-range benefits to society. Is this another opportunity for the surveying profession in Michigan to provide responsible leadership? The alternative is for surveyors to become technical subordinates to the engineers and planners who promote use of the system.

A number of those responding to the MSRLS questionnaire who use the MSPCS listed the "Lack of a central data repository" as an obstacle to the use of the MSPCS. While the lack of a central data repository might not prevent any one from using the MSPCS it certainly seems reasonable that the existence of one would foster additional and more efficient uses. If such a central data repository were established the office could also be used effectively to standardize survey methods and procedures for insuring the integrity of the data. Thus data submitted by one could be used with confidence by another and vice versa.

In summary, the obstacles to the use of the MSPCS fall into two broad categories:

1. finding a way to fund the implementation and to coordinate effective use of the MSPCS; and

2. the reluctance of the land surveyors to provide professional leadership in the application of modern technology.

CHAPTER III. A METRIC MAP PROJECTION FOR MICHIGAN

Design Criteria

The reason for considering a revision to the existing MSPCS is that the National Geodetic Survey is redefining the North American Datum and readjusting the entire horizontal geodetic control network throughout the United States. The combined result will be published as the North American Datum of 1983. It is also the policy of the NGS to publish metric state plane coordinates for any horizontal control point defined by the 1983 datum. These two design criteria, the geodetic datum and the length units, have already been determined and need no further consideration.

The NGS has stated that, unless requested by an individual state to make a change, the state plane coordinate system in each state will be based on the projection and defining parameters for the state as listed in USC&GS Special Publication No. 235 (1974 revision). Before the NGS will accept a change for a given state, the state must have amended its legislation to accommodate the requested change. Since the State of Michigan has already adopted the Lambert projection the NGS has stated that Michigan's Transverse Mercator System designed by the USC&GS in 1933 will be eliminated in favor of the legislatively approved Lambert system. Is the existing Lambert system the best map projection to be used with the 1983 NAD or are there changes which could be made to improve the usefulness of the MSPCS? If a change can be made to improve the system, the time to do it is when the transition is made to the use of metric coordinates and the 1983 Datum.

Conformality

One of the most basic design criteria for a map projection for a state plane coordinate system is that of conformality. A result of imposing the condition of conformality is that the shape of any small area on the surface of the earth is unchanged when projected to a plane. This means that the surveyor or engineer using a conformal projection in a limited area has only to concern himself with distortions to the distances as the angles are preserved. The condition of conformality was applied in the design of the original state plane coordinate systems and has been used quite successfully. No reason has been found or will be given for using anything but a conformal map projection for the Michigan Metric Coordinates 1983.

Elevation of Reference Datum

One proposed change is to move the reference datum back to the ellipsoid instead of keeping it at the 800 foot elevation. At the time (1964) Berry proposed the elevated reference surface, electronic distance measuring instruments (EDMI) were not yet commonly available and most surveying computations were performed with mechanical calculators and tables of trigonometric functions. His goal was to reduce the computational burden of the land surveyor using the MSPCS. In that, he was successful.

However, with the advent of EDMI, expanded use of one second theodolites, and powerful computing capabilities; land surveyors who use the MSPCS now routinely perform surveys which have a random error that is smaller than the systematic error introduced by the scale factor and sea level reduction. Consequently, the surveyors using the MSPCS routinely make a correction to the 800 foot elevation (which could just as easily be made to the ellipsoid) before applying the scale factor. The problem Berry sought to eliminate is no longer a problem. There is instead, a problem of confusion over which is the proper scale factor. All the material presented by Berry on using the elevated reference surface is correct and consistent. However, when one uses other sources, a discrepancy is quickly discovered. (See extensive explanation in Appendix C, "The Michigan Scale Factor.") With the reference surface moved back to the ellipsoid, the confusion over the scale factor should be eliminated and computational effort would not be increased.

It is shown in Appendix C that the scale factor is independent of the semi-major axis of the ellipsoid. Therefore, if one wishes to work at some elevation other than on the ellipsoid, the advantages of an elevated reference surface are still available. When used properly, the results are identical.

Maximum Scale Distortion Allowable

As pointed out earlier, there are two conflicting criteria to be considered when designing map projections for the Michigan State Plane Coordinate System. The first criterion is that the maximum scale distortion be less than a certain limit, say 1/10,000. The second criterion is that the state should be covered with one zone. It is not possible to satisfy both criteria for the State of Michigan. If the 1/10,000 criterion is applied a minimum of 3 zones is required. If one zone is used the maximum scale distortion rises to approximately 1/2500. Which criterion is more important and which could have the greatest adverse impact if not satisfied? A lot of arguments could be presented for each side but the issue really boils down to this: What is so sacred about the 1/10,000 limit? Is it inflexible? Is it invalid? Is it obsolete?

The criterion of containing the scale factor distortion within 1/10,000 was arbitrarily chosen in the early 1930's when the original state plane coordinate systems were designed. The reasoning was that a systematic error contributing no more than 1/10,000 could be easily absorbed in the random error of a survey which had a linear misclosure of 1/3000 to 1/5000 or even 1/7500. This meant that the distances and areas shown on ordinary survey maps and engineering drawings could be used as ground distances and areas with the discrepancies attributed to survey procedure instead of map projection or scale distortion. Since then the 1/10,000 criterion has come to be loosely regarded as a yard stick for determining the acceptability of land and construction surveying measurements. Thus its sacredness consists primarily of its past use and acceptance.

The 1/10,000 criterion has been extensively applied to the scale factor distortion to determine the acceptability of one's choice of a map projection. However, of more interest to land surveyors and geodetic engineers is the grid factor which expresses the ratio of the grid distance over the horizontal ground distance. The grid factor distortion has two components, the scale factor distortion and the sea level reduction, and is the total distortion between ground and grid which must be acknowledged and handled acceptably. The contribution of elevation to the total distortion was well known to Adams when he designed the various state plane coordinate systems [2], but his philosophy was that ". . . the importance of having the work tied in with the control net far outweighs the need for exact ground level distances. Actual lengths and areas can easily be determined from a map made on the state grid even though the coordinates may give slightly different results."

Adams' reluctance to consider the total distortion further has lead to confusion over the best way to handle significant distortions when they are encountered. Dracup [18] describes the use of "project datum coordinates" which will give the ground distance from a coordinate inverse but cautions that great care must be taken to assure that project datum coordinates are not used a state plane coordinates. Pryor [8] proposed an approach called "datum adjustment" in which the state plane coordinates are adjusted for the elevation of a given area making it possible to compute correct ground distances from coordinate inverses. Berry [3] designed the existing MSPCS with the idea of controlling the grid factor distortion within 1/10,000. He was successful for the most part; but even so, there are several areas in Michigan where the grid factor distortion is greater than 1/10,000. Thus although the 1/10,000 criterion has been used extensively in the past, it has been applied inconsistently and is not really a true measure of the goal to be achieved.

Is the 1/10,000 distortion criterion obsolete? When the criterion was applied in the 1930's to the state plane coordinate systems, most land surveying fell in the "transit/tape" category. Traverse misclosures of 1/3000 to 1/5000 were normal except for precise or control surveys in which substantial effort was required to obtain results known to be better than 1/10,000. However, the state of the art has changed dramatically over the past 50 years. In place of the transit/tape equipment, most surveyors are using the theodolite/EDMI combination. Normal traverse practice has progressed to where misclosures of 1/10,000 to 1/30,000 are normal; and, if a survey crew exercises a reasonable degree of care, traverse misclosures of 1/50,000 or better are easily attainable. No longer is the scale distortion buried in the random error of the measurements. If that were the goal, the scale distortion would have to be less than 1/50,000 and more like 1/100,000. Since that is not practical, the distortion caused by representing a curved earth on a flat plane will have to be included as a part of the computations on all state plane coordinate surveys.

If the effects of the grid factor distortion are routinely included in all state plane coordinate surveys, then actual ground distances and areas can be readily determined. This has been done successfully by the author on several projects involving hundreds of miles of high voltage transmission lines. The state plane coordinates were published for each point of intersection (PI) of the project center line. The grid factor applicable to that part of the project was shown on each sheet of the plan & profile drawings. The stationing along the center line represented actual ground distance. The only apparent discrepancy was that the coordinate inverse distance between PI's did not agree with the difference in stationing. Actually, no discrepancy existed because the product of the grid factor and the stationing difference gives the coordinate inverse distance. Although the grid factor distortion often exceeded 1/10,000, the difference between grid distance and ground distance was handled routinely. Thus, there is no limit to the grid distortion which can be realistically imposed as a criterion on selection of a map projection.

Coordinate Origin

The last criterion to be presented for a map projection to be used for the Michigan Metric Coordinates is that of range of coordinate values which will occur. When Berry designed the existing MSPCS in 1964, he chose an "X" value of 2,000,000 feet for the false easting on the central meridian of each zone. This meant that the "X" coordinate could always be distinguished from the "Y" coordinate because the "Y" coordinate (excluding Isle Royale) never exceeded 1,000,000 feet. The range of "X" values and "Y" values to be used with the Michigan Metric Coordinates 1983 should be chosen as follows:

1. The X & Y coordinate values should be distinguishable by inspection of their respective magnitudes.
2. The 1983 datum coordinates should be distinguishable from the 1927 datum coordinates by a simple rule.
3. The zone in which a pair of coordinates is located should be unique.
4. The magnitude of the coordinates should be kept as small as possible for convenience in computing area from coordinates.

Stoughton [21] has suggested that the above criteria be achieved for Lambert zones by assigning specified values of X to the central meridian. Thus if the values of 4,000,000 meters, 6,000,000 meters, and 8,000,000 meters were assigned as false eastings to the central meridian of three new zones, all of the above would be satisfied except number 4. Problems could arise if one tried to compute the area of a small tract accurately using such large coordinate values. (Normal practice is to avoid the problem by subtracting off a large constant before computing the area.) Otherwise, all criteria have been met and no change in the latitude of the origin (y=0) is required.

However, if it is decided to go to a projection which covers the entire state with one zone, the coordinate origin can be selected such that no coordinate value is over a million and all the criteria can be met. The first criterion is met because the "X" coordinate is larger than the "Y" coordinate in the lower peninsula and the "Y" coordinate is larger than the "X" coordinate in the upper peninsula.

Mr. Stem who is on the Director's staff at the NGS has suggested in a letter to the author that criterion number 4 is more important than the first three and has offered a less simple method of distinguishing datums and zones than is proposed by Stoughton. Anyone desiring information on his method can obtain the same from Mr. Stem at NGS in Rockville, Maryland.

Candidate Map Projections

With the design criteria that have been discussed, what are the alternatives as far as actual map projections are concerned? A list of possible conformal map projections including the number of zones required to cover the State of Michigan and the minimum scale factor on the axis of the projection is given below.

Table 1 Conformal Map Projections

Projection	Number of Zones Req'd	Minimum Scale Factor
Transverse Mercator	3	0.9999
UTM (6^{o} zones)	3	0.9996
UTM (2^{o} zones)	4	0.9999
Lambert Conformal	3	0.9999
Oblique Mercator	1	0.9996
Combination of types	2	
Upper Peninsula (Lambert)		0.9999
Lower Peninsula (Trans. Mercator)		0.9998

The transverse Mercator projections designed by the USC&GS in the 1930's were eliminated in favor of the Lambert conic conformal projections as described previously. Nothing has been discovered or will be presented to suggest that the transverse Mercator map projection should be considered again.

The Universal Transverse Mercator (UTM) map projections were designed and implemented by the U.S. military for world wide coverage. A UTM projection is a zone 6° in width which extends from 80° south latitude to 80° north latitude. The scale factor on each central meridian is 0.9996 giving a distortion of -1/2500.

At one time the UTM system was proposed to replace the existing state plane coordinate when the 1983 NAD was implemented. The advantages quoted were that the UTM system is well established and that the transformation formulas are identical from zone to zone. However, the proposal was dropped because of anticipated resistance to the greater scale distortion and due to the fact that UTM zone boundaries are meridians and do not follow topographical and/or political boundaries. For example, even with the greater zone width of the UTM system, the State of Michigan lies in three different UTM zones (See Figure 6). If three zones must be used, the existing Lambert projections become the logical choice. Therefore, even though the UTM coordinates will be shown by the USGS on the new metric topographic maps [10], use of the UTM system for the Michigan Metric Coordinates 1983 is not recommended.

In 1973 Pryor [8] proposed the use of a modified UTM system with 2° zones to be used for engineering and cadastral surveys. The advantages of his proposal were that the scale distortion would be reduced to

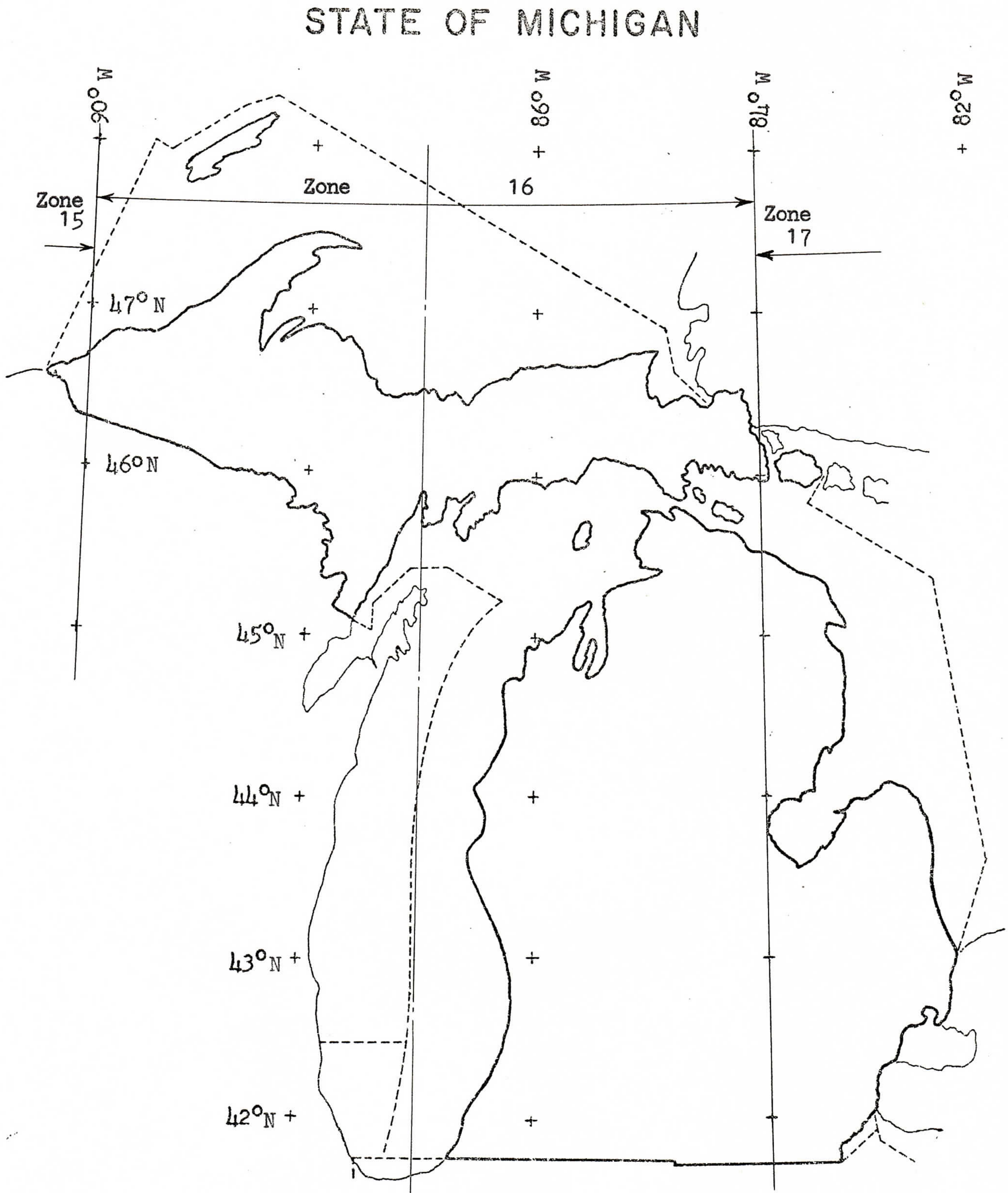

Figure No. 6
Universal Transverse Mercator Zones For Michigan

1/10,000 and that the transformation formulas would be the same from zone to zone. According to Dracup, [18] the idea of the 2° zone never caught on because it too followed meridional boundaries rather than political boundaries.

The existing MSPCS is comprised of three Lambert conic conformal projections designed by Professor Berry and adopted by the Michigan Legislature in 1964. The system has been used quite successfully and certainly warrants consideration for continued use. Although it has been proposed to lower the reference datum back to the ellipsoid, some have argued for continued use of the Lambert system because users are familiar with it. Several respondents raised the question, "What next?" The system designed in the 1930's was eliminated in favor of a new one in 1964. Now the MSPCS is being changed again. "Why can't it stay the same?" Of course, the type of projection can stay the same. But since the geodetic datum is being redefined and metric units are being adopted, a break with the past will be made. Whichever system is adopted, it should remain in use as long as the geodetic datum remains unchanged.

A drawback to the use of the Lambert projections is that the entire state can <u>not</u> be conveniently covered with one zone. However, there is a one zone projection designed by Berry [4] which covers the entire state and jurisdictional waters (See Figure 7). It is called the "Michigan GEO-REF System" and uses an oblique Mercator map projection. It uses metric units and has a scale factor of 0.9996 on the axis of the projection. The magnitude of the coordinates is always less than 1,000,000 meters and except for a small area on the eastern end of the upper peninsula the X coordinate is always larger in the lower peninsula and the

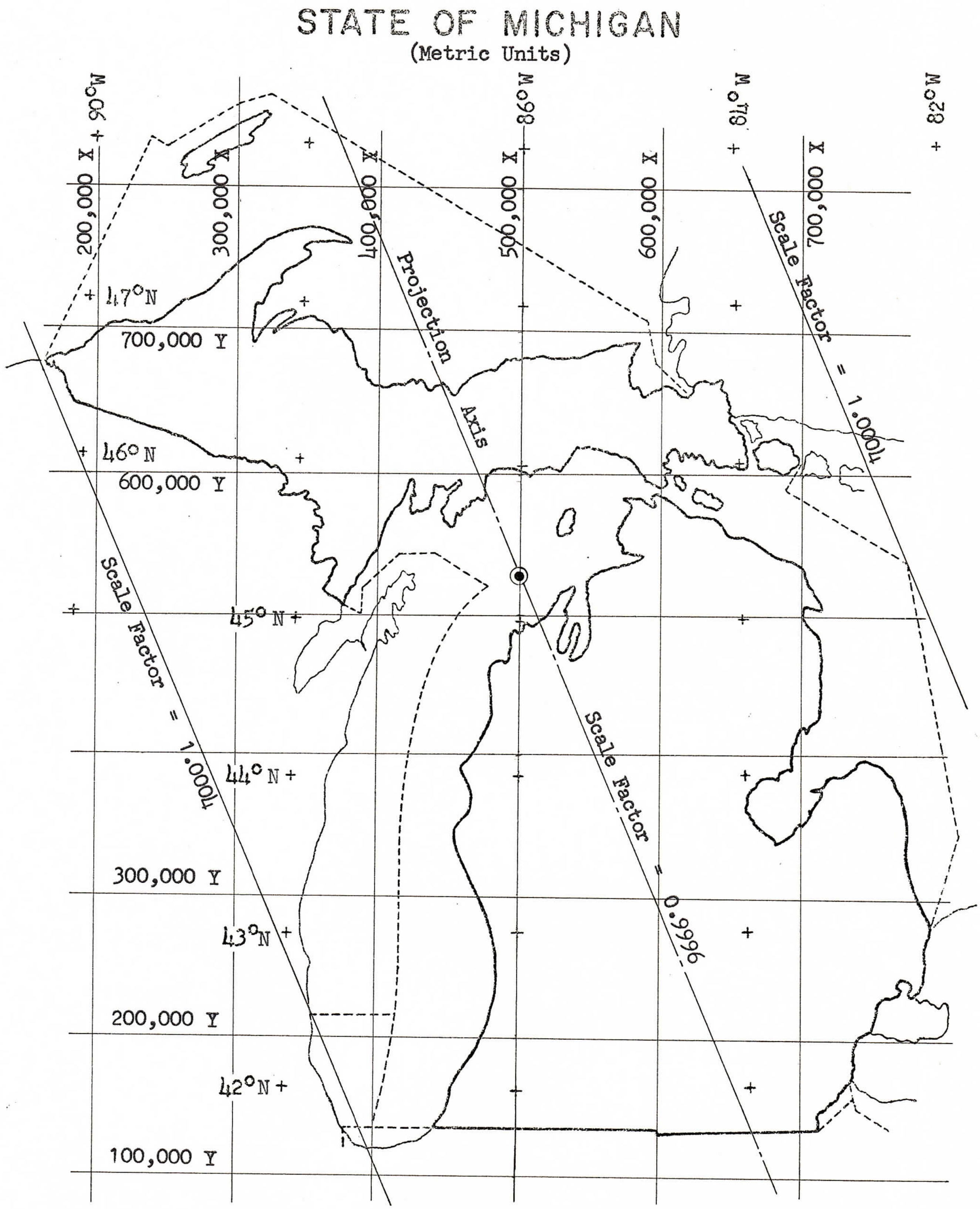

Figure No. 7 Michigan GEO-REF Coordinate System

Y coordinate is always larger in the upper peninsula.

The most obvious objection to the GEO-REF system is that it doesn't meet the 1/10,000 scale factor distortion criterion. However, it has been shown that the 1/10,000 criterion is an arbitrary one and is not really a measure of the goal to be achieved. What then is the maximum grid factor distortion for the GEO-REF system? Very little of the state is over an elevation of 300 meters. At the 300 meter elevation on the axis of the projection the grid factor is:

$$0.9996 * \frac{6378.332}{6378.332 + .300} = 0.9995530$$

where

0.9996 = scale factor on the projection axis.

6378.332 = radius of curvature of the ellipsoid in kilometers at $\emptyset = 45^{\circ}\ 18'\ 53''$.

0.300 = elevation of point in kilometers.

This means that the approximate worst grid factor distortion is 1/2161. It certainly falls short of the 1/10,000 goal adopted by the USC&GS for the state plane coordinate systems; but if the elevation is also considered, the grid factors will not differ that much. Since the grid factor distortion is handled by appropriate computations, no positional accuracy is lost and other benefits can be achieved. In short, the GEO-REF system:

1. covers the entire state and jurisdictional waters with one zone;

2. has only one set of constants and transformation formulas;

3. uses metric units and has no coordinate values over 1,000,000 meters; and

4. maintains positional accuracy of all survey points.

Another possible alternative (especially if the upper peninsula is serious about becoming the State of Superior) would be to use a combination of two projections, one for each peninsula. Since the upper peninsula is already covered by a Lambert zone the only change would be to design a single transverse Mercator projection for the lower peninsula. The scale factor distortion on the central meridian would be approximately 1/5000 and the grid factor distortion at the 300 meter elevation would be about 1/4000. Other than eliminating one zone in the lower peninsula there is little benefit to be derived from using such a combination of projections.

Of the alternatives considered, the three Lambert zones are attractive due to their acceptance and present use while the GEO-REF projection is attractive because it covers the entire state with one zone.

Revisions to Existing Law

Given that the 1983 NAD is being prepared and that metric units will be used for state plane coordinates, what changes should be made to the existing Michigan Coordinate System Law? Whether use of the Lambert projections is continued or whether the "GEO-REF" system is adopted, the following changes should be made:

1. The North American Datum of 1983 will have to be specified in place of the North American Datum of 1927 and the Clarke Spheroid of 1866.

2. The units of the coordinates will have to be meters to conform to the policy adopted by the National Geodetic Survey. As suggested by Stoughton [21] the relationship between the meter and the American Survey Foot should also be included (1 meter = 39.37/12 feet).

3. The reference datum in the State of Michigan should be returned to the ellipsoid.

4. A transition period for making the change to Michigan Metric Coordinates 1983 should be established and written into the law. Mr. Stem of the NGS writes that NGS ". . . adopts the philosophy that both the SPC system on the 1927 datum and the SPC system on the 1983 datum should be legally recognized from the time of passage of the act to some date in the future - - after which time the 1927 datum coordinates should not be used. I feel 1990 would permit a reasonable transition period."

In addition to the above changes which have to do with the actual map projection and design of the Michigan Metric Coordinates 1983, there are several other changes which have been suggested to facilitate implementation of the new system:

1. Clarify the accuracy specifications for surveys contained within the law.

2. Assign greater weight to Michigan Metric Coordinates as evidence for corner location on boundary surveys.

3. Establish a funding mechanism within the law for implementing the Michigan Metric Coordinates 1983.

The accuracy specifications in the existing Michigan Coordinate System Law are given by:

> The position of the Michigan Coordinate System shall be marked on the ground by triangulation or traverse stations established in conformity with standards adopted by the USC&GS for first-order and second-order geodetic control surveys, whose geodetic positions have been rigidly adjusted on the North American Datum of 1927, and whose coordinates have been computed on the system herein defined.

The law does not specify whether the standards to be applied are those existing at the time the law was passed or at the time the survey is performed. The specifications issued by NGS have been revised from time to time to response to the dynamic nature of modern survey practice. As Stoughton [21] points out, the law should not mandate a set of standards applicable at a given time but should accommodate inevitable change by stating:

> ". . . in conformity with standards adopted by the Federal Geodetic Control Committee and successors at the time the surveys are executed."

Under the category of "desired changes" in the MSRLS questionnaire, the item of assigning greater weight to state plane coordinates as evidence received more responses than any other. Under the existing law the coordinate position of a boundary corner is to be regarded as supplemental to the position of the boundary corner as determined by reference to a line or corner of the U.S. Public Land Surveys. The existing law was written as it is for a reason; however as technology and modern survey practice continue to progress, more impetus will be gained for accepting coordinates as evidence for property location and description. These legal concepts are very important and should be studied throughly; however, they are beyond the scope of this thesis.

In the past several years, the benefits of using the MSPCS have become more and more evident. Although the existing law provides no funds for implementation of the MSPCS, the Plane Coordinate Committee of the MSRLS has been active in promoting the concept of a centralized surveying & mapping organization within the state government. The "Michigan Survey Authority" would be responsible for coordinating implementation of the MSPCS with numerous users and would serve as a data repository. In July 1976 MSRLS sponsored a meeting of state officials and interested professionals with Captain Leonard S. Baker, then Director of the National Geodetic Survey. At that time Mr. John P. Woodford, Director of the Michigan Department of Transportation, indicated his willingness to have such a survey organization within his department <u>if</u> the state legislature would provide the funding. Establishing such an authority could provide an efficient method for implementing the Michigan Metric Coordinates 1983 and should be considered. However, justifying and presenting arguments for such a change is beyond the scope of this thesis.

CHAPTER IV. SUMMARY AND RECOMMENDATIONS

Summarizing the material presented in this thesis, "A Metric Map Projection for the State of Michigan," the following recommendations are made:

1. The Michigan Coordinate System Law should be revised to specify use of the 1983 NAD and metric coordinates.

2. Additional effort should be made by the Michigan Society of Registered Land Surveyors (MSRLS) to contact potential users for valuable input on selection of the projection to be used.

3. MSRLS should conduct open discussion in a public forum on the merits of the alternatives herein presented.

4. One of the two following map projections should be adopted to define a new state plane coordinate system to be known as "Michigan Metric Coordinates 1983."
    a. three Lambert conic conformal projections
    b. one oblique Mercator projection

In separate, but concurrent action, it is recommended that a "Michigan Survey Authority" be established within an existing state agency. The responsibilities of the authority would be:

1. to serve as a central repository for geodetic control information and section corner location data.

2. to provide a funding mechanism for the establishment and maintenance of geodetic control.

3. to standardize surveying procedures for geodetic control.

4. to publish and enforce accuracy specifications for geodetic control surveys.

5. to conduct training seminars for other state agencies on the use of Michigan Metric Coordinates 1983.

6. to disseminate geodetic control data and section corner data to users within state government, federal agencies, county and municipal governments, utility companies, and the survey, engineering, and planning professions.

Included in Appendix D, "Model Law for Michigan Metric Coordinates 1983," is a model law developed by the National Geodetic Survey and adapted to the State of Michigan by the author. The model law incorporates use of the 1983 NAD and metric units. It presumes use of three Lambert conic conformal projections with the standard parallels of each zone located at the same geodetic latitude as the Michigan Coordinate System of 1927. The elevated reference surface has been eliminated and all scale factors will be computed on the ellipsoid. (See Appendix C "The Michigan Scale Factor.") The latitude of the origin for the "Y" coordinate has not been changed; however, the location of the central meridian in the south and central zones has been moved slightly to coincide more closely with the Michigan Principal Meridian [6]. The false easting on the central meridian of each zone was chosen such that:

1. the X & Y coordinates are distinguishable by inspection of their magnitudes.

2. there is no overlap of 1983 datum coordinates with the 1927 datum coordinates.

3. there is no overlap of coordinate pairs in the three zones.

The defining parameters, computed projection constants, and transformation formulas for the Michigan Metric Coordinates 1983 are listed in

Appendix E, "Parameters, Constants, and Formulas for Michigan Metric Coordinates 1983." The direct formulas are closed-form, giving an exact solution; however, the inverse transformation formulas contain a power series developed by Adams [1]. Any inverse transformation should be tested with the direct computation to confirm that the 0.0001 meter criterion established by NGS [17] for the transformations has been met.

If it is decided to use an oblique Mercator projection, a new model law, projection constants, and transformation formulas will have to be published.

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REFERENCES

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May 12, 1986

Mr. James Stem
National Geodetic Survey
Rockville, MD 20852

Dear Mr. Stem:

The proposed amendments to Michigan Act 9, P.A. 1964, have finally been released by the Legislative Service Bureau and were introduced in the House of Representatives on April 23, 1986 and referred to the House Committee on State Affairs. A copy of the bill, HB 5518, is enclosed for your perusal.

Currently the Legislature is debating appropriations and liability insurance, and it appears doubtful anything can be accomplished on this piece of legislation prior to the Summer recess, and as this is an election year, it will be early Fall probably before they reconvene. This is the second year of a biennial session, and should the legislation die in Committee, we will arrange to have it introduced again as soon as possible in the next session.

If you have any questions or comments, please do not hesitate to contact us.

Very truly yours,

MICHIGAN SOCIETY OF REGISTERED LAND SURVEYORS

Mary Lou Conlin
Administrative Secretary

cc: RELomax
RBurtch
PBLapham
EFBurkholder

ASCE

EXTRA PAGE...
- ADDED SEVERAL YEARS LATER
- DOCUMENTS ACTION BY MICHIGAN LEGISLATURE.

APPENDICES

APPENDIX A

TABULATION OF QUESTIONNAIRE RESPONSES FROM MEMBERS OF THE MICHIGAN SOCIETY OF REGISTERED LAND SURVEYORS

In August, 1979, a questionnaire was sent to all members of the Michigan Society of Registered Land Surveyors (M.S.R.L.S.) in an effort to determine:

1. the level of usage of the Michigan State Plane Coordinate System (MSPCS).
2. obstacles to greater use of the MSPCS.
3. changes which should be considered in a revision to the Michigan Coordinate System Law.
4. preference for a projection which should, or should not, be used in a metric projection utilizing the North American Datum of 1983.

The members of M.S.R.L.S. are persons who are licensed to practice Land Surveying in the State of Michigan. However, not all persons licensed by the state as Land Surveyors are members of M.S.R.L.S. In August, 1979 there were 991 Land Surveyors licensed to practice in the State of Michigan. Of that number, 692 were members of M.S.R.L.S. and received a copy of the questionnaire.

There were 50 responses received to the questionnaire which was sent to M.S.R.L.S. members. A summary of those responses is shown in the following pages.

MICHIGAN SOCIETY OF REGISTERED LAND SURVEYORS

(300 West Grand River Avenue, Suite E, Lansing)
Mailing Address: P.O. Box 11104, Lansing, MI 48901
Phone 517 484-2413

August - 1979

TO: Members of M.S.R.L.S.

FROM: Earl F. Burkholder, Chairman, Plane Coordinate Committee

RE: Questionnaire on Michigan Coordinate System

I have been registered as a Land Surveyor in Michigan since 1974 and have been active on the Plane Coordinate Committee since its inception in 1975. In August, 1978, I enrolled in the Graduate School of Purdue University and plan to obtain a Master's Degree in Geodesy and Surveying in May, 1980.

For my thesis project I am studying the mapping projections used for the state plane coordinate systems, the one for Michigan in particular. I would like to get information from MSRLS members, so I asked the MSRLS Board of Directors for permission to send you the enclosed questionnaire. I need information on:

-- the level of use of the existing system
-- problems encountered in using the existing system
-- features you would like to see in a new metric projection
-- any other items you think should be included in a revision to the existing Michigan Coordinate System statute

The existing Michigan Coordinate System is defined in feet units on three Lambert Conformal Projections of the Clarke Spheroid of 1866 (modified), and referenced to the North American Datum of 1927. However, the National Geodetic Survey (NGS) is preparing a readjustment of the horizontal control network which will be known as the North American Datum of 1983. According to a policy statement published by the NGS in the Federal Register, Volume 42, No. 57, March 24, 1977, pages 15913 to 15914, they will publish plane coordinates in two different systems, the UTM system and a metric version of the existing state plane coordinate systems. They will wait, however, until 1982 to publish new constants for the metric system to allow sufficient time for the individual states to amend their existing plane coordinate legislation. This provides an opportunity for Michigan surveyors to have a voice in determining which system is adopted for the State of Michigan. My goal is to gather information from interested professionals, evaluate your concerns, and to propose a system to be adopted.

I would like to have replies from as many Michigan surveyors as possible, even if you answer only items 2 & 3 (item 1 is optional). Please read the questionnaire very carefully and answer the questions by placing checks in the appropriate spaces. Also feel free to include additional comments, adding extra pages if necessary.

Please return your questionnaire by September 15, 1979 (or as soon as possible) to: Earl F. Burkholder, R.L.S., Civil Engineering Dept. Purdue University, West Lafayette, Indiana 47907.

Thank you for your participation and assistance.

MICHIGAN COORDINATE SYSTEM QUESTIONNAIRE

1. (optional) Name:____________________Address:________________________

   Firm:____________________Position:________________________

2. Member of: M.S.R.L.S.______ A.S.C.E.______ M.S.P.E.______
   A.C.S.M.______ N.S.P.E.______ Other________

3. Extent that you have used the existing Michigan Coordinate System.

   Never______ Once or twice______ Occasionally______
   Usually______ Frequently ______ Always (almost)______

4. What features of the existing system have been an obstacle to <u>your</u> use of it?

   _____ Lack of request or acceptance by client or employer
   _____ Lack of instrumentation to meet accuracy specifications
   _____ Sea level reduction to "800 foot elevation"
   _____ Inconvenience caused by crossing zone boundaries
   _____ Confusion on determination of proper grid factor or scale factor
   _____ Lack of familarity in the use of the system
   _____ Inadequate number of control points in the area
   _____ Difficulty obtaining control data from others
   _____ Acceptance and/or use of your data by others
   _____ Lack of central data repository
   _____ Other

5. What features would you like to see included in a revision to the existing Michigan Coordinate System Law?

   _____ Use of the North American Datum of 1983
   _____ Use of metric (SI) units
   _____ Projection defined at sea level rather than at present 800 foot level
   _____ Accuracy specifications stated more specifically
   _____ Greater weight assigned to state plane coordinates as evidence
   _____ Elimination of scale factors (impossible if angles are to be preserved)
   _____ Reduction in the number of zones within the state
   _____ Specification of penalty for improper use and/or publication of data
   _____ Comments

6. On a scale of 1 (indifferent) to 10 (very strongly), how strongly do you feel as to which projection should, or should not, be used for a metric system? Place a number in each space. For example, if you have no preference either way, place the number 1 in each space.

Should be used		Should <u>not</u> be used
________	Transverse Mercator	________
________	Lambert Conic Conformal	________
________	Oblique Mercator	________
________	UTM with 6° zones	________
________	UTM with 2° zones	________

Comments and/or reasons for your preferences:

Summary of M.S.R.L.S. Questionnaire Responses

Total number of Michigan Surveyors - August, 1979		991
Total members of M.S.R.L.S. - August, 1979		692
Total number of responses to questionnaire		50
From M.S.R.L.S. members	47	
From M.S.R.L.S. associate members	2	
From licensed non-member	1	

MEMBER OF:	NUMBER OF RESPONSES	% OF TOTAL
M.S.R.L.S.	49	98
A.S.C.E.	7	14
M.S.P.E.	9	18
A.C.S.M.	26	52
N.S.P.E.	10	20
Other	12	24

LEVEL OF USE	NUMBER OF RESPONSES	% OF TOTAL
Never	26	52
Once or twice	10	20
Occasionally	9	18
Usually	0	0
Frequently	3	6
Always (almost)	2	4

OBSTACLES TO GREATER USE	NUMBER OF RESPONSES	% OF TOTAL
Lack of request or acceptance by client or employer	34	68
Lack of instrumentation to meet accuracy specifications	3	6
Sea level reduction to "800 foot elevation"	2	4
Inconvenience caused by crossing zone boundaries	0	0
Confusion on determination of proper grid factor or scale factor	4	8
Lack of familarity in use of the system	27	54
Inadequate number of control points in the area	32	64
Difficulty obtaining control data from others	5	10
Acceptance and/or use of your data by others	8	16
Lack of central data repository	11	22
Other (comments)	11	22

DESIRED CHANGES	NUMBER OF RESPONSES	% OF TOTAL
Use of the North American Datum of 1983	11	22
Use of metric (SI) units	6	12
Projection defined at sea level rather than at the present 800 foot elevation	14	28
Accuracy specifications stated more specifically	11	22
Greater weight assigned to state plane coordinates as evidence	23	46
Elimination of scale factors (impossible if angles are to be preserved)	1	2
Reduction in the number of zones within the state	1	2
Specification of penalty for improper use and/or publication of data	5	10
Other (comments)	11	22

PREFERENCE FOR MAP PROJECTION (should be used)

PROJECTION	NUMBER OF RESPONSES	SUM OF RESPONSES	MEAN FOR CATAGORY
Transverse Mercator	20	74	3.70
Lambert Conic Conformal	21	98	4.67
Oblique Mercator	18	21	1.17
UTM with 6^o zones	19	38	2.00
UTM with 2^o zones	21	78	3.71

PREFERENCE FOR MAP PROJECTION (should NOT be used)

PROJECTION	NUMBER OF RESPONSES	SUM OF RESPONSES	MEAN FOR CATAGORY
Transverse Mercator	14	41	2.93
Lambert Conic Conformal	12	16	1.33
Oblique Mercator	17	53	3.12
UTM with 6^o zones	15	42	2.80
UTM with 2^o zones	13	35	2.69

BREAKDOWN OF RESPONSES BY LEVEL OF USE

OBSTACLES TO GREATER USE	Never	Once or twice	Occasionally	Frequently	Always (almost)	Total
- Lack of request or acceptance by client or employer	19	9	5		1	34
- Lack of instrumentation to meet accuracy specifications	2		1			3
- Sea level reduction to "800 foot elevation"				1	1	2
- Inconvenience caused by crossing zone boundaries						0
- Confusion on determination of proper grid factor or scale factor	1	2	1			4
- Lack of familarity in the use of the system	19	5	3			27
- Inadequate number of control points in the area	16	6	7	2	1	32
- Difficulty obtaining control data from others	1	2	1		1	5
- Acceptance and/or use of your data by others	4	2	2			8
- Lack of central data repository	3	2	4	1	1	11
- Other (comments)	3	2	3	1	2	11

DESIRED CHANGES	Never	Once or twice	Occasionally	Frequently	Always (almost)	Total
- Use of the North American Datum of 1983	2	2	4	2	1	11
- Use of metric (SI) units	3	1	2			6
- Projection defined at sea level rather than at present 800 foot elevation	7	1	2	3	1	14
- Accuracy specifications stated more specifically	3	2	4	1	1	11
- Greater weight assigned to state plane coordinates as evidence	6	4	8	3	2	23
- Elimination of scale factor (impossible if angles are to be preserved)	1					1
- Reduction of number of zones within the state	1					1
- Specification of penalty for improper use and/or publication of data	3			2		5
- Other (comments)	4	1	4	1	1	11

COMMENTS IN RESPONSES TO M.S.R.L.S. QUESTIONNAIRE

Comments From Those Who Have Never Used the Michigan State Plane Coordinate System

Obstacles to Use:

Our firm is engaged in Civil Engineering, Land Surveying and other services in the _______ area of Michigan. In the past 25 years we have never had a request to utilize the coordinate system. To do so without direct instruction would place an additional and unnecessary cost upon the client. For this reason we have become a little rusty on the use and necessity of the system. Act 74, Michigan P.A., 1979 (sic) has proved to be a great success in the perpetuation of government control corners and I am sure that the establishment of a practical coordinate system will be of equal benefit to land surveyors and those who utilize their services.

I am unaware of what state plane coordinate data is available.

A majority of surveys performed are not large enough to warrant the additional expense incurred to tie into existing control.

All section corners should be remonumented and tied to the state plane coordinates.

We have never used the state plane coordinate system as we never have been in a position where it would be advantageous to us. We therefore are unfamilar with the system.

For small surveys it does not become feasible to use the state plane coordinate system. When counties are on a remonumentation program, assign the state plane coordinates. The result would be greater use statewide.

Changes Which Should be Made

A revision to the existing law should include a provision for local enforcement so clients understand the need to pay extra.

I think the state plane coordinates should be required by state law for large projects.

I think we should change from the survey foot to the standard foot if we use feet.

Preferences On Map Projections

I have no preference for which map projection is used because I am not familar with the state plane coordinate system.

I am not familar enough with the different map projection systems to state a preference.

The U.S.G.S. quad maps have Lambert and 2° UTM grid lines on the margins. It would outdate existing maps to change to anything else. (Note: The map boundaries and grid ticks on all USGS maps based on the 1927 datum will be outdated by the NAD 1983.)

I do not know enough about practical use and applications of different map projections for our type of work.

In the conduct of a professional Land Surveying business in the urban area for private clients, the difference in map projections is meaningless.

Comments From Those Who Have Used the
Michigan State Plane Coordinate System Once or Twice

Obstacles to Use:

Once a control point is found the vegetation growth over the years makes it difficult to use the point.

If every section corner had state plane coordinates on it, the system would be used much more.

Changes Which Should be Made:

Control points should be established at closer intervals and laws enacted pertaining to specific use of the system as to subdivisions, sections, etc.

Preferences On Map Projections:

I have not used it enough to make a comment.

I don't care which projection is used, just so there is a standard system to be useful for everyone.

Comments From Those Who Have Occasionally Used the Michigan State Plane Coordinate System

Obstacles to Use:

An obstacle is the excessive cost to the individual client in obtaining control.

The extra cost which the client is not willing to pay in light of competition from other surveyors in the area who are not familar with the use of the system curtails its use.

Control monuments must be more readily available and education in the use of the state plane coordinate system must be achieved before the benefits of using the system can be realized. We have seen very little use of the system on subdivisions. It appears that most either lack the knowledge of use or are hampered by the scarcity of information. The workshops are based on the assumption that all have knowledge of use. It is hoped that general education as to the "hows and whys" could be made available. I realize there is no way surveyors can be forced to obtain this, but I believe it should be made available. By accepting and using it, we could provide for future mapping of descriptions, thus providing more of a professional service than just a technical one. If a local system would be required to conform to the state system it would further acceptance and use.

Changes Which Should be Made:

I am against using the metric system as this will mess up the present feet scale.

The MSPCS should be used as a data base for the Modernization of Land Data Systems (MOLDS). Then each surveyed parcel could enjoy absolute location and description.

Leave it as it is because I trust Professor Ralph M. Berry.

Preferences On Map Projections:

I would rather go back to the Transverse Mercator Projection because I am more familar with it.

The Lambert system works very well for Michigan. A change would mean more confusion for surveyors who have enough trouble understanding the present system. My second choice would be the UTM system for uniformity.

Comments From Those Who Frequently or Always Use the Michigan State Plane Coordinate System

Obstacles to Use:

Without adequate control monuments in an area, the system is useless.

We need a central data repository for work done by ourselves as well as others. I am also concerned with the quality of work done by others. We need accepted standards and specifications.

More training is needed to fully implement any system into standard practice.

Changes Which Should be Made:

You should investigate the deletion of the 1½ mile requirement for control. Most private surveyors now have the equipment to accurately run control for 5 to 15 miles or more.

Requirements to tie surveys to the state plane coordinate system should be established.

Preferences For Map Projections:

Michigan's upper peninsula is best adapted to the Lambert Projection while the lower peninsula adapts well to the Transverse Mercator Projection. Other states use such a mixture of systems. The UTM is available, but most cumbersome to use. Since most firms have their own or time-sharing computers to use, any grid system can be used if the control monuments and geodetic coordinate data are available. The state of the art has the ability to use grids far more than is currently in practice, but the level of understanding is not ready to take advantage of them.

More and more surveyors are becoming comfortable and/or familar with the present system. Major revisions will damage acceptance and use of the Michigan State Plane Coordinate System.

APPENDIX B

TABULATION OF CONTACTS AND SUMMARY OF RESPONSES FROM STATE AGENCIES, COLLEGES AND UNIVERSITIES, UTILITY COMPANIES, AND RELATED PROFESSIONALS

Letters of inquiry on the possible use of the Michigan State Plane Coordinate System (MSPCS) were sent to various Michigan state agencies, colleges and universities, utility companies, and related professionals who might share a concern in the use of the MSPCS. A typical letter of inquiry is shown on page 61, a list of those persons contacted is given beginning on page 63, and a summary of the individual replies begins on page 65.

MICHIGAN SOCIETY OF REGISTERED LAND SURVEYORS

(300 West Grand River Avenue, Suite E, Lansing)
Mailing Address: P.O. Box 11104, Lansing, MI 48901
Phone 517 484-2413

(Date)

(Name and Title)
(Organization)
(Address)

RE: THE MICHIGAN STATE PLANE COORDINATE SYSTEM

Dear Sir;

I am writing to request information relating to the use of the Michigan Coordinate System within (your organization) and to solicit your assistance in a study I am making of a metric map projection for the State of Michigan.

I have been a member of the Michigan Society of Registered Land Surveyors since 1974 and have been active on our "Plane Coordinate Committee" since it was formed in 1975 to represent M.S.R.L.S. on a study committee organized by the Michigan Department of Transportation to study the "Benefits to the State of Michigan of a Statewide Coordinate System." In August, 1978, I enrolled in the Graduate School of Purdue University and plan to obtain a Master's Degree in Geodesy and Surveying in May, 1980. The study I am doing on map projections for state plane coordinate systems is for my Master's Degree thesis.

As you may be aware, the National Geodetis Survey (N.G.S.) is preparing a readjustment of the national geodetic control network to be known as the North American Datum of 1983. When the adjustment is completed, the plane coordinates of the horizontal control points will be published by the N.G.S. in metric units (Federal Register, Vol. 42, No. 57, March 24, 1977, pages 15913 to 15914). Since the Michigan Coordinate System law will have to be revised to accommodate the change, the scope of my thesis includes a study of those changes required to make the transition as efficiently as possible and to propose a projection which will meet the needs of the users.

It is my goal to contact users of the Michigan Coordinate System, to ask them what features of the existing system have been obstacles to effective utilization of it, and to provide an opportunity for all concerned to express what they think should be included in a revision to the Michigan Coordinate System law. I have already circulated a questionnaire to all members of M.S.R.L.S. and am now contacting various state agencies, utility companies, corporations, and related professionals who may be active or potential users.

(Name)
Michigan Coordinate System
Page 2

Is it possible that you, or that someone in (your organization) is well versed in the use of the existing Michigan Coordinate System who might also be interested in whatever metric system is adopted? If so, would you be willing to describe in what way the Michigan Coordinate System is being used and how its use may have benefitted you? Additionally, I would like you to list any features of the existing projections or of the statute itself which have caused problems or have been an obstacle to your use of the Michigan Coordinate System. Since the law will have to be revised to accommodate the change, it could be an opportunity to make other changes as well. What changes to the existing Michigan Coordinate System law would be beneficial to your use of the Michigan State Plane Coordinate System?

I know I have asked for information which may take some time to compile, but since your input could have a significant impact on the outcome of my study, I trust you will give my request serious consideration. If you are willing to help, would two weeks be sufficient time to compile a detailed reply?

Thank you for your interest and consideration.

Yours truly,

Earl F. Burkholder

Earl F. Burkholder, RLS, PE
Chairman, Plane Coordinate Committee

Mailing Address:

Earl F. Burkholder, RLS, PE
Department of Civil Engineering
Purdue University
West Lafayette, Indiana 47907

EFB/efb

LIST OF SPECIFIC CONTACTS

1. Mr. John P. Woodford, Director
   Michigan Department of Transportation

2. Mr. Howard A. Tanner, Director
   Michigan Department of Natural Resources

3. Environmental Research Institute of Michigan
   Ann Arbor, Michigan

4. Dr. William C. Taylor, Chairman
   Department of Civil Engineering, Michigan State University

5. Dr. Larry W. Tombaugh, Chairman
   Department of Forestry, Michigan State University

6. Dr. William J. Johnson, Dean
   School of Natural Resources, University of Michigan

7. Professor Robert Hanson, Director
   Department of Civil Engineering, University of Michigan

8. Professor Tapan K. Datta, Chairman
   Department of Civil Engineering, Wayne State University

9. Dr. Gorden T. Krueter, Head
   Department of Civil Engineering, Michigan Technological Univ.

10. Professor Gene A. Hesterberg, Head
    Forestry Department, Michigan Technological University

11. Mr. James B. Shane, AIA, Head
    Construction Department, Ferris State University

12. Professor Jarl Roine, Head
    Department of Geography, Earth Science, and Conservation
    Northern Michigan University

13. Dr. Wayne Kiefer, Chairman
    Department of Geography, Central Michigan University

14. Professor Elwood Kureth, Head
    Department of Geography, Eastern Michigan University

15. Dr. Joseph Stoltman, Chairman
    Department of Geography, Western Michigan University

16. Dr. John D. Nystuen, Chairman
    Department of Geography, University of Michigan

17. Professor Robert D. Swartz, Chairman
Department of Geography, Wayne State University

18. Dr. Gary Manson, Chairman
Department of Geography, Michigan State University

19. Mr. Loren Green, State Treasurer
Michigan Department of Treasury

20. Mr. Patrick C. Babcock, Director
Michigan Department of Labor

21. Mr. Gus Harrison, Commissioner
Michigan Bureau of State Lottery

22. Dr. Maurice S. Reizen, Director
Michigan Department of Public Health

23. Mr. Dean M. Pridgeon, Director
Michigan Department of Agriculture

24. Mr. Richard H. Austin, Secretary
Michigan Department of State

25. Colonel Gerald L. Hough, Director
Michigan Department of State Police

26. Mr. William F. McLaughlin, Director
Michigan Department of Commerce

27. Mr. John T. Dempsey, Director
Michigan Department of Social Services

28. Mr. Walter Williams, Engineer of Surveys
City of Detroit

29. Mr. Don Winningham, Cartographic Unit
Detroit Edison Company

30. Mr. Wayne Potter, Civil Engineering Department
Consumers Power Company

31. Abrams Aerial Survey Corporation
Lansing, Michigan

32. Michigan Society of Planning Officials
Detroit, Michigan

SUMMARY OF RESPONSES FROM STATE AGENCIES

Mr. John P. Woodford, Director — October 15, 1979
Michigan Department of Transportation (MDOT)

Response by: Mr. Doug Hooth, Survey Supervising Engineer
November 7, 1979 Signed by: Mr. Woodford

When existing monumentation is sufficiently available the Design Division of the MDOT uses the MSPCS as a data base for plan development of proposed improvements to existing and proposed truck lines. If Second Order monumentation is not available, the design is based on "Project Datum" coordinates. Mr. Hooth states, "In either case, the coordinate datum system established during the design survey phase is the system used in subsequent phases of highway development such as serial mapping, construction alignment computations, right of way line locations and monumentation, and construction layout survey."

The MDOT also uses the MSPCS in their county mapping program, environmental studies data filing and land use filing systems. In addition, the land use information for the Great Lakes Coastal Areas is provided to the MDNR by the MDOT using the MSPCS based on the U.S.G.S. quad system.

The MDOT has made significant commitments in the use of the MSPCS and supports use of the Lambert Conformal Projection as designed by Professor Ralph M. Berry for the reasons given at the time the present law was passed in 1964. Mr. Hooth states, "Any proposed changes in use of map projections for the state should be carefully reviewed to determine whether substantial benefits will result."

As a closing comment, Mr. Hooth points out that the existing Michigan Coordinate System Act failed to provide funding for densification of survey control or to establish an authority within an existing state department for that purpose.

Mr. Howard A. Tanner, Director — October 15, 1979
Michigan Department of Natural Resources

Although several telephone discussions were held with Mr. Ronald Webster, no formal response has been received as of March 1, 1980.

Mr. Dean M. Pridgeon, Director
Michigan Department of Agriculture

November 12, 1979

Response by: Mr. Dean Pridgeon, Director
November 26, 1979

At present time no one in department is using the MSPCS. If a need arises, assistance will be obtained from the MDOT.

Mr. Loren Green, State Treasurer
Michigan Department of Treasury

November 12, 1979

Response by: Mr. Richard Lomax, Manager
January 28, 1980 Plat Section

The Treasury Department does not currently utilize the MSPCS although a number of Divisions could use it for subdivisions, land parceling, checking exemptions, and tax maps and boundaries.

The basic problem is a lack of funding to support a central mapping agency similar to the one in Missouri or Arkansas.

Mr. Patrick C. Babcock, Director
Michigan Department of Labor

November 12, 1979

Response by: Mr. Bill B. Moyer, Acting Director
December 11, 1979

The Labor Department has not utilized the MSPCS as we are unaware of any function within the department which would benefit from using the system.

Mr. Gus Harrison, Commissioner
Michigan Bureau of State Lottery

November 12, 1979

Response by: Mr. Laurence R. Curtis, Marketing Manager
November 26, 1979

The distribution of lottery tickets is handled through the bank courier system. Winning ticket information is coded by county and city and stored in our computers. This procedure has proven to be quite efficient for us.

Dr. Maurice S. Reizen, Director
Michigan Department of Public Health

November 12, 1979

No response received as of March 1, 1980.

Mr. Richard H. Austin, Secretary
Michigan Department of State

November 12, 1979

Response by: December 3, 1979 — Mr. LeRoy Barnett, Reference Archivist

None of the divisions within the Department of State are using the MSPCS. In fact, according to contacts within state government, the only Departments using the MSPCS are the MDOT and the MDNR.

Colonel Gerald L. Hough, Director
Michigan Department of State Police

October 15, 1979

Response by: December 5, 1979 — Mr. Richard A. Groop, F/LIEUTENANT
Acting Commanding Officer
Executive Division

We do not use the MSPCS, but we have developed the Michigan Accident Location Index (MALI) which is a county based system utilizing highway intersections and their connecting roads as a means of locating traffic accidents.

Mr. William F. McLaughlin, Director
Michigan Department of Commerce

November 12, 1979

Response by: December 4, 1979 — Mr. Wayne L. Workman, Director
Office of Economic Development

The MSPCS is not used by the MDOC. However, we are interested in possible future applications.

Mr. John T. Dempsey, Director
Michigan Department of Social Services

November 12, 1979

Response by: December 7, 1979 — Mr. John T. Dempsey, Director

We do not use the MSPCS because it is oriented primarily towards Land Surveying and descriptions and does not appear to offer the type of information we need for social services. However, the potential of using a statewide data base for planning and management activities involving location of events and objects does offer intriguing possibilities. Please keep us informed of developments in this area.

SUMMARY OF RESPONSES FROM COLLEGES AND UNIVERSITIES

Mr. James B. Shane, AIA, Head — October 15, 1979
Construction Department, Ferris State College

Response by: Professor Jens Otto Rick, Director
October 24, 1979 Surveying Program

A two page reply was summarized by Professor Rick as follows: "The current Lambert Conic Conformal Projection as designed by Professor Ralph M. Berry offers many tailored features which provide for convenient coordinate use by surveyors and engineers. A revision of this coordinate system should consider seriously the alternate advantages. Michigan experienced a new coordinate system adoption in 1965 (Transverse Mercator to Lambert Conic) which is confusing to many users today. Mapping and civil engineering projects initiated prior to 1965 have these older coordinates which frustrate users familar only with the present Lambert coordinates. Introducing a third system would compound an already delicate situation. Conversion to the metric values of X and Y coordinates may alter some of the intrinsic advantages to the present system, but this does not change the basic zone, size, and orientation or the use of the Michigan Spheroid.

Professor Gene A. Hesterberg, Head — October 15, 1979
Department of Forestry, Michigan Technological University

Response by: Professor Charles E. Hein
October 30, 1979

The Land Surveying program here is still in its infancy so the MSPCS is not presently being used.

Dr. William C. Taylor, Chairman — October 15, 1979
Department of Civil Engineering, Michigan State University

Response by: Professor Leo V. Nothstine
February 7, 1980

We would like to have state plane coordinates of visible points available so students in the second surveying course could do a "remote point coordinate determination" using their own observations. Mr. Nothstine also states, "I'm on a committee that has the objective to get all the section corners in Ingham County recorded with ties and hopefully coordinates. This is a CETA program. I doubt if the coordinates shall be finally obtained, but could have been readily if our state plane system was adequately available."

Dr. Larry W. Tombaugh, Chairman October 15, 1979
Department of Forestry, Michigan State University

Response by: Dr. Tombaugh
October 29, 1979

"We do not make enough use of the Michigan Coordinate System . . . . to be of help in your survey."

Dr. William J. Johnson, Dean October 15, 1979
School of Natural Resources, University of Michigan

Response by: Professor Charles E. Olsen, Jr.
December 14, 1979

The Michigan Coordinate System is not being used currently. However, the UTM grid is used as a base for land use mapping. Since the State of Michigan falls into several UTM zones, a metric coordinate system with no seams or boundaries would be beneficial.

Professor Robert Hanson, Director October 15, 1979
Department of Civil Engineering, University of Michigan

Response by: Professor Hanson
November 7, 1979

The University of Michigan no longer offers the Geodesy program and does not have a faculty member who could contribute to a metric projection system project.

Professor Tapan K. Datta, Chairman October 15, 1979
Department of Civil Engineering, Wayne State University

No response received as of March 1, 1980.

Dr. Gorden T. Krueter, Head October 15, 1979
Department of Civil Engineering, Michigan Technological University

Response by: O. D. Boutilier
October 26, 1979 Administrative Assistant

No one in the Department of Civil Engineering has been involved with the Michigan Coordinate System. Contact the Department of Forestry and/or Mr. Collins in the Department of Civil Technology.

Dr. Wayne Kiefer, Chairman October 15, 1979
Department of Geography, Central Michigan University

No response received as of March 1, 1980.

Professor Elwood Kurth, Head October 15, 1979
Department of Geography, Eastern Michigan University

Response by: Dr. Laurence Ogden, Professor of Geology
January 24, 1980

We have no use for the MSPCS, have had no experience with it, and feel there is no input we can provide to your study of it.

Professor Jarl Roine, Head October 15, 1979
Department of Geography, Earth Science, and Conservation
Northern Michigan University

Response by: Professor J. Patrick Farrell
January 23, 1980

The state plane coordinates are taught in a course called, "Maps: Reading, Analysis and Interpretation." It is important for students to understand and to be able to use the numbers appearing on the topographical maps. We have approximately 150 students exposed to the state plane coordinate system each year.

Another use of the MSPCS is in the area of Planning and computer digitized location of features within a given political jurisdiction.

There are advantages to using the metric system, but it will be unfortunate if the present system of eastings and northings in feet is abolished.

Dr. Joseph Stoltman, Chairman October 15, 1979
Department of Geography, Western Michigan University

Response by: Professor Thomas W. Hodler
October 29, 1979

We use the MSPCS as well as the UTM system as an instructional aide in our cartographic classes for locating points and areas of interest on the 7½ minute and the 15 minute USGS topographical maps.

I also use the system for Geocoding various point and area data into a computerized format. One example is inventorying utility poles and hardware for a small utility company in Indiana.

It would be convenient to have one metric system covering the contiguous 48 states.

Professor Robert D. Swartz, Chairman October 15, 1979
Department of Geography, Wayne State University

No response received as of March 1, 1980.

Dr. John D. Nystuen, Chairman — October 15, 1979
Department of Geography, University of Michigan

Response by: Dr. John D. Nystuen
January 24, 1980

We have used the MSPCS in geographical research associated with the State of Michigan. The principal use has been to employ the system as X & Y coordinates in a computer data base where geographical locations and relative positions were important in the analysis. The existing system is adequate for the accuracies required in this type of work.

Dr. Gary Manson, Chairman — October 15, 1979
Department of Geography, Michigan State University

Response by: Professor Gary Manson

No one in our department is an expert with the MSPCS. We do a lot of thematic mapping, but geodesy is not one of our activities.

SUMMARY OF RESPONSES FROM RELATED PROFESSIONALS

Mr. Walter Williams, Engineer of Surveys — October 22, 1979
City of Detroit

No response received as of March 1, 1980.

Mr. Don Wimmingham, Cartographic Unit — October 22, 1979
Detroit Edison Company

No response received as of March 1, 1980.

Abrams Aerial Survey — January 18, 1980
Lansing, Michigan

No response received as of March 1, 1980.

Environmental Research Institute of Michigan — October 15, 1979
Ann Arobr, Michigan

No response received as of March 1, 1980.

Michigan Society of Planning Officials
Detroit, Michigan

January 30, 1980

Response by: Mr. Terry L. Jerrens, Executive Director
February 25, 1980

The Michigan Society of Planning Officials has not addressed the issue of use of the Michigan State Plane Coordinate System and has no policy on current or future use. Since the sophistication of local planning operations varies greatly, it may be more useful for you to contact the U.S. Bureau of the Census for the names of Michigan planning agencies which utilize the Duplicate Independent Mapping Encoding (DIME) File. Users of this system will be affected by changes in the Michigan State Plane Coordinate System.

Dr. Wayne D. Potter, Supervisory Civil Engineer
Consumers Power Company

October 22, 1979

Response by: Mr. Wayne D. Potter
November 15, 1979

Our use of the MSPCS has been limited due to little need for it and due to the scarcity of control monuments which makes it difficult and time-consuming to use the system. However, we have used the system on several power plant sites and on several transmission lines. In some cases the MSPCS is required on permit applications and in other cases the control monuments are available in the area of interest.

APPENDIX C

The Michigan Scale Factor

A Technical Paper Presented at

The 40th Annual Meeting of

The American Congress On Surveying & Mapping

Held at

St. Louis, Missouri, March 10-14, 1980

By

Earl F. Burkholder, RLS, PE

Department of Civil Engineering

Purdue University

West Lafayette, Indiana 47907

THE MICHIGAN SCALE FACTOR

Earl F. Burkholder, RLS, PE
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BIOGRAPHICAL SKETCH

Earl F. Burkholder is currently enrolled in the Graduate School of Purdue University where he plans to obtain a Master's Degree in Geodesy & Surveying in May, 1980. He graduated "cum laude" from the University of Michigan in 1973 with a Bachlor of Science Degree in Civil Engineering. From 1973 to 1978 he worked for an international consulting firm, Commonwealth Associates, Inc. of Jackson, Michigan. He has been registered as a Land Surveyor in Michigan, New York, Minnesota, North Dakota, and Nebraska and is registered as a Professional Engineer in Michigan. He is a member of the American Congress on Surveying and Mapping, the American Society of Civil Engineers, the American Society of Photogrammetry, and the Michigan Society of Registered Land Surveyors.

ABSTRACT

The Michigan Coordinate System, which utilizes three Lambert Conic Conformal Projections, was formally adopted by the Michigan Legislature in 1964. The projections are based on the Clarke Spheroid of 1866, modified to place the reference surface at an elevation of approximately 800 feet above sea level. This was done in an effort to eliminate the need for the sea level reduction on most surveys since most of the land surface in Michigan is not far from the 800 foot elevation. When the sea level reduction is eliminated, the grid distance becomes the product of the horizontal ground distance and the scale factor. However, if one computes the scale factor for a point in Michigan according to the formula found in C&GS Publication 62-4, "State Plane Coordinates by Automatic Data Processing", or if one uses a calculator or a computer programmed to use the same formula, the result is not the same as is found in the "Plane Coordinate Projection Tables" for the State of Michigan. The published formula gives a scale factor which is valid on the sea level reference surface while the Michigan projection tables give the correct scale factor for the reference surface at the 800 foot elevation. The formula can be easily modified to give the correct scale factor; however, either scale factor will work if used in conjunction with the proper sea level factor. A similar problem is encountered in the Michigan Lambert zones when one attempts to use the constant "L_5" from Publication 62-4 for "k_o" in the formulas for computing scale factors from state plane coordinates as given by Professor Ralph Moore Berry in 1972. Again, correct determination of the Michigan scale factor is assured by using the correct constants in the published formulas.

DEFINITIONS

Sea level factor - that factor by which a short horizontal ground distance is multiplied to determine the corresponding distance on a reference surface. In this paper two reference surfaces are considered, the surface of the Clarke Spheroid of 1866 and the Michigan Spheroid. The difference between them is approximately 800 feet in elevation.

Scale factor - that factor by which a distance on a reference surface (spheroid) is multiplied to determine the corresponding distance on the projection surface.

Grid factor - the product of the sea level factor and the scale factor. The grid factor is constant for a given spheroid, zone, elevation and location.

These definitions are intended to be consistent with, although not as inclusive as, the definitions for the same terms as found in, "Definitions of Surveying and Associated Terms", [7].

INTRODUCTION

The geometry of the distance reduction for both the sea level reference surface and the 800 foot elevation reference surface is shown in Figure C1 where the horizontal ground distance, D_1, is reduced to D_2 and D_2' by;

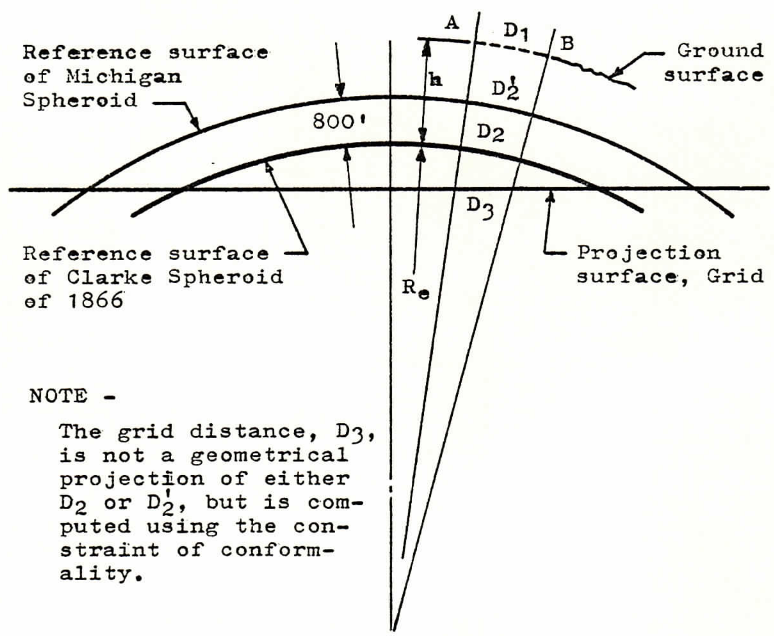

Figure No. C1 Distance Reduction from Ground to Grid

$$D_2 = D_1 * R_e/(R_e + h) \tag{1}$$

$$D_2' = D_1 * (R_e + 800)/(R_e + h). \tag{2}$$

The reduction of the distance on the reference surface to its corresponding grid distance, D_3, on the projection surface is given by;

$$D_3 = D_2 * SF_s = D_2' * SF_e \quad \text{where,} \tag{3}$$

- D_1 = the horizontal ground distance between the plumb lines at points "A" and "B".
- D_2 = the horizontal ground distance reduced to the surface of the Clarke Spheroid of 1866.
- D_2' = the horizontal ground distance reduced to Michigan Spheroid.
- R_e = the radius of curvature of the spheroid at a given latitude, often taken to be 20,906,000 feet, but see equations (10)&(13).
- h = the height of the horizontal ground distance above the Clarke Spheroid of 1866.
- D_3 = the grid distance on the projection surface.
- SF_s = the scale factor for a point on the sea level reference surface, Clarke's Spheroid.
- SF_e = the scale factor for a point on the elevated reference surface, the Mich. Spheroid.

In equation (1) the sea level factor is unity when $h = 0$, but in equation (2) the sea level factor is unity when $h = 800$ feet. The distance on the reference surface is either longer or shorter than the corresponding horizontal ground distance depending on whether the ground elevation is below or above the reference surface.

Substituting equations (1) and (2) into equation (3) gives;

$$D_3 = D_1 * \left(\frac{R_e}{R_e + h}\right) * SF_s = D_1 * \left(\frac{R_e + 800}{R_e + h}\right) * SF_e \tag{4}$$

from which the grid factor is determined by;

$$\frac{D_3}{D_1} = \left(\frac{R_e}{R_e + h} \right) * SF_s = \left(\frac{R_e + 800}{R_e + h} \right) * SF_e \tag{5}$$

and the ratio of the scale factors is given by;

$$SF_s/SF_e = (R_e + 800)/R_e. \tag{6}$$

However, in determining the size of the Michigan Spheroid, the ratio in equation (6) was held to be 1.0000382 exactly (page 1 of [10]). Hence equation (6) can be rewritten as;

$$SF_s = 1.00003820 * SF_e. \tag{7}$$

The purpose of this paper is to:

1. show that the scale factors obtained in Michigan by using the formula in Publication 62-4 (page 4 of [5]) differ from the scale factors listed in the projection tables by a factor of 1.00003820.

2. analyze the reason for the discrepancy.
3. demonstrate proper use of the Michigan scale factor.
4. show how scale factors in Michigan can be computed correctly from state plane coordinates using the formulas given by Berry [4].

BACKGROUND

The surface of the Earth is approximated by rotating an ellipse about its minor axis. The ellipse shown in Figure C2 represents a meridian section of the Earth. The size and shape of the Michigan Spheroid are specified by the length of the semi-major axis, a = 20,926,631.530789 American Survey feet, and the eccentricity, e = 0.08227 18542 23003 8 (page 1 of [10]). The location of a point on a meridian is specified by its latitude, ϕ, or by its co-latitude, $P = 90^{\circ} - \phi$. Other quantities which are derived from these are;

- b = the semi-minor axis of the ellipse.
- M = the radius of curvature in the meridian section.
- N = the radius of curvature in the prime vertical, perpendicular to the meridian section.
- R_e = the geometrical mean radius of curvature for the surface of the spheroid at a given latitude.

$$b^2 = a^2 * (1 - e^2)$$

$$M = a * (1 - e^2)/(1 - e^2 \sin^2\phi)^{3/2} \qquad (8)$$

$$N = a/(1 - e^2 \sin^2\phi)^{\frac{1}{2}} \qquad (9)$$

$$R_e = (M*N)^{\frac{1}{2}} = (a * (1-e^2)^{\frac{1}{2}})/(1-e^2 \sin^2\phi) \qquad (10)$$

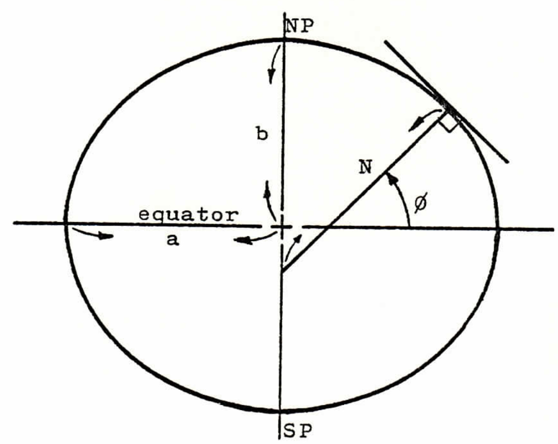

Figure No. C2 Elements in the Meridian Section Ellipse

The North American Datum of 1927 (NAD 1927) is the national reference for the horizontal control network which defines the coordinates used in the state plane coordinate systems. The NAD 1927 is, in turn, computed on the Clarke Spheroid of 1866. However, in an effort to simplify computations by eliminating the need for the sea level reduction on most surveys, the three Michigan Lambert projections were defined on a spheroid having a reference surface approximately 800 feet above the surface of the Clarke Spheroid of 1866. This was achieved by multiplying the semi-major axis of the Clarke Spheroid by 1.0000382 (exact) and holding the eccentricity unchanged [9] [10].

Table C1 shows a comparison of the ellipsoidal parameters and derived values for "b" and for "R_e" at the central parallel, ϕ_o, of each of the three Michigan zones. Although the values are listed for both spheroids to show the difference, the values of the Michigan Spheroid should be used in <u>all</u> state plane coordinate computations in Michigan.

TABLE C1 COMPARISON OF SPHEROIDS (Part 1)

	Clarke, 1866	Michigan	Ratio	Difference
a	20,925,832.16'	20,926,631.53'	1.00003820	799.37'
e	0.082271854223	0.082271854223	1.00000000	none
b	20,854,892.01'	20,855,688.67'	1.00003820	796.66'
South Zone, $\phi_o = 42^\circ\ 53'\ 06''.055446$				
R_e	20,920,471.45'	20,921,270.62'	1.00003820	799.17'
Central Zone, $\phi_o = 44^\circ\ 56'\ 36''.092428$				
R_e	20,925,571.01'	20,926,370.37'	1.00003820	799.36'
North Zone, $\phi_o = 46^\circ\ 17'\ 07''.101225$				
R_e	20,928,899.07'	20,929,698.56'	1.00003820	799.49'

ANALYSIS OF SCALE FACTOR FORMULA

The formula for the scale factor in a Lambert Zone as given by Claire in "State Plane Coordinates by Automatic Data Processing" (page 4 of [5]) is;

$$k = \frac{L_6 * R_m * (1 - 0.0067686580 * \sin^2\phi)^{\frac{1}{2}}}{20{,}925{,}832.16 * \cos\phi} \qquad (11)$$

which can be restated as;

$$k = \frac{L_6 * R_m * (1 - e^2 * \sin^2\phi)^{\frac{1}{2}}}{a * \cos\phi}\text{, where} \qquad (12)$$

L_6 = the projection constant, ℓ, computed from the basic equations for the Lambert projections with two standard parallels (page 11 of [12]).

R_m = the map radius of a given parallel. This value is tabulated for each minute of latitude in the projection tables.

ϕ = the latitude of a given point.

e^2 = 0.0067686580, the square of the eccentricity of both spheroids, Clarke 1866 and Michigan.

a = 20,925,832.16 feet, the semi-major axis of the Clarke Spheroid of 1866.

When the value of "a" of the Clarke Spheroid of 1866 is used in equation (12) instead of the value of "a" for the Michigan Spheroid, an erroneous scale factor is obtained. Since the ratio of the two values of "a" is 1.0000382, the resulting scale factors are of the same ratio. The correct scale factor is applicable to the reference surface at the 800 foot elevation and agrees with the scale factors in the projection tables [10]. By equation (7) the erroneous value of the scale factor is applicable to the sea level reference surface.

CHOOSING THE SEA LEVEL FACTOR

The sea level factor is determined from the elevation of a point and the radius of curvature of the spheroid at the same point. As mentioned earlier the value of R_e is often taken to be 20,906,000 feet, although it could be computed using equation (10). However, since the ratio in equation (6) was held to be 1.0000382 exactly, a value of R_e for the entire state is obtained by solving equation (6) for R_e.

$$R_e = 800/(1.0000382 - 1) = 20{,}942{,}400 \text{ feet} \qquad (13)$$

The values of R_e at the central parallel of each of the three Michigan zones are listed in Table 1. These values of R_e could also be used.

The grid factor is defined as the product of the scale factor and the sea level factor, but since the grid factor is also the ratio of the grid distance to the ground distance, it remains constant for a given location and elevation. Thus, it is a matter of choice whether one uses the correct scale factor at the 800 foot elevation with the elevated sea level factor, SLF_e,

$$SLF_e = \frac{R_e + 800}{R_e + h} = \frac{20{,}943{,}200}{20{,}942{,}400 + h} \qquad (14)$$

or if one uses the erroneous scale factor as computed by the formula in Publication 62-4 with the conventional sea level factor, SLF_s,

$$SLF_s = \frac{R_e}{R_e + h} = \frac{20{,}942{,}400}{20{,}942{,}400 + h}. \qquad (15)$$

Note that equation (14) reduces the horizontal ground distance to the 800 foot elevation and that equation (15) reduces the horizontal ground distance to sea level. Thus, when the ground elevation is acceptably near the 800 foot elevation, equation (14) becomes unity and the grid factor equals the correct scale factor.

EXAMPLE OF GRID FACTOR COMPUTATION

The following is an example of determining the grid factor in Michigan at a latitude of 45° and at an elevation of 1200 feet above mean sea level. The scale factor formula given in Publication 62-4 and restated in equation (12) is;

$$k = \frac{L_6 * R_m * (1 - e^2 * \sin^2\phi)^{1/2}}{a * \cos\phi}, \text{ where}$$

L_6	=	0.7064074100 (central zone)	page 7 of [10]
R_m	=	20,981,064.925	page 20 of [10]
e^2	=	0.006768657997	Table 1
a	=	20,926,631.53 (Michigan)	Table 1
a	=	20,925,832.16 (Clarke, 1866)	Table 1

Using the Michigan Spheroid value of "a", k is;

$$\frac{(0.7064074100)(20981064.925)(1-0.006768657997*\sin^2 45^\circ)^{1/2}}{20,926,631.53 * \cos 45^\circ}$$

$$k = \frac{14,796,078.60}{14,797,363.06} = 0.9999131966 \text{ (correct).} \qquad (16)$$

Using the Clarke Spheroid of 1866 value of "a", k is;

$$\frac{(0.7064074100)(20981064.925)(1-0.006768657997*\sin^2 45^\circ)^{1/2}}{20,925,832.16 * \cos 45^\circ}$$

$$k = \frac{14,796,078.60}{14,796,797.83} = 0.9999513933 \text{ (erroneous).} \qquad (17)$$

The elevated sea level factor by equation (14) is;

$$SLF_e = \frac{R_e + 800}{R_e + h} = \frac{20,943,200}{20,943,600} = 0.9999809011 \qquad (18)$$

and the conventional sea level factor by equation (15) is;

$$SLF_s = \frac{R_e}{R_e + h} = \frac{20,942,400}{20,943,600} = 0.9999427033 \qquad (19)$$

The grid factor, being the product of the scale factor and the appropriate sea level factor, can be obtained by using either scale factor. The correct scale factor at the 800 foot reference surface (16) times the elevated sea level factor (18) is;

$$GF = (0.9999131966)(0.9999809011) = 0.9998940994 \qquad (20)$$

and the erroneous scale factor (17) times the conventional sea level factor is;

$$GF = (0.9999513933)(0.9999427033) = 0.9998940994 \qquad (21)$$

Although the same grid factor was obtained separately using both scale factors, it will be shown in the next section that there is only one "correct" scale factor. It is recommended that the correct scale factor be used in all cases.

FURTHER ANALYSIS OF THE SCALE FACTOR FORMULA

The analysis of the scale factor formula is not complete until each term in the equation is examined and until it is assured that all spheroidal differences have been taken into account. Investigating equation (12) term by term, L_6 is derived by Thomas (page 117 of [12]) and shown by Berry [3] to be;

$$L_6 = \frac{\ln (N_s * \sin P_s / N_n * \sin P_n)}{\ln \tan(Z_s/2) - \ln \tan(Z_n/2)}, \quad \text{where} \tag{22}$$

N_s = the radius of curvature in the prime vertical at the south standard parallel.

N_n = the radius of curvature in the prime vertical at the north standard parallel.

P_s = the colatitude of the south standard parallel.

P_n = the colatitude of the north standard parallel.

Z_s = the conformal colatitude of the south standard parallel.

Z_n = the conformal colatitude of the north standard parallel.

The conformal colatitude which appears in the denominator of equation (22) is derived by Thomas (page 87 of [12]) as;

$$\tan(Z/2) = \tan(P/2) * \left[\frac{1 + e * \cos P}{1 - e * \cos P}\right]^{e/2}. \tag{23}$$

When the eccentricity, e, is held constant the conformal colatitude is independent of the semi-major axis, a, of the spheroid.

Considering the numerator of equation (22) and recalling from equation (9) that,

$$N = a/(1 - e^2 * \sin^2\phi)^{1/2} \tag{24}$$

the numerator of equation (22) becomes;

$$\ln\left[\frac{a * \sin P_s}{(1 - e^2 * \sin^2\phi_s)^{1/2}} \Big/ \frac{a * \sin P_n}{(1 - e^2 * \sin^2\phi_n)^{1/2}}\right]$$

$$= \ln\left[\frac{\sin P_s}{\sin P_n} * \frac{(1 - e^2 * \sin^2\phi_n)^{1/2}}{(1 - e^2 * \sin^2\phi_s)^{1/2}}\right] \tag{25}$$

which is also independent of the semi-major axis, "a", when the eccentricity is held constant. Since the numerator and the denominator of equation (22) are both independent of the semi-major axis of the spheroid, the term, L_6, is also independent of the semi-major axis when the eccentricity is held constant.

Next, consider the map radius, R_m, which according to Adams and Claire (page 6 of [2]) is;

$$R_m = K * (\tan Z/2)^{L_6} \tag{26}$$

where K is the map radius of the equator as shown by Berry [3] and is computed by;

$$K = \frac{N_s * \sin P_s}{L_6 * (\tan Z_s/2)^{L6}} = \frac{N_n * \sin P_n}{L_6 * (\tan Z_n/2)^{L6}}. \qquad (27)$$

Substituting for "N" from equation (9),

$$K = \frac{a * \sin P_s}{(1 - e^2 * \cos^2 P_s)^{1/2} * L_6 * (\tan Z_s/2)^{L6}}. \qquad (28)$$

Substituting equation (28) into equation (26),

$$R_m = \frac{a * \sin P_s * (\tan Z/2)^{L6}}{(1 - e^2 * \cos^2 P_s)^{1/2} * L_6 * (\tan Z_s/2)^{L6}}. \qquad (29)$$

Substituting equation (29) into equation (12), the scale factor becomes;

$$k = \frac{L_6 * a * \sin P_s * (\tan Z/2)^{L6} * (1 - e^2 \sin^2 \phi)^{\frac{1}{2}}}{L_6 * a * \cos \phi * (\tan Z_s/2)^{L6} * (1 - e^2 \cos^2 P_s)^{\frac{1}{2}}}$$

$$= \frac{\sin P_s * (\tan Z/2)^{L6} * (1 - e^2 \sin^2 \phi)^{1/2}}{\cos \phi * (\tan Z_s/2)^{L6} * (1 - e^2 \cos^2 P_s)^{1/2}} \qquad (30)$$

which is also independent of the semi-major axis for a given eccentricity. The scale factor depends only on the latitude of a given point once the eccentricity of the spheroid is determined and the location of the standard parallels is selected. Therefore, there is only one "correct" scale factor for a given latitude in a given Michigan Lambert zone. Although equation (30) is not very efficient for routine computation of the scale factor, the results from equation (30) are consistent with the scale factors published in the projection tables [10].

DETERMINATION OF THE MAPPING RADIUS, R_m

The value of the mapping radius can be computed directly using equation (26). However, eleven significant figures are required to get R_m correct to three decimal places of feet. Since the tenth significant figure of some calculators is not to be trusted, it is even more important to use another method to compute R_m. Rather than "looking it up in the tables", Adams and Claire [2] developed a way to compute the distance from the central parallel which is a much smaller number. This method is used by Claire in Publication 62-4 as;

$$R_m = L_3 + s\,L_5 \left[1 + \left(\frac{s}{10^8}\right)^2 \left[L_9 - \left(\frac{s}{10^8}\right) L_{10} + \left(\frac{s}{10^8}\right)^2 L_{11}\right]\right] \qquad (31)$$

where,

$$s = 101.2794065\left[60 * (L_7 - \phi') + L_8 - \phi'' + \left[1052.893882 - (4.483344 - 0.023520 \cos^2\phi) \cos^2\phi\right] \sin\phi \cos\phi\right],$$

the length of the meridian arc from ϕ_0 to ϕ and, (32)

L_3 = the map radius of the central parallel, ϕ_o.

L_5 = the scale factor along the central parallel.

L_7 = the degrees and minutes portion of the rectifying latitude for the central parallel (minutes).

L_8 = the seconds portion of the rectifying latitude for the central parallel.

L_9 =
L_{10} =
L_{11} =
coefficients for the series expansion of the change in the map radius between ϕ and ϕ_o as given in equation (403) of Thomas [12].

ϕ' = the degrees and minutes portion of ϕ (minutes).

ϕ'' = remainder of ϕ expressed in seconds.

Figure C3 shows some of the elements listed above on a developed Lambert conic conformal projection and helps to illustrate the method used by Claire. Instead of computing R_m directly, he starts with R_o, the mapping radius of the central parallel of the zone and adds algebraically (north is minus) the distance along the central meridian to the parallel of latitude which goes through a given point.

The concept of a rectifying sphere is used in equation (32) to determine the meridian arc distance between ϕ and ϕ_o. A rectifying sphere is a sphere which has the same circumference as the ellipse of a meridian section of a sphereoid. On a sphere the latitude increases linearly with the arc

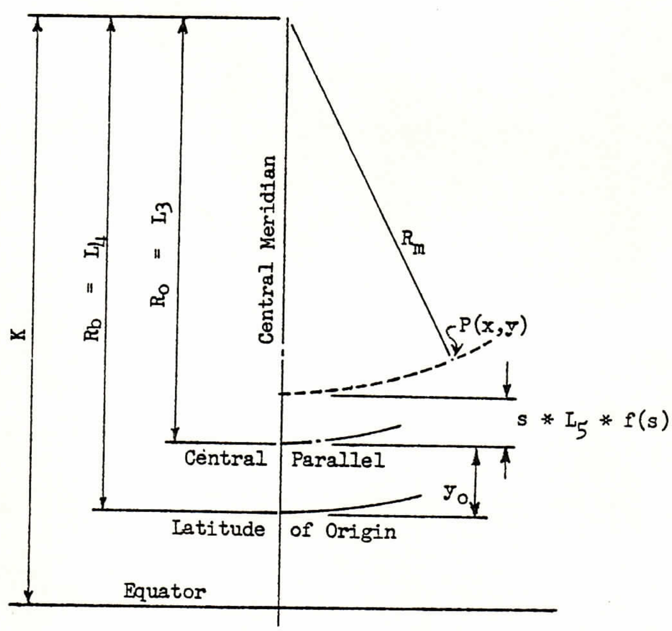

Figure No. C3 Developed Lambert Conic Conformal Projection distance. However, on an ellipse the relationship is more complex. Adams (Appendix of [1]) derived the relationship between the geodetic latitude, ϕ, and the rectifying latitude, ω, and Claire (page 42 of [5]) restates it as;

$$\omega'' = \phi'' - (1052''.893882 - (4''.483344 - 0''.023520*\cos 2\phi)*\cos^2\phi)*\sin\phi*\cos\phi \quad (33)$$

The numerical values in equation (33) were determined using the eccentricity of the Clarke Spheroid of 1866. However, since the Michigan Spheroid has the same eccentricity, the rectifying latitude for a given geodetic latitude is the same on either spheroid.

By equation (32) the meridian distance, s, is the product of the length per second of arc times the number of seconds of arc of rectifying latitude which corresponds to the interval of geodetic latitude, $\phi_o - \phi$. The seconds of rectifying latitude is given by equation (33) and Claire (page 43 of [5]) gives the quantity 101.2794065 as ". . . the length, in feet, of one second of arc on a sphere whose circumference equals the meridional arc of the Clarke 1866 ellipsoid." A different length factor is required for use with the Michigan Spheroid which has a longer meridional arc.

Since an ellipse is symmetrical to both axes, the length of an ellipitical quadrant is equal to the length of a quadrant of its rectifying sphere. The length factor is computed by dividing the ellipitical quadrant arc by 32,400 seconds per quadrant. Adams (page 122 of [1]) gives the length of a meridian arc of an ellipse as;

$$M = a(1 - e^2)\int_0^{\phi} \frac{d\phi}{(1 - e^2 \sin^2\phi)^{3/2}} \quad (34)$$

which, when evaluated for limits of 0° to 90° with $e^2 = 0.006768657997$ and using the formula given by Clark (page 405 of [6]) and coefficient for e^8 as given by Jordan (page 67 of [8]), gives;

$$s, (0^\circ - 90^\circ) = 1.568134898 * a. \quad (35)$$

Since equation (35) is linear in "a", the resulting length factor will be of the same ratio as the values for the semi-major axis of the two spheroids. Table C2 gives a comparison of the values for the meridian quadrant and the length factor for both spheroids.

TABLE C2 COMPARISON OF SPHEROIDS (Part 2)

	Clarke, 1866	Michigan	Ratio	Difference
a, (Table 1)	20,925,832.16'	20,926,631.53'	1.00003820	799.37'
s, (0° - 90°)	32,814,527.69'	32,815,781.20'	1.00003820	1253.51'
length factor	101.2794065'/"	101.2832753'/"	1.00003820	.0038688'/"

Since the distance "s" as given by equation (32) uses the length factor for the Clarke Spheroid of 1866, the value of "s" will be too small by a factor of 1.00003820 when used in any of the three Michigan Lambert zones. Claire circumvents the problem by using the "sea level" value of the scale factor of the central parallel, L_5, in equation (31) which is too large by a factor of 1.00003820 (see equation (7)). Thus, the product of "s*L_5" in equation (31) is unchanged and the same formula can be used for the Michigan zones as is used for Lambert zones in other states. Claire's value of L_5 is to be used with the length factor for the Clarke Spheroid of 1866, but the "correct" scale factor for the central parallel of the zone is to be used with the length factor for the Michigan Spheroid. The correct scale factor for the central parallel of each Michigan zone and Claire's value of L_5 are listed in Table C3.

TABLE C3 COMPARISON OF CENTRAL PARALLEL SCALE FACTORS

	Correct Scale Factor @ ϕ_o	Claire's L_5	Ratio
South Zone	0.99990 68822	0.99994 50783	1.00003820
Central Zone	0.99991 27095	0.99995 09058	1.00003820
North Zone	0.99990 28379	0.99994 10344	1.00003820

Formulas for the constants, L_9, L_{10}, and L_{11} in equation (31) are derived by Thomas [12] and restated by Claire [5],

$$L_9 = \frac{1}{6 * R_o * N_o} * 10^{16} \qquad (36)$$

$$L_{10} = \frac{(5 * R_o - 4 * N_o) * \tan \phi_o}{24 * R_o^2 * N_o^2} * 10^{24} \qquad (37)$$

$$L_{11} = \frac{(5 + 3 * \tan^2 \phi_o)}{120 * R_o * N_o^3} * 10^{32}, \quad \text{where} \qquad (38)$$

R_o = M = the radius of curvature in the meridian section at the central parallel.

N_o = N = the radius of curvature in the prime vertical at the central parallel.

Although these constants should be computed using the value of "a" for the Michigan Spheroid, it turns out that the difference in R_m caused by using Claire's constants, which are computed on the Clarke Spheroid of 1866, is less than 0.01 foot at a distance of 500,000 feet from the central parallel. As one gets closer to the central parallel, the difference becomes even smaller. The values of L_9, L_{10}, and L_{11} for each Michigan zone are listed in Table C4 for both spheroids. Claire's values from Publication 62-4 are also listed. Since the difference is so small, either set of constants can be used; however, use of the correct constants is recommended.

Since the method of computing R_m used by Claire gives the same value as one obtains by using equation (26) little harm would have been done if the value of L_5 would have been noted as <u>not</u> being the scale factor of the central parallel for the three Michigan Lambert zones. However, as noted in the next section, proper use of L_5 is crucial in determining the correct scale factor from state plane coordinates.

TABLE C4 COMPARISON OF CONSTANTS L_9, L_{10}, AND L_{11}

	Clarke, 1866	Michigan	Claire 62-4
South Zone			
L_9	3.808078	3.80779	3.80808
L_{10}	4.157064	4.15659	4.15706
L_{11}	32.89009	32.8851	33
Central Zone			
L_9	3.806222	3.80593	3.80622
L_{10}	4.468752	4.46824	4.46875
L_{11}	34.60000	34.5947	35
North Zone			
L_9	3.805012	3.80472	3.80501
L_{10}	4.684299	4.68376	4.68430
L_{11}	35.85450	35.8490	36

COMPUTATION OF SCALE FACTOR FROM STATE PLANE COORDINATES

An engineering approach to computing scale factors was presented by Professor Ralph M. Berry in 1972 [4]. His formulas are tailor made for computer processing and the results are generally reliable to seven or eight significant figures. The equation given by Berry for the scale factor is;

$$k = k_o * (1 + K * q^2) \text{ with } q = s/10^6 \quad \text{where,} \tag{39}$$

k_o = the scale factor of the central parallel of a Lambert zone, generally Claire's value of L_5.

K = an empirical constant for a given zone. These constants are tabulated for all zones in the appendix of [4].

q = a function of the distance from the central parallel.

s = the distance from the central parallel.

The correct value of the scale factor of the central parallel as listed in Table 3 must be used to compute a correct scale factor in Michigan. If Claire's value of L_5 for Michigan Lambert zones is used, the resulting scale factor is too large by a factor of 1.00003820. From equation (7) one can see that the erroneous scale factor is really a "sea level" scale factor. Although the correct grid factor could still be obtained using equation (15), use of the correct scale factor is recommended. If the ground elevation is far enough from 800 feet MSL to make the sea level reduction significant, the elevated sea level factor should be used with the correct scale factor to compute the correct grid factor.

CONCLUSION

Although inferences could be made as to the merits of the use of project datums, zone datums, and sea level reductions (especially when incorporating the 1983 NAD), that is sufficient for another paper and beyond the scope of this paper on the Michigan scale factor.

The Michigan scale factor is applicable to a reference surface 800 feet above sea level and should be used for distances which are near (or have been reduced to) that reference surface. The Michigan scale factor can be determined correctly by;

1. Scaling the latitude of a point from an appropriate topographic map and using the latitude as an argument to select the corresponding scale factor from the projection tables.
2. Using the formula listed in Publication 62-4, modified to use the Michigan Spheroid value of "a" rather than the value of "a" for the Clarke Spheroid of 1866.
3. Computing the scale factor from state plane coordinates according to the procedure given by Professor Ralph M. Berry [4]. However, the correct value of the scale factor for the central parallel of the zone must be used for k_o (see Table 3).

The author has encountered several cases where the wrong scale factor has been used due to confusion caused by the discrepancies discussed in this paper. It is also disconcerting to discover that commercial computer programs are available which give a sea level scale factor in Michigan. However, the point is made that it is our responsibility as professionals to understand and to verify the answers obtained from a "black box". It is hoped that this paper will increase our understanding of the Michigan scale factor and give it a better chance of being used correctly.

ACKNOWLEDGMENT

In addition to the support of the faculty at Purdue University the author would like to acknowledge the assistance and wholehearted support of Professor Ralph Moore Berry of the National Geodetic Survey who designed the existing Michigan Coordinate System while Professor of Geodetic Engineering at the University of Michigan.

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APPENDIX D

MODEL LAW FOR MICHIGAN METRIC COORDINATES 1983

An act to describe, define, and officially adopt a system of metric coordinates for designating the geographic position of points on or near the surface of the earth within this state, and to repeal the existing Michigan Coordinate System Act, being sections 54.231 - 52.239 of Michigan Compiled Laws Annotated.

The people of the State of Michigan enact:

Sec. 1. (1) The systems of plane coordinates which have been established by the National Ocean Survey/National Geodetic Survey (formerly the United States Coast and Geodetic Survey) or its successors for defining and stating the geographic positions or locations of points on or near the surface of the Earth within the State of Michigan are hereafter to be known and designated as the Michigan Coordinate System of 1927 and the Michigan Metric Coordinates 1983, or "MCS 27" and "MMC 83."

(2) For the purpose of the use of these systems the state is divided into a north zone, a central zone and a south zone.

(3) The area now included in the following counties constitutes the north zone: Gogebic, Ontonagon, Houghton, Keweenaw, Baraga, Iron, Marquette, Dickinson, Menominee, Alger, Delta, Schoolcraft, Luce, Chippewa and Mackinac.

(4) The area now included in the following counties constitutes the central zone: Emmet, Cheboygan, Presque Isle, Charlevoix, Leelanau, Antrim, Otsego, Montmorency, Alpena, Benzie, Grand Traverse, Kalkaska, Crawford, Oscoda, Alcona, Manistee, Wexford, Missaukee, Roscommon, Ogemaw, Iosco, Mason, Lake, Osceola, Clare, Gladwin and Arenac.

(5) The area now included in the following counties constitutes the south zone: Oceana, Newaygo, Mecosta, Isabella, Midland, Bay, Huron, Muskegon, Montcalm, Gratiot, Saginaw, Tuscola, Sanilac, Ottawa, Kent, Ionia, Clinton, Shiawassee, Genesee, Lapeer, St. Clair, Allegan, Barry, Eaton, Ingham, Livingston, Oakland, Macomb, Van Buren, Kalamazoo, Calhoun, Jackson, Washtenaw, Wayne, Berrien, Cass, St. Joseph, Branch, Hillsdale, Lenawee and Monroe.

Sec. 2. (1) As established for use in the North Zone, the Michigan Coordinate System of 1927 or the Michigan Metric Coordinates 1983 shall be named; and in any land description in which it is used, it shall be designated the "Michigan Coordinate System of 1927, North Zone" or "Michigan Metric Coordinates 1983, North Zone."

(2) As established for use in the Central Zone, the Michigan Coordinate System of 1927 or the Michigan Metric Coordinates 1983 shall be named; and in any land description in which it is used, it shall be designated the "Michigan Coordinate System of 1927, Central Zone' or "Michigan Metric Coordinates 1983, Central Zone."

(3) As established for use in the South Zone, the Michigan Coordinate System of 1927 for the Michigan Metric Coordinates 1983 shall be named; and in any land description in which it is used, it shall be designated the "Michigan Coordinate System of 1927, South Zone" or "Michigan Metric Coordinates 1983, South Zone."

Sec. 3. The plane coordinate values for a point on or near the Earth's surface, used to express the geographic position or location of such point in the appropriate zone of this system, shall consist of two distances expressed in U.S. Survey Feet (1 meter = 39.37/12 feet) and decimals of a foot when using the Michigan Coordinate System of 1927 and expressed in meters and decimals of a meter when using the Michigan Metric Coordinates 1983. One of these distances, it be known as the "x-coordinate," shall give the position in an east-and-west direction; the other, to be known as the "y-coordinate," shall give the position in a north-and-south direction. These coordinates shall be made to depend upon and conform to plane rectangular coordinate values for the monumented points of the North American Horizontal Geodetic Control Network as published by the National Ocean Survey/National Geodetic Survey (formerly the United States Coast and Geodetic Survey), or its successors, and whose plane coordinates have been computed on the systems defined in this act.

Sec. 4. When any tract of land to be defined by a single description extends from one into the other of the above coordinate zones, the positions of all points on its boundaries may be referred to either of the two zones, the zone which is used being specifically named in the description.

Sec. 5. (1) For purposes of more precisely defining the Michigan Coordinate System of 1927, the following definition by the United States Coast and Geodetic Survey (now National Ocean Survey/National Geodetic Survey) is adopted:

(a) The Michigan Coordinate System, North Zone, is a Lambert conformal projection of the Clarke Spheroid of 1866, magnified in linear dimension by a factor of 1.0000382, having standard parallels at north latitudes 45 degrees 29 minutes and 47 degrees 5 minutes, along which parallels the scale shall be exact. The origin of coordinates is at the intersection of the meridian 87 degrees zero minutes west of Greenwich and the parallel 44 degrees 47 minutes north latitude. This origin is given the coordinates : x = 2,000,000 feet and y = 0 feet.

(b) The Michigan Coordinate System, Central Zone, is a Lambert conformal projection of the Clarke Spheroid of 1866, magnified in linear dimension by a factor of 1.0000382, having standard parallels at north latitude 44 degrees 11 minutes and 45 degrees 42 minutes, along which parallels the scale shall be exact. The origin of coordinates is at the intersection of the meridian 84 degrees 20 minutes west of Greenwich and the parallel 43 degrees 19 minutes north latitude. This origin is given the coordinates: x = 2,000,000 feet and y = 0 feet.

(c) The Michigan Coordinate System, South Zone, is a Lambert conformal projection of the Clarke Spheroid of 1866, magnified in linear dimension by a factor of 1.0000382, having standard parallels at north latitude 42 degrees 6 minutes and 43 degrees 40 minutes along which parallels the scale shall be exact. The origin of coordinates is at the intersection of the meridian 84 degrees 20 minutes west of Greenwich and the parallel 41 degrees 30 minutes north latitude. This origin is given the coordinates: x = 2,000,000 feet and y = 0 feet.

(2) For purposes of more precisely defining the Michigan Metric Coordinates 1983, the following definition by the National Ocean Survey/ National Geodetic Survey is adopted:

(a) The Michigan Metric Coordinates 1983, North Zone is a Lambert conformal projection of the North American Datum of 1983, having standard parallels at north latitudes 45 degrees 29 minutes and 47 degrees 5 minutes, along which parallels the scale shall be exact. The origin of coordinates is at the intersection of the meridian 87 degrees zero minutes west of Greenwich and the parallel 44 degrees 47 minutes north latitude. This origin is given the coordinates: x = 8,000,000 meters and y = 0 meters.

(b) The Michigan Metric Coordinates 1983, Central Zone is a Lambert conformal projection of the North American Datum of 1983, having standard parallels at north latitude 44 degrees 11 minutes and 45 degrees 42 minutes, along which parallels the scale shall be exact. The origin of coordinates is at the intersection of the meridian 84 degrees 21 minutes 53 seconds west of Greenwich and the parallel 43 degrees 19 minutes north latitude. This origin is given the coordinates: x = 6,000,000 meters and y = 0 meters.

(c) The Michigan Metric Coordinates 1983, South Zone is a Lambert conformal projection of the North American Datum of 1983, having standard parallels at north latitudes 42 degrees 6 minutes and 43 degrees 40 minutes, along which parallels the scale shall be exact. The origin of coordinates is at the intersection of the meridian 84 degrees 21 minutes 52 seconds west of Greenwich and the parallel 41 degrees 30 minutes north latitude. This origin is given the coordinates: x = 4,000,000 meters and y = 0 meters.

(3) The position of the Michigan Coordinate System of 1927 shall be as determined from horizontal geodetic control points established throughout the state in conformity with the standards of accuracy and specifications for first-order and second-order geodetic surveying as prepared and published by the Federal Geodetic Control Committee (FGCC) of the United States Department of Commerce, whose geodetic positions have been rigidly adjusted on the North American Datum of 1927, and whose coordinates have been computed on the Michigan Coordinate System of 1927. Standards and specifications of the FGCC (or its successors) in force on date of said survey shall apply.

(4) The position of the Michigan Metric Coordinates 1983 shall be as determined from horizontal geodetic control points established throughout the state in conformity with the standards of accuracy and specifications for first-order and second-order geodetic surveying as prepared and published by the Federal Geodetic Control Committee (FGCC) of the United States Department of Commerce, whose geodetic positions have been rigidly adjusted on the North American Datum of 1983, and whose coordinates have been computed on the Michigan Metric Coordinates 1983. Standards and specifications of the FGCC (or its successors) in force on date of said survey shall apply.

Sec. 6. No coordinates based on either coordinate system herein described purporting to define the position of a point or land boundary corner shall be presented to be recorded unless the recording document shall contain an estimate (one standard deviation) of the positional tolerance of the coordinates being recorded. The recording document shall also contain a description of the nearest first or second-order horizontal geodetic control monument from which the coordinates being recorded were determined and the method of survey for such determination. If the position of the described first or second-order geodetic control monument is not published by the National Geodetic Survey (or its successors) the recording document shall contain a certification signed by a Michigan Registered Land Surveyor stating that the subject control monument and its coordinates have been established and determined in conformance with the specifications given in section 5 of this act.

Sec. 7. (1) The use of the term "Michigan Coordinate System of 1927" on any map, report of survey, or other document, shall be limited to coordinates based on the Michigan Coordinate System of 1927 as defined in this act.

(2) The use of the term "Michigan Metric Coordinates 1983" on any map, report of survey, or other document, shall be limited to metric coordinates based on the Michigan Metric Coordinates 1983 as defined in this act.

Sec. 8. (1) For purpose of describing the location of any survey station or land boundary corner in the State of Michigan, it shall be considered complete, legal, and satisfactory description of such location to give the position of said survey station or land boundary corner by Michigan Metric Coordinates 1983.

(2) Wherever Michigan Metric Coordinates 1983 are used to describe any tract of land which in the same document is also described by reference to any subdivision, line or corner of the United States public land surveys, or to any subdivision plat duly recorded in accordance with Act No. 172 of 1929, as amended, being sections 560.1 to 560.80 of the Compiled Laws of 1948, the description by coordinates shall be construed as supplemental to the basic description of such subdivision, line or corner contained in the official plats and field notes filed of record, and in the event of any conflict the description by reference to the subdivision, line or corner of the United States public land surveys, or recorded subdivision plat, shall prevail over the description by coordinates.

Sec. 9. If any provision of this Act shall be declared invalid, such invalidity shall not affect any other portion of this Act which can be given effect without the invalid provision; and to this end, the provisions of this act are declared to be severable.

Sec. 10. The Michigan Coordinate System of 1927 shall not be used after January 1, 1990; the Michigan Metric Coordinates 1983 will be the sole system after this date.

APPENDIX E

PARAMETERS, CONSTANTS, AND FORMULAS FOR MICHIGAN METRIC COORDINATES 1983

This appendix contains the parameters for the Michigan Metric Coordinates 1983 based on three Lambert conic conformal projections. Formulas for the projection constants and computed constants are given for the three zones. Listed lastly is a summary of transformation formulas given by Berry in the text of a Geodetic Control Seminar sponsored by the Michigan Society of Registered Land Surveyors in December, 1971, at the University of Michigan. The information contained herein is sufficient to compute Lambert conic conformal projection transformations for the following three Lambert zones of the 1983 NAD.

	North Zone	Central Zone	South Zone
Standard Parallels, Ø(N) =	47° 05' 45° 39'	45° 42' 44° 11'	43° 40' 42° 06'
Central Meridian, λ(W) =	87° 00'	84° 21' 53"	84° 21' 53"
Latitude of Origin (Y = 0)	44° 47'	43° 19'	41° 30'
False Easting on Central Meridian	8,000,000 m	6,000,000 m	4,000,000 m

The parameters of the North American Datum of 1983 are;

a = 6,378,137 meters (semi-major axis of reference ellipsoid),

$1/f$ = 298.257 (flattening of the reference ellipsoid), and

$e^2 = 2f - f^2$ (eccentricity of ellipsoid squared).

The following symbols relate to the defining parameters of the three Lambert zones:

P_s = geodetic co-latitude of the south standard parallel.

P_n = geodetic co-latitude of the north standard parallel.

λ_c = longitude of the central meridian.

P_b = geodetic co-latitude of the origin (y = 0 meters).

C = constant value of "X" assigned to the central meridian to avoid negative "X" coordinates in the zone area and to distinguish Michigan Metric Coordinates 1983 from coordinates based on the Michigan Coordinate System of 1927.

The following zone constants are computed for the design parameters given and need to be computed only once for each zone:

Z_n & Z_s = conformal co-latitude of the north standard parallel and the south standard parallel respectively, defined by:

$$\tan(Z_i/2) = \tan(P_i/2)\left[\frac{1 + e\cos P_i}{1 - e\cos P_i}\right]^{e/2}. \qquad (1)$$

N_n & N_s = length in meters of the prime vertical at the north standard parallel and the south standard parallel respectively, defined by:

$$N_i = a / (1 - e^2 \cos^2 P_i)^{\frac{1}{2}}. \qquad (2)$$

ℓ = Lambert projection zone constant, computed by:

$$\ell = \frac{\log(N_s \sin P_s / N_n \sin P_n)}{\log\tan(Z_s/2) - \log\tan(Z_n/2)} \qquad (3)$$

ϕ_o = latitude of the central parallel, computed by:

$$\phi_o = \arcsin(\ell) \qquad (4)$$

K = map radius of the equator in meters, computed by:

$$K = \frac{N_s \sin P_s}{\ell\,(\tan(Z_s/2))^{\ell}} \quad \text{or} \quad \frac{N_n \sin P_n}{\ell\,(\tan(Z_n/2))^{\ell}} \qquad (5)$$

R_b = mapping radius of the latitude of the origin in meters, computed by:

$$R_b = K\ (\tan\ (Z_b/2))^{\ell} \qquad (6)$$

R_o = mapping radius of the central parallel in meters, computed by:

$$R_o = K\ (\tan\ (Z_o/2))^{\ell} \qquad (7)$$

Y_o = Y coordinate of the central parallel on the cnetral meridian in meters, computed by:

$$Y_o = R_b - R_o \qquad (8)$$

The values of the zone constants for the three Lambert zones for the Michigan Metric Coordinates 1983 are listed below:

	North Zone	Central Zone	South Zone
ℓ	0.72278 99347 32933	0.70640 74068 62361	0.68052 92599 12149
K (meters)	11,779,843.7567	11,878,338.0026	12,061,671.8246
$\emptyset_o$	46° 17' 07".1002234	44° 56' 36".0915270	42° 53' 06".0544886
R_b (meters)	6,275,243.8512	6,581,660.2399	7,031,167.2984
R_o (meters)	6,108,308.6116	6,400,902.4479	6,877,323.4179
Y_o (meters)	166,935.2396	180,757.7920	153,843.8846

Using the zone constants listed above, the Michigan Metric Coordinates 1983 can be determined for any point within a zone from its latitude and longitude. The transformation formulas are derived by Thomas in Special Publication No. 251 and restated by Berry as follows:

$P = 90^\circ - \emptyset$, the geodetic co-latitude; (9)

$\Delta\lambda = \lambda c - \lambda$, delta lambda in seconds; (10)

$\theta = \ell * (\Delta\lambda)$, the convergence at $(\emptyset,\lambda)$; (11)

$R = K\ (\tan\ (Z/2))^{\ell}$, radius of the projected parallel; (12)

$$X' = R * \sin\theta \tag{13}$$

$$X = X' + C, \qquad \text{X coordinate at point } (\phi,\lambda); \tag{14}$$

$$Y' = R * \cos\theta \tag{15}$$

$$Y = R_b - Y', \qquad \text{Y coordinate at point } (\phi,\lambda); \tag{16}$$

$$k = \ell * R\,/(N \sin P), \text{ the scale factor at } (\phi,\lambda). \tag{17}$$

The latitude and longitude of a point whose Michigan Metric Coordinates 1983 are known can be determined from the inverse transformation as follows:

$$X' = X - C \tag{18}$$

$$Y' = R_b - Y \tag{19}$$

$$\theta = \arctan\,(X'/Y'), \text{ the convergence at point } (X,Y); \tag{20}$$

$$\Delta\lambda = \theta\,/\ell \tag{21}$$

$$\lambda = \lambda_c - \Delta\lambda \qquad \text{the longitude of point } (X,Y); \tag{22}$$

$$R = (X'^2 + Y'^2)^{\frac{1}{2}} \tag{23}$$

$$Z = 2\arctan\,(R/K)^{1/\ell}, \text{ the conformal co-latitude}; \tag{24}$$

$$\chi = 90^{\circ} - Z, \text{ the conformal latitude.} \tag{25}$$

The difference between the latitude of the point and the conformal latitude of the same point is given by the $(\phi-\chi)$ series developed by Adams in Special Publication No. 67. Berry extended Adams' formula to include powers of e^{10} and rearranged it in the following form:

$$(\phi-\chi) = P\sin\chi\cos\chi + Q\sin\chi\cos^3\chi + R\sin\chi\cos^5\chi + S\sin\chi\cos^7\chi + T\sin\chi\cos^9\chi \tag{26}$$

$$(\phi-\chi) = \sin\chi\cos\chi(P + \cos^2\chi(Q + \cos^2\chi(R + \cos^2\chi(S + T\cos^2\chi)))) \tag{27}$$

$$\phi = \chi + (\phi-\chi), \text{ the geodetic latitude of the point } (X,Y). \tag{28}$$

The coefficients in Berry's formula for the ($\phi - \chi$) series have been computed for the North American Datum of 1983 as follows:

P = 0.00668 69258 929	radians	=	1379.27747 36923	seconds
Q = 0.00005 20146 604	"	=	10.72879 38593	"
R = 0.00000 05544 592	"	=	0.11436 54278	"
S = 0.00000 00067 177	"	=	0.00138 56241	"
T = 0.00000 00000 891	"	=	0.00001 83734	"

Errata for Burkholder's 1980 Thesis
April 2019

The values for reciprocal flattening for the GRS 1980 used in this thesis were as supplied by NGS (see next page) but, unfortunately, those values were premature – see subsequent article by Earl F. Burkholder, also included herein.

UNITED STATES DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
NATIONAL OCEAN SURVEY
Rockville, Md. 20852

April 24, 1980 OA/C131x1:EJM

Mr. Earl Burkholder
Purdue University
Department of Civil Engineering
West Lafayette, Indiana 47907

Dear Mr. Burkholder:

To confirm the information given to you by telephone on April 10, 1980, please be assured that the values of the parameters for the GRS 80 ellipsoid will be held fixed in the NAD 83 Adjustment. Those values were agreed upon at the December 1979 meeting of the International Association of Geodesy in Canberra, Australia. They are:

semi-major axis (a) = 6,378,137 meters
flattening (f) = 1/298.257

Your question concerning whether the ellipsoid in the NAD 27 Datum was merely coincident at the datum point, MEADES RANCH--a zero geoid height--or actually tangent--a zero geoid height and the astronomic azimuth corrected for the deflection of the vertical--was referred to Mr. James Petty, Chief of the Gravity and Astronomy Section. Deflections of the vertical computed from observed astronomic positions were not known at MEADES RANCH until the late 1930's or early 1940's when astronomic positions were measured. Therefore, since the geodetic azimuth was set equal to the astronomic azimuth, the ellipsoid is coincident and not tangent at MEADES RANCH. Should you have further questions pertaining to this subject, please contact Mr. Petty, (301) 443-8620.

From our conversation about your work, you may find an article by Herbert W. Stoughton entitled "The Surveyor and the Law" to be of interest. The article appears in the March 1980 issue of the ACSM publication, Surveying and Mapping.

We hope this answers your questions, but should you need additional assistance, please do not hestitate to contact us.

Sincerely yours,

John G. Gergen

John G. Gergen, Chief
Horizontal Network Branch
National Geodetic Survey

Geometrical Parameters of the Geodetic Reference System 1980

by Earl F. Burkholder

Abstract. The four defining parameters of the Geodetic Reference System 1980 were adopted by the XVII General Assembly of the International Union of Geodesy and Geophysics meeting in Canberra, Australia in December 1979. Since only one of the four defining parameters (a, the semimajor axis of the ellipsoid) is an element of the geometrical ellipsoid, a second geometrical parameter must be derived. The defining parameters adopted by the IUGG are listed and formulas are quoted for computing values of various geometrical elements to any accuracy desired. Finally, values of various geometrical elements of the Geodetic Reference System 1980 are listed to 16 significant figures.

Introduction

The parameters of the Geodetic Reference System of 1980 (GRS 1980) were adopted by the XVII General Assembly of the International Union of Geodesy and Geophysics meeting in Canberra, Australia in December 1979. The four defining parameters are elements of physical geodesy, one of them being "a," the semimajor axis of the ellipsoid. The second defining parameter normally required for geometrical geodesy, i.e., e^2 or 1/f, is not one of the four defining parameters, but must be computed from them. The purpose of this article is to list formulas for computing e^2 and various other geometrical elements of the reference ellipsoid and to give values for them to 16 significant figures.

Definitions and Formulas

The definitions and formulas in this section are as given by Moritz (1980). The defining parameters of the Geodetic Reference System 1980 (GRS 1980), which are held exact, are:

a	$= 6378137$ m	the equatorial radius of the earth.
GM	$= 3986005 \times 10^8 M^3/S^2$	the geocentric gravitational constant of the Earth, including the atmosphere.
J_2	$= 108263 \times 10^{-8}$	the dynamical form factor of the Earth, excluding permanent tidal deformation.
ω	$= 7292115 \times 10^{-11}$ rad/S	the angular velocity of the Earth.

The closed form computational formulas for computing the square of the eccentricity of the ellipsoid, e^2, are given as:

$$e^2 = 3J_2 + (4/15)(\omega^2 a^3/GM)(e^3/2_{q_0}) \quad \text{where} \tag{1}$$

$$2_{q_0} = (1 + 3/e'^2)\arctan(e') - 3/e', \quad \text{and} \tag{2}$$

$$e' = e/(1 - e^2)^{1/2} \tag{3}$$

Since equation (1) has "e" on both sides of the equals sign, it must be solved iteratively even though it is in closed form.

Constraints

Moritz (1980) also gives the value of e^2 (and other computed ellipsoid elements) to 12 significant figures which is accurate enough for most geometrical geodesy applications. However, if one is not satisfied (for whatever reason) with 12 significant figures and has access to a suitable computer, the ellipsoidal elements may be computed to any desired accuracy.

The limit to the number of significant figures attainable from equation (1) is deter-

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mined by the term 2_{q_0} which is defined in equation (2) and is the difference of two separate terms. Since the first five digits in each of the two terms in equation (2) are the same, the number of significant digits in the value 2_{q_0} is 4-5 less than the number of significant digits in the value of e′. Therefore, it takes a 16-17-digit computer to compute e^2 to 12 significant figures and the 20-digit computer used by this author will give a value of e^2 to 15-16 significant figures. Linear regression and numerical analysis of the residuals of the computed values were used to confirm the 16th significant figure which may vary slightly if e^2 is computed on a computer having greater significant figure capacity. The computational accuracy of all subsequent derived values is dependent on the accuracy of e^2, the significant digit capacity of the computer and on the number of terms included in any equation containing an infinite series.

Additional Formulas

The equations listed in this section are all in closed form except for the equation for the length of the meridian quadrant. The equation for the meridian quadrant is one given by Schmid (1971) and can be extended to any accuracy desired. The e^{14} term contributes less than 0.0000 0000 4 m to the meridian length and was omitted from the value given in the next section.

The equation for R_2 is a closed form equation derived from the formula for the surface area of the ellipsoid given by Jordan (1962).

$b = a(1-e^2)^{1/2}$, the semiminor axis. (4)

$c = a^2/b$, the polar radius of curvature. (5)

$f = (a - b)/a$ the flattening. (6)

$E = (a^2 - b^2)^{1/2}$, the linear eccentricity. (7)

$$Q = a(1-e^2)(\pi/2)(1 + (3/4)e^2 + (45/64)e^4 + (175/256)e^6 + (11025/16384)e^8 + (43659/65536)e^{10} + (693693/1048576)e^{12} + (2r+1)/2^{4r}\binom{2r}{r}^2 e^{2r} + \ldots), \quad (8)$$

the length of the meridian from the equator to the pole.

$R_1 = (a + a + b)/3$, the arithmetic mean radius (9)

$$R_2 = [a(1-e^2)^{1/2}/\sqrt{2}][1/(1-e)^2) + (1/2e)\ln\{(1+e)/(1-e)\}]^{1/2}, \quad (10)$$

the radius of a sphere having the same surface area as the ellipsoid.

$R_3 = \sqrt[3]{a^2 b}$, the radius of a sphere having the same volume as the reference ellipsoid. (11)

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$$\left.\begin{array}{l} a = 6{,}378{,}137 \text{ m (exact)} \\ e^2 = 0.00669\ 43800\ 22903\ 416 \end{array}\right\} \text{parameters} \quad (1)$$

$e'^2 = 0.00673\ 94967\ 75481\ 622$

$e = 0.08181\ 91910\ 42831\ 85$ (2)

$e' = 0.08209\ 44381\ 51933\ 42$ (3)

$b = 6{,}356{,}752.31414\ 0347$ m (4)

$c = 6{,}399{,}593.62586\ 4032$ m (5)

$f = 0.00335\ 28106\ 81183\ 637$ (6)

$1/f = 298.25722\ 21008\ 827$

$E = 521{,}854.00970\ 03544$ m (7)

$Q = 10{,}001{,}965.72922\ 984$ m (8)

$R_1 = 6{,}371{,}008.77138\ 0116$ m (9)

$R_2 = 6{,}371{,}007.18088\ 3514$ m (10)

$R_3 = 6{,}371{,}000.78997\ 4137$ m (11)

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