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**3-D Coordinates - A Universal Rectangular Coordinate
System for a GIS and Other Spatial Databases**

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Abstract

Rules of solid geometry and 3-dimensional (3-D) rectangular coordinate systems are well known and used in many applications. Points on or near the earth's surface have long been described with rectangular coordinates; plane coordinates for horizontal position and elevation for the third dimension. When required to accommodate earth's curvature, standard practice is to describe horizontal position using geodetic coordinates of latitude/longitude. The third dimension is height above/below the ellipsoid. The 3-D geocentric coordinate system recommended for a GIS accommodates both local rectangular coordinates and geodetic latitude/longitude/height coordinates on a global scale without introducing distance distortions inherent in the use of a map projection.

Introduction

Convergence of modern technologies; the computer revolution, global positioning system (GPS) surveying, and geographic information systems (GIS's), has created both the demand and tools for using the 3-D geocentric coordinate system. Routine output of a GPS surveying system is a space vector between points on the ground. A GPS vector can be very precise (within millimeters) and is defined by rectangular components ($\Delta X/\Delta Y/\Delta Z$) in the geocentric coordinate system. Coupled with the X/Y/Z coordinates of points already part of the National Geodetic Reference System (NGRS), those components can be used very efficiently with rules of solid geometry for definition of location, computations, and analysis in three dimensional space. The 3-D geocentric coordinate system does not replace existing systems, but can provide underlying geometrical integrity to all spatial data.

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Past practice has used 2-dimensional plane coordinates for most engineering drawings with the third dimension included in a profile view. Modern practice includes conventional 3-dimensional geometry for many engineering drawings with elaborate computer aided drafting (CAD) packages available for modeling, computation, and display (from any angle). Spatial computation and representation is essentially a solved problem. The remaining challenge addressed in this paper is to link 3-D geometry to the three-dimensional earth in a mathematically defensible way which preserves geometrical integrity.

The physical earth is curvilinear and location on it is specified with geodetic latitude/longitude coordinates in units of degrees, minutes, and seconds. Elevation is the third dimension and given in length units of meters. While a correct 3-dimensional representation can be made, it is awkward to work with a mixture of angular and length units--latitude/longitude on the ellipsoid and height.

Two dimensional representation of the earth's curved surface has been accomplished with conformal mapping. When needed, elevation or height has been included for the third dimension. The resulting coordinate system does not enjoy strict 3-dimensional integrity for two reasons; 1) mathematical set theory (range/domain) used in conformal mapping does not define a third dimension and 2) elevation on a map is referenced to a curved surface, typically the geoid. Rules of solid geometry can be used only to the extent one can safely assume a flat earth.

Need for an Improved Mathematical Model

Perhaps the most compelling reason to consider using the 3-D system is that modern measurement technology has outgrown conventional mathematical models. Four examples are:

1. Distances are routinely measured to an accuracy approaching 1:100,000 or better using electronic distance meters (EDM) and to 1:1,000,000 or better using GPS. As noted in Burkholder (1991), existing models for defining horizontal distance can be ambiguous and subject to misinterpretation. Horizontal distance as used in the 3-D system is the same as HD(1), the right triangle component of slope distance as described in that paper.
2. Modern total station surveying instruments make three basic measurements:
 - Slope (slant) distance,
 - Zenith (or vertical) direction, and
 - Given an appropriate backsight, azimuth (or direction) of a line.These measurements (along with instrument/target heights) yield local geodetic horizon coordinate differences which feed directly into the 3-D coordinate system via a rotation matrix. Conventional models separate field survey data into 2-D coordinates which may lack connection to the NGRS and trig height elevations which may not be accurate due to earth's curvature.

3. GPS surveying is a three dimensional operation but many users force GPS results into a 2-D data base. One dimension of information is potentially lost or wasted. Geocentric coordinate differences are the primary output of a GPS survey and the 3-D model accommodates that information. And, the 3-D model does not distort any geometrical elements as conformal mapping distorts distances. The 3-D mathematical model uses all three dimensions and does not "pollute" the quality of any measurements.
4. Conventional photogrammetric mapping is tied to ground control defined by "pseudo 3-D" coordinates, i.e. map projection coordinates and elevations. That is changing with the advent of GPS surveying. A recent issue of Photogrammetric Engineering & Remote Sensing contains 9 separate articles on the use of GPS in photogrammetry and most authors discuss some aspect of using the 3-D coordinate system. Guest Editor, Novak (1993) states in the Foreword, "It is the first time that the world community has accepted and can practically utilize a common global coordinate system."

Coordinate Systems

This section is a summary of material presented in a previous paper, "Using GPS Results in a True 3-D Coordinate System," (Burkholder 1993a). However, additional emphasis is given here to the geocentric coordinate system as the basis of the 3-D system being discussed. See also Soler & Hothem (1988).

The **geocentric coordinate system**, also called earth-centered earth-fixed (ECEF) coordinates, is a system of right-handed three-dimensional rectangular cartesian coordinates which uses length units and has its origin at the earth's center of mass. As shown in Figure 1, the X/Y plane is coincident with the plane of the equator and the Z axis is coincident with the earth's mean spin axis. Rules of solid geometry apply throughout to all geometrical elements such as lines, planes, circles, spheres and other shapes.

A triplet of 12 digit numbers (X/Y/Z coordinates) can be used to describe the location of any point within 99,999,999.9999 meters of the origin to the nearest 0.1 millimeter (normal double precision on most computers routinely handles 16 digits). Or, stated differently, the distance between any two points within 50,000 km of the origin can be computed within 0.1 mm using a pair of 12 digit X/Y/Z coordinates. That range extends beyond the 26,500 km orbit of the NAVSTAR GPS satellites.

Figure 1 also shows how **geodetic coordinates** can be used to describe the location of a point with respect to the curved surface of the ellipsoid. Latitude, ϕ , gives the angular distance north or south of the equator while longitude, λ , (either east or west) gives location with respect to the Greenwich Meridian. Although spherical in nature, geodetic coordinates are 2 dimensional. Height, h , above or

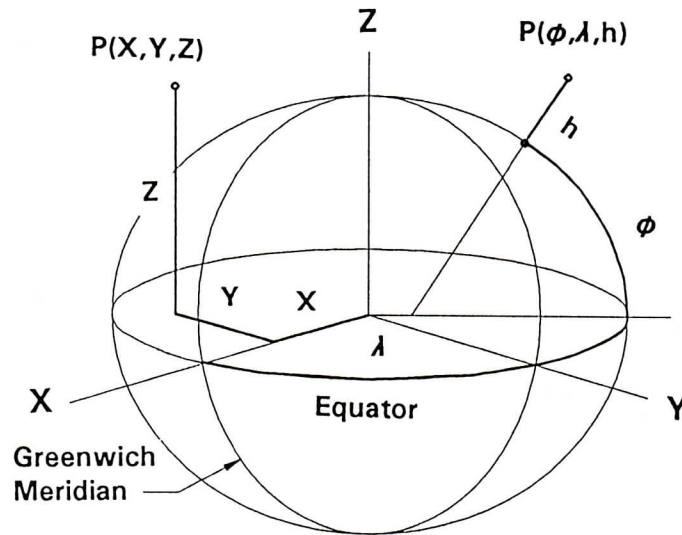


Fig. 1 Geocentric X,Y,Z and Geodetic ϕ,λ,h Coordinates

below the ellipsoid is required for the third dimension. True 3-D integrity can be preserved in using geodetic coordinates, but working with mixed (angular and length) units is somewhat cumbersome and avoided by many.

Because of the difficulties of working with angular geodetic coordinates, **map projection (state plane) coordinates** were devised to permit 2-D latitude/longitude positions to be expressed equivalently with 2-D plane coordinates having length units. Conformal mapping equations are used to establish a one-to-one correspondence between points on the curved earth with those on a flat map and a high degree of geometrical consistency is preserved in two dimensions. However, as shown in Figure 2, the third dimension is not mathematically defined. Hence, adding elevations to map projection coordinates does not constitute using a true rectangular 3-D cartesian coordinate system.

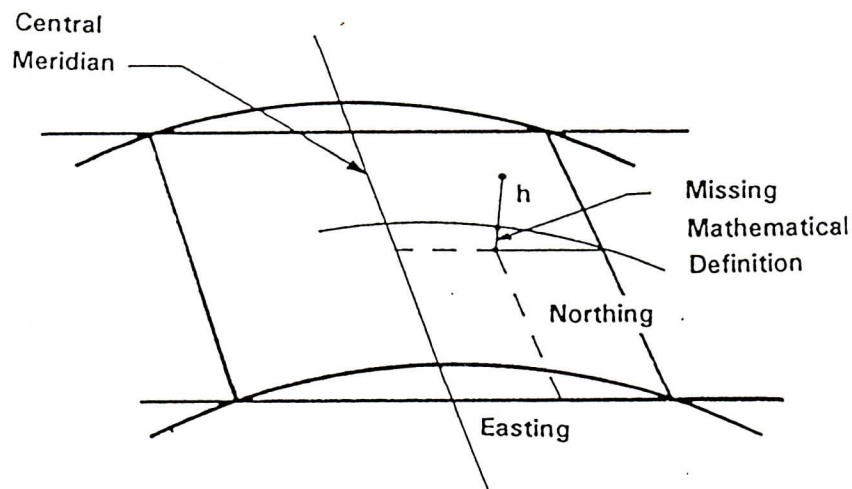
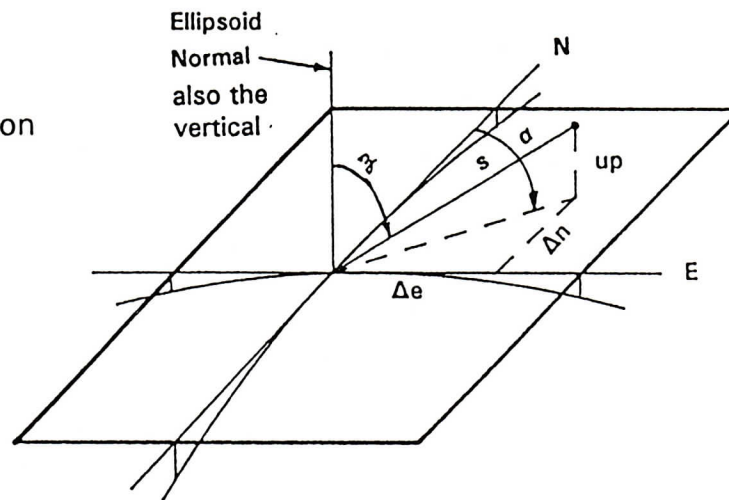


Fig. 2 Map Projection (State Plane) Coordinates and Missing Vertical

Figure 3 depicts a **local geodetic horizon coordinate system** which has many familiar features. It is a two-dimensional plane of North & East (or X & Y) distances related to real or assumed coordinates at the origin. The third dimension is vertical and, to the extent a flat earth can be assumed, corresponds to elevation. Such a system has been universally used by many disciplines, including engineers, surveyors, cartographers, and GIS professionals, to identify spatial relationships. This paper focuses on the fact that many land parcels have been surveyed and mapped using such a system of plane coordinates and that many problems arise when attempting to aggregate parcels, surveys, and disparate coordinate systems into one GIS data base.

Fig. 3
Local Geodetic Horizon
Coordinate System



A Better Way: Using 3-D Coordinates

The normal solution to make the pieces fit better has been to start with a precise survey based upon latitude/longitude positions of the NGRS and to densify the existing network such that local corners, surveys and land parcels can all be related to the NGRS. Generally, computations are performed using geodetic, state plane, or local map projection coordinates. Many such 2-dimensional surveys have been performed in support of various mapping projects and establishment of geographic information systems. Leveling for elevation may/may not be included.

However, given current computer resources, 3-D GPS positioning technology, information management practices, and the need to share reliable spatial data, a better computational model is needed. The local geodetic horizon coordinate system and 3-D geocentric coordinates--part of the 3-D Geodetic Model described by Leick (1990)--meets those needs. The model is completely rigorous, 3-dimensional, and universal because one system includes the entire world.

The 3-D Geodetic Model utilizes many plane surveying practices and concepts. Latitude & departure from one point to another form the local 2-dimensional component and, just as in plane surveying (Burkholder 1994),

horizontal distance is the right triangle component of slope distance. There is no reduction to sea level or state plane scale factor to worry about because each slope distance is used in 3-dimensional space and right triangle components of that distance are used to describe local horizontal and vertical lengths.

In the 3-D system, vertical is not necessarily the difference in elevation, but is always the perpendicular distance from a point to the local tangent plane through the instrument station (also called the standpoint). Because the components are strictly spatial, rectangular, and 3-dimensional, there are no level surface computations in the 3-D system. However, elevations can be readily obtained at any point as the difference of the ellipsoid height (computed from X/Y/Z coordinates) and the geoid height (being obtained from active geodetic research and programs such as GEOID93). Of course, conventional bench marks, levels, and elevations can also continue to be used.

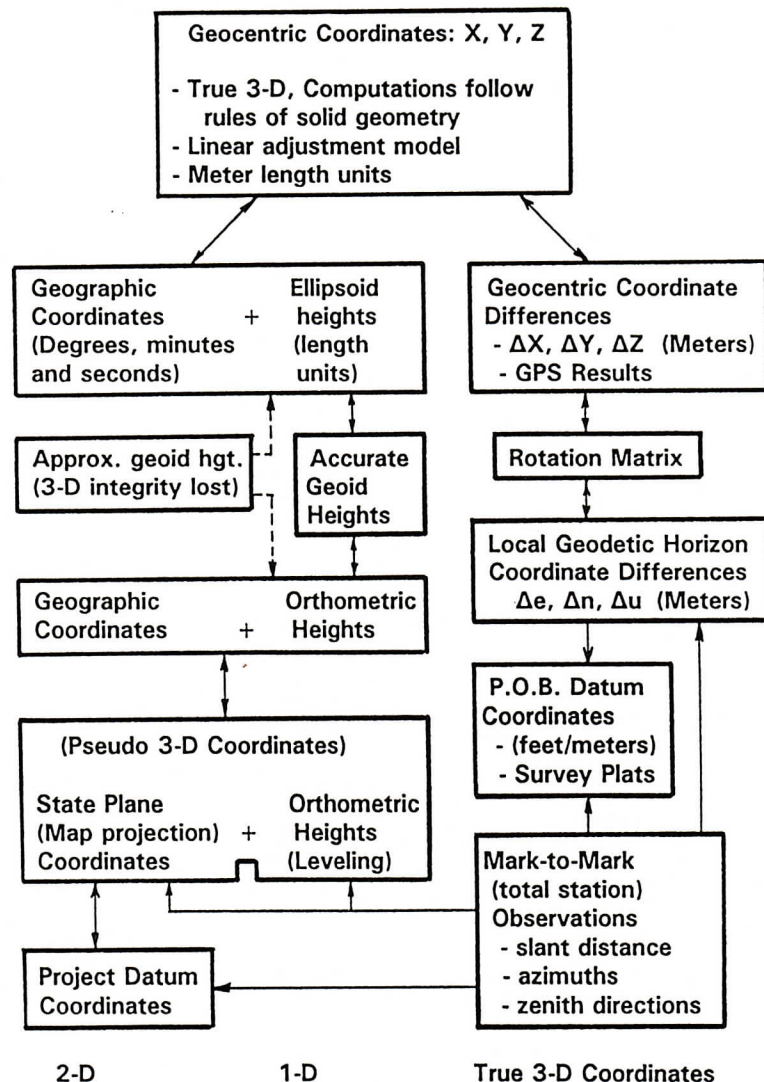


Fig. 4 Diagram Showing Relationship of Coordinate Systems

The connection between local geodetic horizon and 3-D geocentric coordinates is a rotation matrix described in detail by Burkholder (1994) and (1993a). Mathematical equations for transforming between the various systems are listed in both. The diagram in Figure 4 appears in both articles but is repeated here for convenience. Points to be made with reference to the diagram are:

1. The 3-D system fully incorporates use of existing state plane and geodetic coordinates. It is not a matter of discarding one system for another, but a matter of using a better model and which preserves valuable 3rd dimension information routinely collected during a GPS survey or aerial mapping.
2. Modern total station surveying instruments routinely collect the three basic measurements needed for 3-D computations--slope distance, zenith direction and azimuth. Yes, instrument/reflector heights are also specifically needed.
3. A master file of X/Y/Z coordinates is the most efficient way of storing spatial data. Answers in any other system can be obtained by identifying the system and converting data to that format--geodetic coordinates, state plane coordinates, UTM coordinates can all be obtained with equal ease. The same primary data are viewed or used in a chosen derivative system.

Philosophical Considerations & Conclusions

Consider how spatial data are used. If a single point location is needed--say for inventory purposes--uniqueness of location connected to some other attribute is more important than precise coordinates. Using the 3-D system is appropriate because it covers the entire world and near space. A master file of 3-D coordinates can be used and the unique tag brought out of the data base as X/Y/Z, geodetic, state plane, or UTM coordinates.

Many times, it is important to know where one point is with respect to another. That means one point comes out of the master file as the standpoint, shown as the P.O.B. in Figure 4. The location of the second point is brought out of the data base (master file) with respect to the first. The local geodetic horizon coordinate direction and distance (from Δe & Δn plane coordinates) provides the answer. The local relationship between any pair of points can be obtained from the master file simply by specifying the pair sequence, see Burkholder (1993c).

Mapping will be revolutionized. One method of mapping will be to choose a P.O.B. at the center of the map. Then all points on the map will be plotted with respect to the map center. It is the same as making each map on its own tangent plane projection. For large scale maps one P.O.B. per map will be sufficient as radial distortion grows to 1:1,000,000 at a distance of about 9 km from the P.O.B., Burkholder (1993b). Another method for small scale maps will be to choose a number of P.O.B.'s from which "neighborhood" points are plotted. The

criterion could be that the spacing between P.O.B. points could be increased until the images (plot) of a common point with respect to adjacent P.O.B.'s are separated by a measurable difference, say 0.25 mm. The point is that a map is made as a derivative product and point definition within the master file is unchanged. Many map sizes and scales can be made from the same master file.

The consequence for GIS's and spatial information management is that issues of compatibility are largely resolved. Once spatial location is defined in the 3-D system, it can be used in any of various formats. The technology for defining 3-D locations (GPS, total station surveying instruments, photogrammetric mapping) is already in place and being used. The added cost to define a point in the 3-D system is minimal. In some cases the data already exist.

This paper focuses on the appropriateness and geometrical integrity of the 3-D model. The issue of data quality is not addressed in this paper but is the topic of a previous paper in this same session. Of the many statements which can be made about data quality, one is that standard error propagation equations can be used to track error components of each coordinate triplet and provide error ellipses (standard deviations) with each answer derived from data in the master file.

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