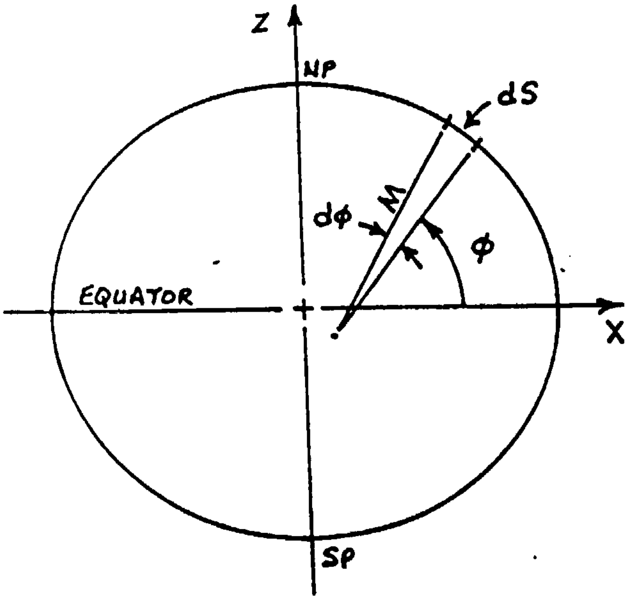
Meridian Arc Length

Early determinations of earth's size were made by measuring a portion of a meridian arc and comparing that length to the angle (usually found by astronomical observations) subtended at earth's center. Having any two of the three; arc length, central angle, or radius; the other is readily computed. Different values of arc length per degree of latitude at various latitudes implied (as stated separately by Newton) that the earth is ellipsoidal. But, having selected an ellipsoid model for the earth, the length of a meridian quadrant, or portion thereof, is computed by integrating differential geometry elements. Arc length of a differential element, <u>ds</u>, equals the instantaneous radius of curvature, <u>M</u>, times the differential increment in geodetic latitude, <u>$d\phi$</u> ($d\phi$ in radians).



$$ds = M\,d\phi \qquad (1)$$

S, the meridian arc length from one latitude to another, is obtained by integrating equation (1) between selected latitude limits. The meridian quadrant limits are 0° & 90° (in radian measure, 0 to $\pi/2$).

$$S_{\phi_1-\phi_2} = \int_{\phi_1}^{\phi_2} M\,d\phi \qquad (2)$$

The expression for M is substituted into equation (2) and the constant portion moved outside the integral to get:

$$S_{\phi_1-\phi_2} = a\,(1 - e^2) \int_{\phi_1}^{\phi_2} (1 - e^2 \sin^2 \phi\,)^{-3/2}\,d\phi \qquad (3)$$

Equation (3) is an elliptical integral and can't be integrated in closed form. That is, the expression inside the integral must be expressed in a series expansion containing ever smaller terms which can then be integrated individually. A solution is obtained by including all those terms which make a difference in the answer to the accuracy desired. Terms beyond those are dropped.

Either a binomial expansion or MacLaurian Series can be used with identical results. The derivation can be found in Rapp (1979). The resulting equation on the next page can be used between any specified latitude limits.

© 2001 Global COGO, Inc.