

$$D = 3 e^2 \sin \phi_1 \cos \phi_1 / (2 \rho (1 - e^2 \sin^2 \phi_1)) \quad \text{per second}$$

$$E = (1 + 3 \tan^2 \phi_1)(1 - e^2 \sin^2 \phi_1) / (6 a^2) \quad \text{per meter}^2$$

The constants B, C, and D are computed using  $\rho = 206264.8062470964$  seconds per radian,  $M$  = radius of curvature in the meridian,  $N$  = the ellipsoid normal, and  $e^2$  = the ellipsoid eccentricity squared. Note that  $(\Delta\phi'')$  appears on both sides of the equation for  $\Delta\phi''$  requiring an iterative solution. Use  $\Delta\phi'' = 0.0$  for the first iteration.

With the latitude of Point 2 known, the longitude (east) is calculated.

$$\lambda_2 = \lambda_1 + \Delta\lambda'' \quad \text{where (in seconds)}$$

$$\Delta\lambda'' = S \rho \sin \alpha_1 / (N_2 \cos \phi_2)$$

The geodetic line azimuth at Point 2 (add  $180^\circ$  to get the back azimuth) can be computed using Clairaut's Constant, however, the Puissant formulas use:

$$\alpha_2 = \alpha_1 + \Delta\alpha'' \quad \text{where (in seconds)}$$

$$\Delta\alpha = \Delta\lambda'' \sin \phi_m / \cos (\Delta\phi/2) + (\Delta\lambda'')^3 \sin \phi_m \cos^2 \phi_m / (12 \rho^2)$$

$$\text{and } \phi_m = (\phi_1 + \phi_2)/2$$

Inverse: Given the latitude and longitude of two points, it is required to find the geodetic line azimuth at each point and the distance (along the geodetic line) from one point to the other. For exacting applications, long lines or cases where the distance along the normal section line may be preferred see references such as Rapp (1979), Jank/Kivioja (1980), Burkholder (1987) or other standard geodesy texts.

$$\Delta\phi = \phi_2 - \phi_1 \quad \text{and} \quad \Delta\lambda = \lambda_2 - \lambda_1 \quad (\text{east longitude})$$

$$x = (\Delta\lambda'' N_2 \cos \phi_2) / \rho = S \sin \alpha_1$$

$$y = (1/B)[\Delta\phi'' + C x^2 + D (\Delta\phi'')^2 + E (\Delta\phi'') x^2] = S \cos \alpha_1$$

$$\tan \alpha_1 = (S \sin \alpha_1 / S \cos \alpha_1) = x/y \quad (\text{from north})$$

$$S = \sqrt{(x^2 + y^2)} \quad \text{in meters}$$