Horizontal Distance Options Supported by the 3-D Global Spatial Data Model (GSDM)

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Short Bio Earl F. Burkholder is a professional surveyor, professional engineer, and Fellow of the American Society of Civil Engineers. He taught surveying at New Mexico State University for 12 years and retired from teaching in 2010. Previously he worked 5 years for an international engineering firm, taught for 13 years at the Oregon Institute of Technology, and was self-employed for 5 years in the 1990s. He wrote a book, "The 3-D Global Spatial Data Model: Foundation of the Spatial Data Infrastructure," published by CRC Press in 2008. Since retirement he has continued to promote use of the 3-D model and, at the request of the publisher, authored a 2nd Edition of the 3-D book – published with a 2018 copyright date.

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Abstract

Existing definitions of horizontal distance are used extensively and successfully. But in a broader context, traditional definitions can be ambiguous. Therefore, the goal of this article is to use the global spatial data model (GSDM) to enhance the clarity of horizontal distance concepts. Horizontal distance is often defined as perpendicular to a plumb line at one point (in a flat plane). Or, horizontal distance may be defined as continuously perpendicular to the plumb line (ocean surface). A different issue, but closely related, is whether the plumb lines at either end of a horizontal distance are taken to be parallel or converging. Parallel plumb lines imply a flat Earth but, if the plumb lines are not taken to be parallel, the length of a horizontal distance changes with elevation. A longer horizontal distance is correlated with a higher elevation. Finally, is a horizontal distance referenced to the plumb line or to the ellipsoid normal? Physical measurements are typically referenced to the plumb line while computations using geospatial data are generally referenced to the ellipsoid normal. This article looks at horizontal distances as computed from stored geospatial data with a focus on those options directly supported by the 3-D GSDM.

Key Words Horizontal Distance, Plumb Line, Geometrical Geodesy, Ellipsoid Normal, Earth-centered Earth-fixed (ECEF), Global Spatial Data Model (GSDM)

Introduction

Traditional concepts of horizontal distance enjoy near universal acceptance and remain valid in many applications. However, additional attention is warranted when working with 3-D digital spatial data - especially when viewed in the context of a more robust geometrical model.

Continued use of a default definition may be acceptable although other options exist. Since geometrical circumstance and user preference are both variables, the choice of definition for horizontal distance, whatever it is, should be documented for the benefit of subsequent users. Clarity of such choices can be enhanced if horizontal distance is considered in the context of the 3-D global spatial data model (GSDM), Burkholder (1997) .

For local applications, the general presumption is that horizontal distance is related to ground level and to the local plumb line. For nominal distances, the right triangle component of a slope distance is used successfully as the horizontal distance – elevation is generally immaterial. The implicit assumption is that plumb lines at either end of a line are parallel, see Figure 1. For longer lines, that assumption is not valid. A better model accommodates converging plumb lines and provides the tangent plane distance between plumb lines, see Figure 2. And, for surveys involving significant elevation differences, standard practice is to reference horizontal distances to a common elevation. In such cases, the elevation of a horizontal distance should not be left to conjecture.

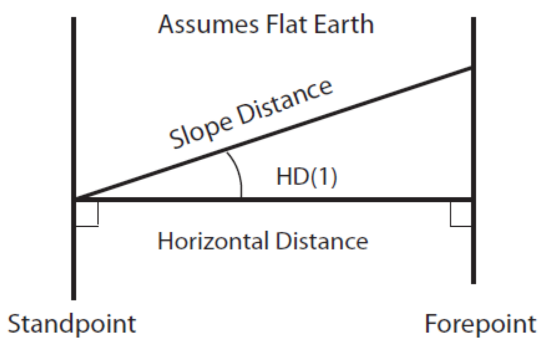


Figure 1,
Right Triangle Component of Slope Distance

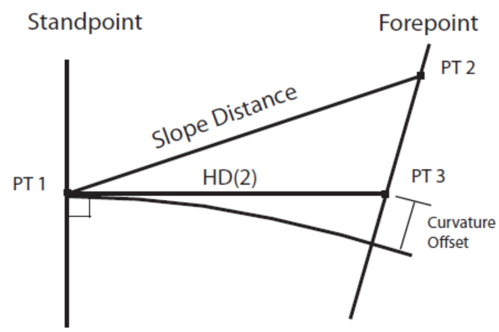


Figure 2
Tangent Plane Distance Between Ellipsoid Normals

Six different definitions for horizontal distance were identified in Burkholder (1991). In that case, all six definitions were referenced to the plumb line and computation of horizontal distance was presumed to originate with physical measurements. In that article, the plumb line and the ellipsoid normal were taken to be coincident. In this article, the definition of horizontal distance is referenced specifically to the ellipsoid normal and horizontal distances are derived from rectangular X/Y/Z Earth-centered Earth-fixed (ECEF) coordinates. Equations for computing various normal-based geometrical relationships are included in Appendix A. As before, the choice of definition of "horizontal" is left to the user.

But this article leaves a gap to be addressed by others. In the context of horizontal distances, what are the consequences, if any, of assuming that the plumb line is coincident with the ellipsoid normal? Note, some of the following are plumb-line-based and others are normal-based.

- Electronic distance measuring instrument (EDMI) and total station measurements are plumb-line-based. Traverses are observed, computed, and results are stored in a file. Those coordinate values are subsequently used to compute angles, distances, areas, and volumes.

- GPS data are collected as either a single point position or as a vector between points. In either case, the GPS observations are reduced to ECEF coordinates or ECEF differences before being expressed in a traditional normal-based coordinate system. GPS results are normal-based.

- Ray tracing and ray intersections are primary operations in photogrammetric mapping. Although orientation of the camera is plumb-line-referenced, the extracted geometry is affected only slightly, if at all, by the direction of the local plumb line. Maps are compiled by fitting ray-tracing geometry and coordinate differences to pre-existing ground control points.

- In LiDAR and terrestrial scanning observations, the primary raw data are 3-D spatial distances to an object that returns the transmitted signal. Although directions are also observed, angular resolution of the scanner is generally considered subordinate to the observed distances. As in photogrammetric practice, maps (or digital point clouds) are compiled by fitting the observed distances and reduced components to independent (pre-existing) control.

- Inertial measurements from gyroscopes and accelerometers are specifically oriented to the plumb line. But those raw data are processed such that the horizontal results of an inertial survey are generally published as normal-based while elevations are published as plumb-line-based.

- Traditional leveling observations are directly associated with the local plumb line. It seems that the importance of elevation data justifies publication of a separate vertical datum (plumb-line-based) rather than incorporating third-dimension observations into a normal-based 3-D database.

Traditional horizontal distance expressions

Burkholder (1991) identifies the following plumb-line-based horizontal distance options:

- HD(1) = the right triangle component of a slope distance.
- HD(2) = the distance between two plumb lines in a plane tangent to the Earth at the instrument station.
- (HD(3) = the chord distance between two plumb lines. The two end points have the same elevation and the chord is perpendicular to the vertical (plumb line) only at the chord midpoint.
- (HD)4 = the arc distance along some level surface between two plumb lines.
- HD(5) = the arc distance at mean sea level between two plumb lines.
- HD(6) = the distance along the geodesic on the ellipsoid surface between two plumb lines.

The intent in listing those six alternatives was to include options from approximate to precise. That is, from plane surveying, HD(1), to geodesy, HD(6). Numerical comparisons and graphs included in the 1991 article show differences between the various options. The convention used then, and repeated here, is that the station occupied (instrument station) is referred to as the standpoint while the other end of the line (reflector) is referred to as the forepoint. An incidental issue is that foot units were used in the 1991 article (either U.S. Survey or International). The GSDM uses meters exclusively – with an exception. When reporting distances computed from ECEF coordinates, the choice of units displayed is the prerogative of the user. The original distance is in meters, but those metric values may be converted to other units as dictated by a given application – see the P.O.B. Datum option (page 6, Burkholder 1997). The underlying X/Y/Z values are metric and remain unchanged.

Fundamental concepts and questions to be clarified

While consensus on a unique definition of horizontal distance may not be possible, the following issues are "put on the table."

- Is horizontal distance perpendicular to the plumb line at a point or perpendicular to a surface at all points? Depending upon circumstances, each definition has been used.
- In what cases is a flat-Earth assumption acceptable? That is, ellipsoid normals at the end points are assumed to be parallel. Some tolerance level may guide a choice - see Table 1 and Figure 6.
- If flat-Earth assumptions are not appropriate, then elevation is an attribute of horizontal distance and should be documented as part of the meta data.
- What difference does it make if horizontal distance is referenced to the plumb line or to the ellipsoid normal? Although some may answer "none," the contrary may be shown.
- As a close approximation for short distances, horizontal distance can be taken to be a chord distance between ellipsoid normals. A more rigorously computed arc distance between points may be a better representation of "horizontal." Either way, elevation is important.
- What difference does it make, if any, whether horizontal distance is the result of a physical measurement or the result of an inverse computation between coordinates? The question really may be, "Which is more ambiguous, the concept of "horizontal" or the definition of "horizontal"?

Computing normal-based horizontal distance from ECEF coordinates

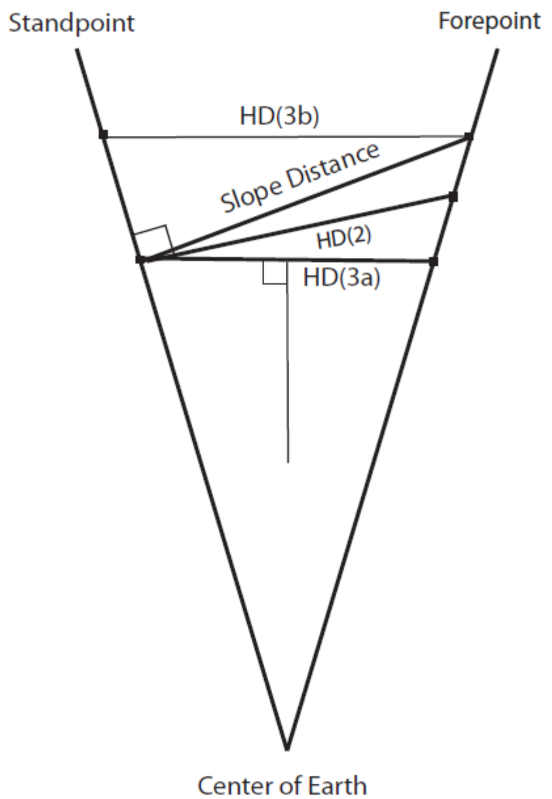
This section identifies various distance computations based on ECEF coordinates. As such, those results all come under the broad umbrella of a generic distance inverse in 3-D space – see Burkholder (2016). The Theorem of Pythagoras is used to compute a generic slope distance as:

$$3\text{–}D\ Dist = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2} \quad \text{where} \tag{1}$$

$$\Delta X = X_{forepoint} - X_{standpoint} \tag{2}$$
$$\Delta Y = Y_{forepoint} - Y_{standpoint} \tag{3}$$
$$\Delta Z = Z_{forepoint} - Z_{standpoint} \tag{4}$$

Depending on the geometry of the standpoint and forepoint, several horizontal distance options can be obtained using equation 1. For example, if the ellipsoid heights of both standpoint and forepoint are the same, the result is HD(3), see Figure 3. If the ellipsoid height for both standpoint and forepoint is zero, the result is a chord distance on the ellipsoid. As illustrated in

Figure 4, the arcs of HD(4) and HD(6) can each be obtained using the standard chord-to-arc procedure, also described in Burkholder (2016). Computation of normal-based sea level arc distance, HD(5), is not addressed in this article.



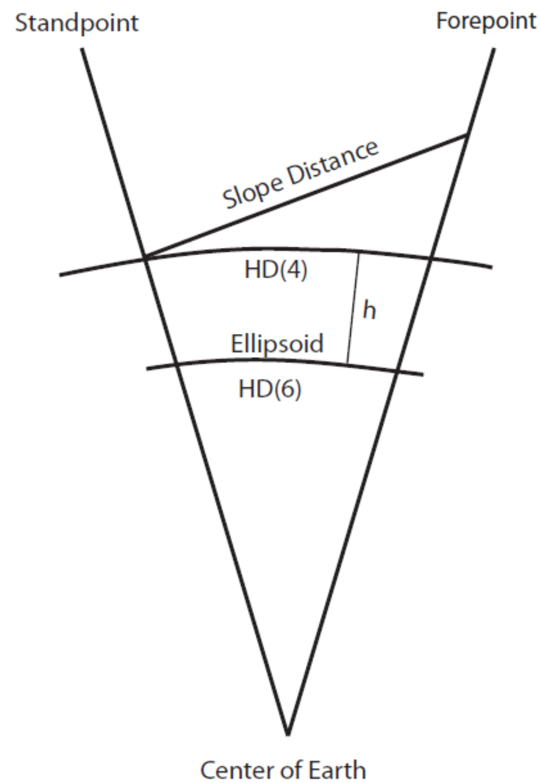


Figure 3
Chord Distances at Standpoint and Forepoint

Figure 4
Arc Distances at Standpoint and on Ellipsoid

HD(1) is the right triangle component of a slope distance and used extensively in local applications. HD(1) is also the horizontal component of an observed GPS vector. The only difference (largely inconsequential) is that HD(1) in the 1991 article is plumb-line based while a GPS vector component is ellipsoid-normal based. Thus, the HD(1) similarity provides a convenient direct link between GPS surveying and plane surveying. The normal-based equations for computing HD(1) from a GPS vector or stored ECEF coordinates are:

$$HD(1) = \sqrt{\Delta e^2 + \Delta n^2} \qquad \text{where} \qquad (5)$$

$$\Delta e = -\Delta X \sin\lambda + \Delta Y \cos\lambda \qquad (6)$$

$$\Delta n = -\Delta X \sin\phi \cos\lambda - \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi \qquad (7)$$

$\Delta X/\Delta Y/\Delta Z$ are geocentric components of a GPS vector standpoint to forepoint.
ϕ and λ are the latitude (north) and longitude (east) of the standpoint.

Equation 5 is quite powerful, but it has limitations. How far can a right triangle horizontal distance be pushed before convergence of the ellipsoid normals becomes an issue? It might not be significant for nominal distances but, the HD(1) distance standpoint to forepoint is not identical to the HD(1) distance from the forepoint to the standpoint. That is because HD(1) lies in a tangent plane through the standpoint. If the computational direction is reversed, the latitude and longitude in equations (6) and (7) are not the same, the tangent plane is not the same, and the horizontal distance between points (forward and back) is slightly different . That difference is discussed later. So, the challenge now is to find a definition for horizontal distance that is the same (or very nearly so) in both directions. According to Armstrong et., al. (2017), a good approximation for a unique bi-directional horizontal distance is:

$$HD = \sqrt{(\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2 - \Delta h^2} \quad \text{where,} \qquad (8)$$

Δh = ellipsoid height difference between standpoint and forepoint.

Equation 8 is recognized as equation 1 modified by the ellipsoid height difference. All terms in equation 8 are squared which means it is immaterial whether any of those differences is positive or negative. The validity of equation 8 is confirmed later in this article even though it contains a geometrical imperfection; h_1 is rarely, if ever, parallel with h_2.

But first, several questions, is there a practical distance limit to the use of equation 8 and what is the geometrical meaning of a "long" distance computed using equation 8? Those answers may also have an impact on one's choice of a definition for horizontal distance. Figure 5 shows a diagram for computing various horizontal distances – all using equation 1 based on geocentric X/Y/Z coordinate values for the numbered endpoints. The presumption is that a standpoint and a forepoint are the basis for subsequent computations. It is further presumed that the standpoint and forepoint have different ellipsoid heights. Using equation 1, a 3-D spatial (slope) distance is computed from the X/Y/Z coordinates of the endpoints. As shown in Figure 5, ECEF coordinates for additional points on the ellipsoid normal are needed to compute additional distances using equation 1.

As shown in Figure 2, Point 3 lies at the intersection of the tangent plane through the standpoint and the ellipsoid normal through the forepoint. The X/Y/Z coordinates for Point 3 are computed using the latitude and longitude of the forepoint. But, the ellipsoid height for Point 3 is the ellipsoid height at the standpoint plus the curvature offset at the forepoint. With X/Y/Z coordinates for both Point 1 and Point 3, the HD(2) horizontal distance is computed using equation 1. See Appendix B for computing the curvature offset. Appendix A includes the equations for computing X/Y/Z coordinates from latitude/longitude/ellipsoid height (called a BK1 transformation) and for computing latitude/longitude/ellipsoid heights from X/Y/Z coordinates (called a BK2 transformation).

The X/Y/Z coordinates of other points in Figure 5 are computed as:

- Point 4 Defined by latitude/longitude at Point 1 and ellipsoid height at Point 2.
- Point 5 Defined by latitude/longitude at Point 2 and ellipsoid height at Point 1.
- Point 6 Defined by latitude/longitude at Point 1 and ellipsoid height of 0 m (on ellipsoid).
- Point 7 Defined by latitude/longitude at Point 2 and ellipsoid height of 0 m (on ellipsoid).

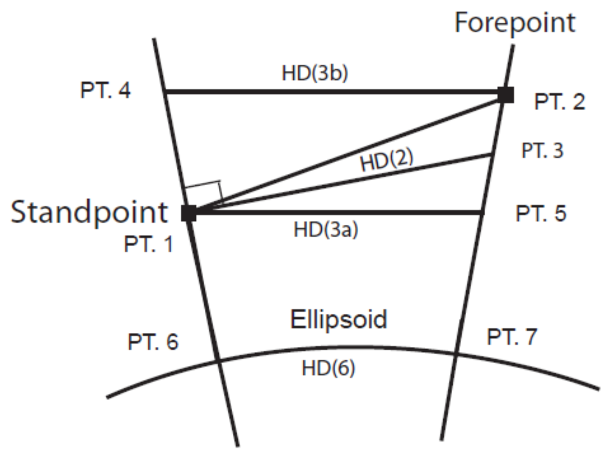


Figure 5, Diagram of Point Numbers

With the X/Y/Z coordinate values of Points 1 thru 7 in hand, equation 1 is used to compute:

HD(2)	Tangent plane distance between ellipsoid normals.
HD(3a)	Chord distance at standpoint ellipsoid height.
HD(3b)	Chord distance at forepoint ellipsoid height.

HD(4)	Arc distance computed from chord distance HD(3a) – Figure 4.
HD(6)	Arc distance computed from ellipsoid chord between Points 6 and 7 – Figure 4.

The horizontal distance designations were chosen to be consistent with the labeling used in Burkholder (1991). As stated previously, HD(5) is not addressed in this article.

HD(3a) and HD(3b) are slightly different due to ellipsoid height differences. A unique horizontal distance between standpoint and forepoint can be computed as the mean of HD(3a) and HD(3b). As it turns out, the mean of HD(3a) and HD(3b) is the same as a horizontal distance computed using equation 8. Incidentally, equation 8 is much easier to use than finding the mean of HD(3a) and HD(3b). The author has attempted, without success, to show a mathematical equivalence between the mean of HD(3a) and HD(3b) and the horizontal distance from equation 8. However, the numerical equivalency is shown in Table 1 and later in this article in the New Mexico Principal Meridian example.

Consequence

If the number of horizontal distance alternatives seems excessive, the reader can be assured that continued use HD(1) is justifiable in many cases. Table 1 was generated by tabulating values for various distance/vertical angle combinations and Figure 6 was developed by plotting associated ratio-of-precision isolines for 1:10,000, 1:20,000, 1:50,000, 1:100,000 and 1:400,000. Figure 6 parallels information shown in Figure 4 of Burkholder (1991) and provides a comparison of

HD(1) with HD(2) for distances up to 5,000 meters. As an example, a survey having a nominal accuracy of 1:50,000 could absorb a horizontal distance systematic error of 1:100,000 without detrimental consequence. As such, limits of distance and vertical angle combinations can be readily deduced from the isolines shown in Figure 6. Although not shown here, similar results could be developed for comparison of HD(1) with HD(3) or with a horizontal distance computed using equation 8.

Table 1 Computation of Horizontal Distances by Various Methods

Slope Distance of 1,000.0 m

Vertical Angle -Deg	HD(1)	HD(2)	HD(3a)	HD(3b)	Mean 3a & 3b	Equation 8 HD	HD(1) - HD(2)	Ratio 1:xx,xxx
All distances in meters.								
1	999.8477	999.8450	999.8449	999.8477	999.8463	999.8463	0.0027	364,991
2	999.3908	999.3854	999.3853	999.3908	999.3881	999.3881	0.0055	182,525
4	997.5641	997.5531	997.5531	997.5641	997.5586	997.5586	0.0109	91,319
8	990.2681	990.2464	990.2464	990.2681	990.2572	990.2572	0.0216	45,772
12	978.1476	978.1157	978.1157	978.1476	978.1316	978.1316	0.0319	30,639

Slope Distance of 2,000.0 m

Vertical Angle -Deg	HD(1)	HD(2)	HD(3a)	HD(3b)	Mean 3a & 3b	Equation 8 HD	HD(1) - HD(2)	Ratio 1:xx,xxx
All distances in meters.								
1	1,999.6954	1,999.6844	1,999.6844	1,999.6954	1,999.6899	1,999.6899	0.0110	182,498
2	1,998.7817	1,998.7598	1,998.7597	1,998.7817	1,998.7707	1,998.7707	0.0219	91,264
4	1,995.1281	1,995.0844	1,995.0843	1,995.1281	1,995.1062	1,995.1062	0.0437	45,660
8	1,980.5361	1,980.4496	1,980.4495	1,980.5362	1,980.4928	1,980.4928	0.0865	22,886
12	1,956.2952	1,956.1675	1,956.1674	1,956.2952	1,956.2313	1,956.2313	0.1277	15,320

Slope Distance of 5,000.0 m

Vertical Angle -Deg	HD(1)	HD(2)	HD(3a)	HD(3b)	Mean 3a & 3b	Equation 8 HD	HD(1) - HD(2)	Ratio 1:xx,xxx
All distances in meters.								
1	4,999.2385	4,999.1700	4,999.1688	4,999.2389	4,999.2039	4,999.2039	0.0685	73,000
2	4,996.9541	4,996.8173	4,996.8161	4,996.9545	4,996.8853	4,996.8853	0.1369	36,506
4	4,987.8203	4,987.5472	4,987.5460	4,987.8206	4,987.6833	4,987.6833	0.2731	18,265
8	4,951.3403	4,950.7995	4,950.7984	4,951.3407	4,951.0696	4,951.0696	0.5408	9,155
12	4,890.7380	4,889.9400	4,889.9389	4,890.7384	4,890.3386	4,890.3396	0.798	6,129

Observations from Table 1 include

1. HD(1) and HD(2) are co-linear. A decision to use HD(2) for horizontal distance as opposed to HD(1) may be influenced by the tolerance requirements of a given application. Also, see qualifying comment in the “specific observations” section.

2. There is very little difference between HD(2) and HD(3a). Since computing HD(2) involves first computing the curvature offset, HD(3a) can often be used in place of HD(2).

3. HD(3a) and HD(3b) are both chords, not horizontal distances. But, the mean of HD(3a) and HD(3b) is numerically the same as the “horizontal distance” computed using equation 8.

Figure 6 is a graph of ratio-of-precision isolines for various combinations of slope distances and vertical angles.

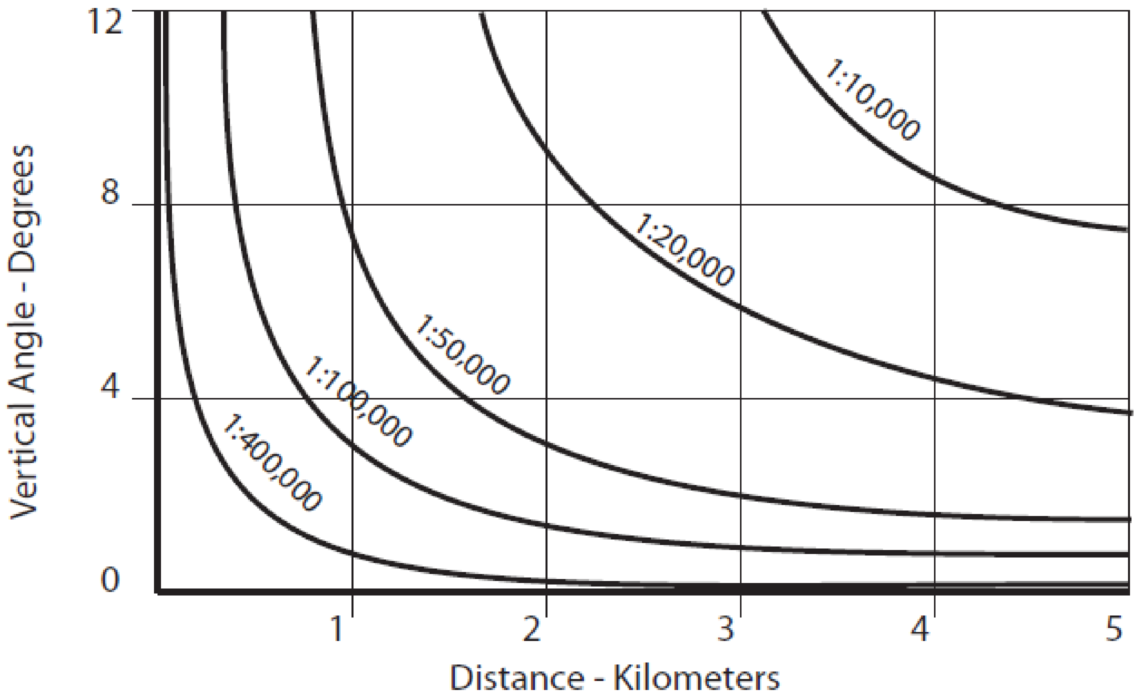


Figure 6, Plot of Comparison of HD(1) Distance with HD(2) Distance

Numerical Example

The project shown in Figure 7 is Example 5 in Chapter 15 of Burkholder (2018). The equation 8 distance in that example is over 222 km and should not be called a horizontal distance.

The New Mexico Initial Point (for the U.S. Public Land Survey System – USPLSS) is Point 201 and taken to be the standpoint. ECEF coordinates for 201 are as published by NGS (PID AI5439). Point 202 is the forepoint and lies on the New Mexico Principal Meridian. It was established by an NMSU class project (see www.globalcogo.com/3DGPS.pdf) as reported in Example 4, Chapter 15 of Burkholder (2018). All ECEF values in this example are NAD 83 (2011).

Initial Point - 201	SW Cor. Sec. 31, T23S-R1E - 202
X = -1,533,309.884 m	X = -1,568,698.064 m
Y = -5,050,681.721 m	Y = -5,167,107.065 m
Z = 3,571,149.193 m	Z = 3,385,214.088 m
ϕ = 34° 15’ 35.”94618 N	ϕ = 32° 15’ 24.”28892 N
λ = 106° 53’ 14.”96154 W	λ = 106° 53’ 16.”54133 W
= 253° 06’ 45.”03846 E	= 253° 06’ 43.”45867 E
h = 1,475.929 m	h = 1,259.566 m

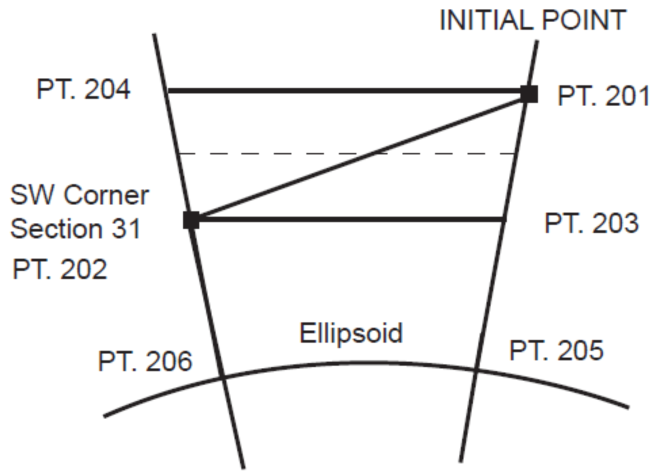


Figure 7,
Diagram of New Mexico Initial Point and SW Corner, Section 31, T23S-R1E,
New Mexico Principal Meridian

Auxiliary points were computed using equations in Appendix A.

Pt 203 was computed using the latitude/longitude of Pt 201 and the ellipsoid height of Pt 202.

Pt 204 was computed using the latitude/longitude of Pt 202 and the ellipsoid height of Pt 201.

Pt 205 was computed using the latitude/longitude of Pt 201 and an ellipsoid height of 0 m.

Pt 206 was computed using the latitude/longitude of Pt 202 and an ellipsoid height of 0 m.

	Point 203	Point 204	Point 205	Point 206
X =	-1,533,257.938 m	-1,568,751.217 m	-1,532,955.528 m	-1,568,388.631 m
Y =	-5,050,510.611 m	-5,167,282.144 m	-5,049,514.482 m	-5,166,087.830 m
Z =	3,571,027.392 m	3,385,329.563 m	3,570,318.320 m	3,384,541.839 m
h =	1,259.566 m	1,475.929 m	0.000 m	0.000 m

Equation 1 was used for each of the following four computations.

3-D slope distance, 201 to 202 – obviously not horizontal	= 222,213.968 m
Chord distance at ellipsoid, 205 to 206	= 222,166.044 m
Chord distance 201 to 204, at ellipsoid height of 201	= 222,217.645 m
Chord distance 202 to 203, at ellipsoid height of 202	= 222,210.080 m

What is the “best” horizontal distance between the Initial Point and the SW Corner Section 31?

Mean chord distance between ellipsoid normals	= 222,213.862 m
Horizontal distance 201 to 202 using equation 8	= 222,213.862 m

Even with the excellent agreement of the equation 8 distance with the mean of the chord distances, that distance is too long to be called a horizontal distance. A better choice would be to determine the arc distance, either on the ellipsoid or at a mean ellipsoid height – both of those choices are documented in Example 5 (Burkholder 2018) and compared to the "horizontal distance" laid out by the original government surveyors.

Ellipsoid distance Pt 201 to Pt 202	= 222,177.307 m
Arc distance at mean ellipsoid height	= 222,225.128 m
Theoretical "ground" distance, 23 townships * 6 miles/township	= 222,089.92 m.

The 1973 BLM Manual stipulates that Principal Meridians are to be laid out within a tolerance of 7 links per 80 chains (0.07/80) or 1:1,143. The approximate ratio or precision for this example is:

$$(222{,}089.92 \text{ m} - 222{,}225.128 \text{ m}) / 222{,}225.128 \text{ m} \quad \text{or} \quad 1{:}1{,}644$$

According to these results, that portion of the New Mexico Principal Meridian was laid out within BLM tolerance. Incidentally, there is some pretty rough terrain in that 222 kilometers (138 miles).

An informative "aside" for this project compares the HD(1) distances standpoint to forepoint with the HD(1) distance forepoint to standpoint. As a matter of interest, those results are:

Initial Point to SW Corner of Section 31, 201 to 202,	HD(1) = 222,176.131 m
SW Corner of Section 31 to Initial Point, 202 to 201,	HD(1) = 222,183.686 m

A legitimate question here is, "What is the significance of the difference and under what circumstances might one HD(1) distance (forward or back) be required over the other?" The following section contains a recommended solution (for reasonable distances). That is, use HD(1) from the selected P.O.B. to each point in the project and use those tangent plane coordinate differences as plane coordinates for the project. Also see comment in "specific observations."

Another scenario

The previous distance examples are all based on a point-pair combination. Another option involves using the P.O. B. Datum coordinates as described in Chapter 1 of Burkholder (2008 and 2018). That option of the GSDM fully supports traditional flat-Earth surveying practices without sacrificing geometrical integrity of 3-D spatial relationships. An example of a 2-D (land surveying) plat based on a 3-D GPS survey is described in the previous link to a class project at NMSU (http://www.globalcogo.com/3DGPS.pdf). In that case, the user selects any point (generally a point local on the survey) for the P.O.B. and computes local east and north differences from the selected P.O.B. to each and any other points in the project. Those local eastings and northings become local tangent plane coordinates that support computation of unique local horizontal distances in that tangent plane and local azimuths with respect to the true meridian through the P.O.B. The example 4, see link above or page 407 in Burkholder (2018), gives the step-by-step procedure for developing a 2-D plat based on a 3-D GPS survey. That survey reports HD(1) distances at the ground level of P.O.B. without distortion. It also gives all azimuths with respect to the true north meridian through the SW Corner of Section 31, the point selected as the P.O.B. Figure 6 in this article provides a visual estimate as to the limit for which one might feel comfortable using those HD(1) distances from the P.O.B. Significant differences in terrain heights can become a factor in making that decision.

Specific observations

1. Equation 1 computes the 3-D spatial distance. It is simple, easy to use, and there is no limit, practical or otherwise, to computing 3-D spatial distances from ECEF coordinates.

2. HD(1) is the simple right triangle component of a slope distance and is useful for both plane and GPS surveying within limitations of flat-Earth assumptions. The disadvantage is that a distance from "here" to "there" is very slightly different from the distance from "there" to "here." But that difference is not a disadvantage if using the P.O.B. datum coordinates (Burkholder 1997) as the basis for creating a 2-D plat of a survey.

3. Equation 8 can be used to compute a unique bi-directional "horizontal distance" at the mean ellipsoid height of standpoint and forepoint for any line length. But, the limit of practical use has not been established. At some undefined limit, the equation 8 distance fails to meet an intuitive definition of horizontal distance.

4. The following are not obvious but need to be acknowledged and discussed as appropriate.

    a. Computation of "new" X/Y/Z coordinates (from either point-position or vector data) from ECEF components based on GPS or photogrammetric data does not involve local plumb line or gravity observations. The definition of horizontal distance is moot.

    b. Local terrestrial observations in 3-D space may need to be modified from plumb-line-based to normal-based (using deflection-of-the-vertical data). But, rarely are horizontal data primary to the determination of a "new" point. The 3-D position of a "new" point is computed from a $\Delta e/\Delta n/\Delta u$ vector (with respect to the standpoint) rotated to $\Delta X/\Delta Y/\Delta Z$ components. Again, the definition of horizontal distance is largely moot.

    c. The user has various options available when computing horizontal distance from stored ECEF coordinates. There are no known universal criteria for determining when to use:

        - HD(1): Most common and enjoys near "universal" application.
        - HD(2): Rarely needed. Appropriately approximated by HD(3).
        - Mean chord distance of HD(3a) and HD(3b): Obtained using equation 8. (This distance is "unique" between points, but its relevance is limited.)
        - HD(4): For long lines, better than chord HD(3) at a given ellipsoid height.
        - HD(6): This distance provides a link to precise geodesy computations on the ellipsoid.

Overview observations

The GSDM (Burkholder 1997) has been identified an appropriate geometrical model for spatial data disciplines worldwide. All location information can be used efficiently within the context of the ECEF coordinate system. The GSDM is prefaced on the assumption of a single origin for geospatial data – Earth's center of mass. Long-standing rules of solid geometry are applied in the context of the ECEF coordinate system and rules of error propagation provide the basis for the stochastic portion of the GSDM. This article uses the Theorem of Pythagoras to compute a distance between any two points having ECEF coordinates. Whether that distance is horizontal or not is determined by the geometry of problem and the choices of the user. Overview observations include:

1. The GSDM embodies the true 3-D characteristics of digital spatial data and supports various options for efficient distance computations. Subsequent users will enjoy benefits to the extent current users include appropriate meta data.

2. When working with spatial and geospatial data, the distinction between plumb-line-based data and normal-based data will, in the future, become more critical in formulation of policies governing efficient use of 3-D digital spatial data.

3. It is possible that the most important impact of this article was omitted – what is the role of gravity? Gravity is the underlying difference between spatial and geospatial data. Are spatial data (data with no gravity considerations) a sub-set of geospatial data or are geospatial (data referenced to planet Earth) a sub-set of spatial data? Is seems that "georeferencing" is the acceptable discriminator between spatial and geospatial data. This author believes that oversimplification belies the important role of gravity. Future standards, specifications, and policy will need to be more specific about the difference gravity makes.

4. A subsequent more theoretical paper on "models" is being developed to provide additional insight. Two important sources include Kuhn (1996) and Pruneau (2017).

Corrections to information in Burkholder (1991)

Note – The following two corrections apply to the information included in Burkholder (1991).

1. Equation 8 on page 108 should read:

$$HD(4) = (R + h_1)\,\theta = 2\,(R + h_1)\,sin^{-1}\left(\frac{HD(3)}{2\,(R + h_1)}\right)$$

2. It is stated incorrectly on page 110, item 3, that HD(1) is co-linear with HD(3). In fact, HD(1) is co-linear with HD(2).

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Appendix A

Material in this Appendix is a sub-set of the equations listed in Chapter 1 of Burkholder (2008 and 2018).

I. Transformations between geodetic and geocentric coordinate values

Bi-directional transformation between geodetic latitude/longitude/ellipsoid height and Earth-center Earth-fixed (ECEF) rectangular X/Y/Z coordinates is required when using the global spatial data model (GSDM). These transformations require a mathematical ellipsoid and a location for the point(s) being transformed. When using the GSDM, the transformation from geodetic to geocentric coordinates is referred to as a BK1 transformation while the geocentric to geodetic transformation is referred to as a BK2 transformation. Equations for both BK1 and BK2 transformations are given in this section.

For purposes of this example, the GRS 80 ellipsoid parameters are used (Burkholder 1984). Other ellipsoids such as the WGS 84 can also be used. Selection of the appropriate ellipsoid is the responsibility of the user.

GRS 80: $a = 6{,}378{,}137$ meters exact
$1/f = 298.2572221008827$
$e^2 = 2f - f^2 = 0.0066943800229034$

<u>BK1 Transformation:</u> Geodetic to Geocentric

Input: Ellipsoid parameters, a and e^2
Geodetic latitude, North is + and south is - ϕ
Geodetic longitude, East is + and west is - λ
Ellipsoid height in meters above the ellipsoid is + and below it is - h

Equations:

$$N = \frac{a}{\sqrt{1-e^2 \sin^2 \phi}} \quad \text{(A1)}$$

$$X = (N + h)\cos\phi\cos\lambda \quad \text{(A2)}$$

$$Y = (N + h)\cos\phi\sin\lambda \quad \text{(A3)}$$

$$Z = (N[1 - e^2] + h)\sin\phi \quad \text{(A4)}$$

<u>BK2 Transformation:</u> Geocentric to Geodetic

Input: Ellipsoid parameters, a and e^2
X/Y/Z geocentric coordinates in meters

Equations:

$$\lambda = tan^{-1}\left(\frac{Y}{X}\right)$$ east longitude with due regard to quadrant. (A5)

$$\emptyset = tan^{-1}\left[\frac{Z}{\sqrt{X^2+Y^2}}\left(1+\frac{e^2\,N\sin\emptyset}{Z}\right)\right] \qquad (A6)$$

Equation A6 is closed form but must be iterated because ϕ appears on both sides of the equals sign. This is not a problem, but it can be a nuisance. An iteration example is given on page 161 of Burkholder (2018) followed immediately by a non-iteration solution of the same problem. Other legitimate non-iterative solutions are readily available – see for example page 77 of Meyer (2010).

After equation A6 is solved, the ellipsoid height is found using equation A7.

$$h = \frac{\sqrt{X^2+Y^2}}{\cos\emptyset} - N \qquad (A7)$$

The choice of using an iteration or another method for performing a BK2 transformation is left to the user. In either case, the user should employ a "confidence builder" as needed to verify integrity of solution. That is, the values found in a BK2 solution can be used in a "reverse" BK1 transformation. The result should come out to be the same as the input values for the BK2 solution.

<u>BK3 and BK4 Transformations:</u> 3D version of Geodetic Forward and Inverse Computations

BK3 (forward)

$$X_2 = X_1 + \Delta X \qquad (A8)$$
$$Y_2 = Y_1 + \Delta Y \qquad (A9)$$
$$Z_2 = Z_1 + \Delta Z \qquad (A10)$$

BK4 (inverse)

$$\Delta X = X_2 - X_1 \qquad (A11)$$
$$\Delta Y = Y_2 - Y_1 \qquad (A12)$$
$$\Delta Z = Z_2 - Z_1 \qquad (A13)$$

<u>BK8 and BK9 Transformations:</u> Rotations between local and geocentric perspectives

BK8 (geocentric to local)

$$\Delta e = -\Delta X\sin\emptyset + \Delta Y\cos\lambda \qquad (A14)$$
$$\Delta n = -\Delta X\sin\emptyset\cos\lambda - \Delta Y\sin\emptyset\sin\lambda + \Delta Z\cos\emptyset \qquad (A15)$$
$$\Delta u = \Delta X\cos\emptyset\cos\lambda + \Delta Y\cos\emptyset\sin\lambda + \Delta Z\sin\emptyset \qquad (A16)$$

BK9 (local to geocentric)

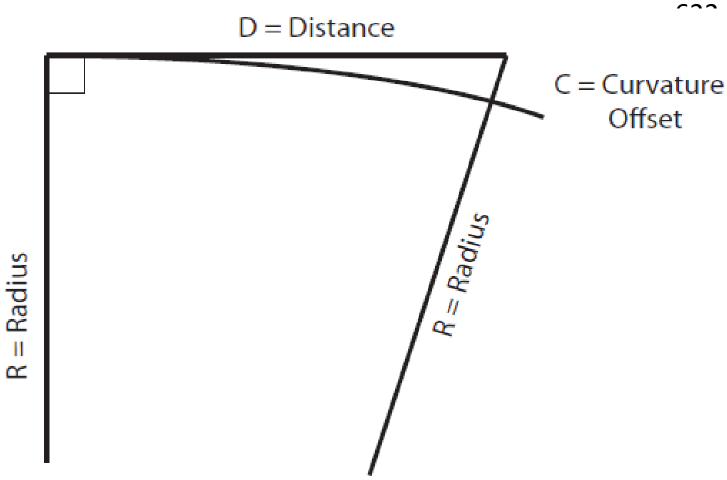
$$\Delta X = -\Delta e\sin\lambda - \Delta n\sin\emptyset\cos\lambda + \Delta u\cos\emptyset\cos\lambda \qquad (A17)$$
$$\Delta Y = \Delta e\cos\lambda - \Delta n\sin\emptyset\sin\lambda + \Delta u\cos\emptyset\sin\lambda \qquad (A18)$$
$$\Delta Z = \Delta n\cos\emptyset + \Delta u\sin\emptyset \qquad (A19)$$

Note, latitude (+N and -S) and longitude (+E and –W) are at standpoint.

Appendix B

Computing the curvature offset

The curvature offset between a tangent plane and the ellipsoid is derived using the Pythagorean Theorem. Strictly speaking, the derivation is valid on the ellipsoid where *h* is 0 m. But, due to the relative magnitude of values involved, the derivation is likewise valid for values of ellipsoid height found on or near Earth's surface. With reference to Figure B1 below, the curvature offset, C, is found from the underlying radius of the Earth, R, and the distance, D, from tangent point to location of offset. The right triangle theorem of Pythagoras and an incidental approximation are used as follows:



Using Pythagorean Theorem,

$$(R + C)^2 = R^2 + D^2 \qquad \text{(B1)}$$

$$R^2 + 2\,R\,C + C^2 = R^2 + D^2 \qquad \text{B(2)}$$

$$2\,R\,C = D^2 - C^2 \qquad \text{B(3)}$$

But, C^2 is much smaller than D^2 and can be discarded. Therefore,

$$C = \frac{D^2}{2\,R} \quad \text{Earth R} \approx 6{,}371 \text{ km} \quad \text{(B4)}$$

Figure B1 Derivation of Curvature Offset