GPS and the GSDM on the New Mexico Principal Meridian

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This is a description of Example 5, Chapter 15, in the 2nd Edition of "The 3-D Global Spatial Data Model." The example consists of two points on the New Mexico Principal Meridian – the Initial Point and the SW Corner of Section 31, T23S – R1E. The current NAD 83 (2011) position of the Initial Point (established by John W. Garretson in 1855) was observed with GPS and published by NGS in June 2012. The GPS position of the SW Corner of Section 31 was established by an NMSU class project as described in Example 4. But note, the class project was completed in 2006 and the results are based on NAD 83 (1992). That 2006 network was later recomputed to obtain the NAD 83 (2011) position of said SW Corner.

Equations from the global spatial data model (GSDM) were used to compute additional points and to perform inverse computations between the Initial Point and said SW Corner. As shown on the accompanying diagram, the two physical points, subsequent auxiliary points, and their identifying point numbers are:

PT 201	Initial Point for the USPLSS in New Mexico.
PT 202	SW Corner Section 31, T23S-R1E, NM Principal Meridian.
PT 203	On ellipsoid normal at latitude/longitude of PT 201, but at ellipsoid height of PT 202.
PT 204	On ellipsoid normal at latitude/longitude of PT 202, but at ellipsoid height of PT 201.
PT 205	On ellipsoid normal at latitude/longitude of PT 201, but on the ellipsoid, h = 0.0 m.
PT 206	On ellipsoid normal at latitude/longitude of PT 202, but on the ellipsoid, h = 0.0 m.

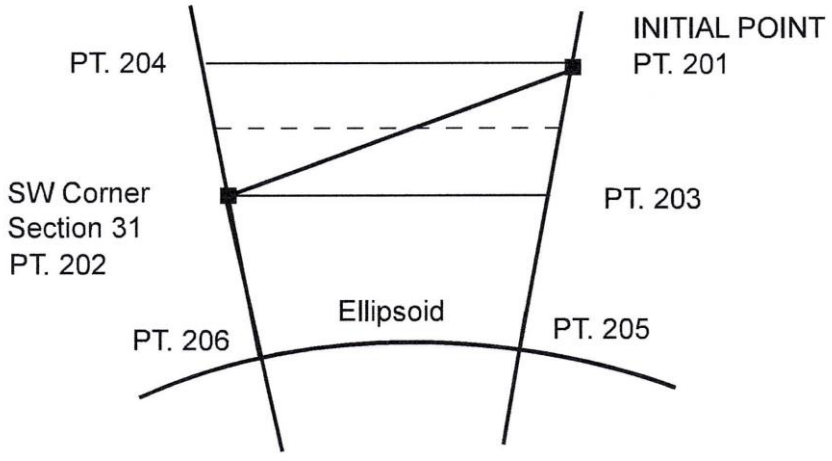

Diagram showing relative locations of points on NM Principal Meridian

Computing the 3-D distance between X/Y/Z points is accomplished using the 3-D Pythagorean equation:

$$D = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2} \qquad (1)$$

A question of interest is. . . What is the horizontal distance (HD) between the two points?

Answer. . . it depends. The user has the option of choosing from several definitions for HD.

The NAD 83 (2011) X/Y/Z and geodetic coordinates of the Initial Point and SW Corner of Section 31 are:

Initial Point	SW Corner, Section 31
X = -1,533,309.884 m	X = -1,568,698.064 m
Y = -5,050,681.721 m	Y = -5,167,107.065 m
Z = 3,571,149.193 m	Z = 3,385,214.088 m
φ = 34° 15′ 35.”94618 N	φ = 32° 15′ 24.”28892 N
λ = 253° 06′ 45.”03846 E	λ = 253° 06′ 43.”45867 E
h = 1,475.929 m	h = 1,259.566 m

Geocentric coordinates and ellipsoid heights for the Auxiliary Points are:

	Point 203	Point 204	Point 205	Point 206
X =	-1,533,257.938 m	-1,568,751.217 m	-1,532,955.528 m	-1,568,388.631 m
Y =	-5,050,510.611 m	-5,167,282.144 m	-5,049,514.482 m	-5,166,087.830 m
Z =	3,571,027.392 m	3,385,329.563 m	3,570,318.320 m	3,384,541.839 m
h =	1,259.566 m	1,475.929 m	0.000 m	0.00 m

Using equation 1, the following distances are computed as:

3-D slope distance PT 201 to PT 202 (obviously not horizontal)	222,213.968 m
Chord distance on ellipsoid, PT 205 to PT 206	222,166.044 m
Chord distance at elevation of Initial Point, PT 201 to PT 204	222,217.645 m
Chord distance at elevation of SW Cor. Sec 31, PT 202 to PT 203	222,210.080 m
Mean chord distance at average ellipsoid height	222,213.862 m

Technically, none of the options listed qualifies for a formal definition of horizontal distance. However, a close approximation of horizontal distance is often computed as:

$$D = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2 - \Delta h^2} \qquad (2)$$

“Gee whiz” observations from these data include:

1. The horizontal distance from equation 2 matches the mean of the chord distances depicted by the dashed line in the diagram.

2. The distance laid out by the original government surveyors was really pretty good. There is some rather rough terrain along the NM Principal Meridian. As stated in the 2nd Edition of the 3-D book, the distance agreement is within the tolerance given in the 1973 BLM Manual for laying out a Principal Meridian.

3. The inverse azimuth from the SW Corner of Section 31 to the Initial Point is within 40 arc seconds of true north. Again, quite impressive given the terrain and other conditions.

More information on horizontal distance options is available in the 2nd Edition of the 3-D book and at http://www.globalcogo.com/HD-Options.pdf. A formal rigorous definition for horizontal distance at elevation is given by Rollins and Meyer when discussing, “Four Methods for Low-Distortion Projections,” ASCE Journal of Surveying Engineering, Vol. 145, No. 4, November 2019.

The INITIAL POINT defines the intersection of the New Mexico Principal Meridian and the Baseline

BLM monument at the SW Corner, Section 31, T23S-R1E, NM Principal Meridian