Datums, Coordinate Systems, Low Distortion Projections (LDP), & the Global Spatial Data Model (GSDM)

Hosted by:
City of Las Cruces Public Works/Project Development
and
Community Development/GIS

April 24, 2015 - 1:00 to 5:00 p.m.

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Presenter: Earl F. Burkholder, PS, PE, F.ASCE

- **Served as NMPS President 2009.**
- **Retired from NMSU Surveying faculty in July 2010.**
- **Is currently Past Chair – ASCE Geomatics Division EXCOM.**
- **2008 Book, The 3-D Global Spatial Data Model (GSDM).**
- **Currently preparing Second Edition of the 3-D book.**
- **Incorporated Global COGO, Inc. in 1996.**
- **BURKORD™ software does 3-D COGO & error propagation.**
- **Significant experience with low distortion projections (LDP).**
- **GSDM was developed as a tool for the end user.**
- **Compare features of a LDP with those of the GSDM.**

4 hr. session is broken into two parts:

- **Datums, Map Projections, and Coordinate Systems**

  - Introduction & Conceptual view of Spatial Data

  - Plane and Solid Geometry relationships

  - Map Projections - Making a 2-D map of a 3-D world

  - Coordinate systems (Questions?)

- **Low Distortion Projections & Global Spatial Data Model**

  - Defining and Designing a Map Projection

  - Computing projection parameters, including a LDP.

  - Definition and geometry of the GSDM.

  - Features of the GSDM, handling directions/distances.

  - Computations in 3-D space (as opposed to ellipsoid/grid).

Surveying & Mapping includes:

- **Making measurements of physical features: lines, distances.**
- **Using written/graphic descriptions of land ownership.**
- **Reconciling measurements with the record. This is HUGE!**
- **Performing services for and providing products to clients.**
- **Although "land" surveying is critical, focus here is technical.**

Spatial data are used to make maps and to describe ownership:

- **Geometrical pieces: distances, angles, orientation, units.**
- **What is difference between horizontal and vertical? (3-D?)**
- **All measurements are "with respect to something."**
- **Distances are between points – monuments.**
- **Coordinates define absolute position relative to a P.O.B.**

What is difference between absolute and relative?

- Absolute is defined as with respect to an origin.
- Relative is with respect to a nearby point/object/feature.
- Relative is difference between two points in same system.
- A DATUM is a predefined "system" which provides context.
- Commonly recognized datums include:
  - Horizontal: NAD 83, NAD 83(xx), NAD27. These are 2-D!
  - Vertical: NAVD88, NGVD29, Sea-level. These are 1-D!
- What is a 3-D datum? 4-D anyone?
  - WGS84 is defined by DoD, NAD83 is defined by NGS.
  - ITRF is defined by the international scientific community.
- In many cases, ITRF & WGS84 can be used interchangeably.

Horizontal Datums:

- **NAD 27 – Clarke ellipsoid, origin at Meade's Ranch, foot units**
- **NAD 83 – GRS 1980 ellipsoid, origin at center of Earth, meters.**
- **Used extensively for mapping and 2-D surveys**

Vertical Datums (Origin is best-fitting equipotential surface):

- **Sea Level Datum of 1929 – name change for NGVD 29.**
- **NAVD 88 origin is 1 arbitrary point – serves US and Canada.**

3-D Datums (Origin is earth's center of mass):

- **NAD 83 is 3-D datum with horizontal & vertical components.**
- **WGS 84 is 3-D datum established by DoD & used for GPS.**
- **ITRF is 3-D datum defined by scientific community.**

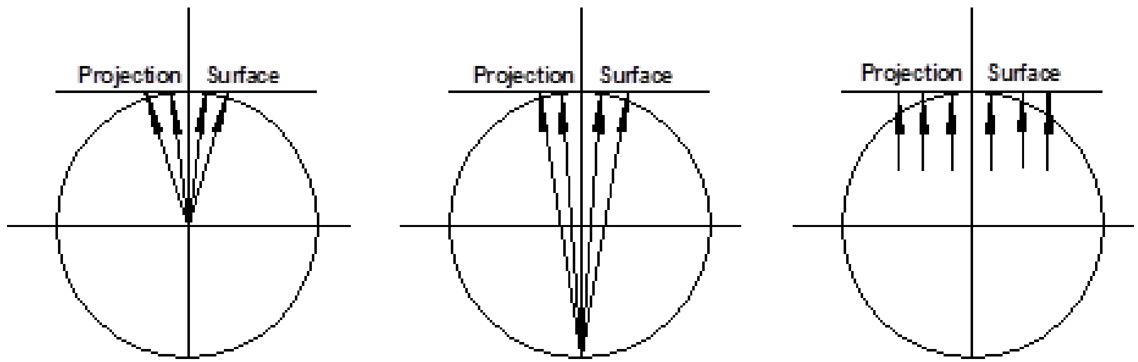
More context to clarify 'spatial data':

- We live in a 3-D world & make 3-D measurements.
- Plane geometry involves easting and northing coordinates.
- For flat-earth view, what is the best label for "up"?
- Horizontal & vertical handled separately – why?
- Maps/plats/drawings show 2-D – or plan/profile.
- Contour lines show elevation. Can you visualize 3-D?
- Using digital technology; flat-earth view is too limiting!
- Need a way to preserve 3-D data using a 2-D map. XX
- Map projections are used to "flatten the earth."
- For abstract considerations, see award-wining paper.

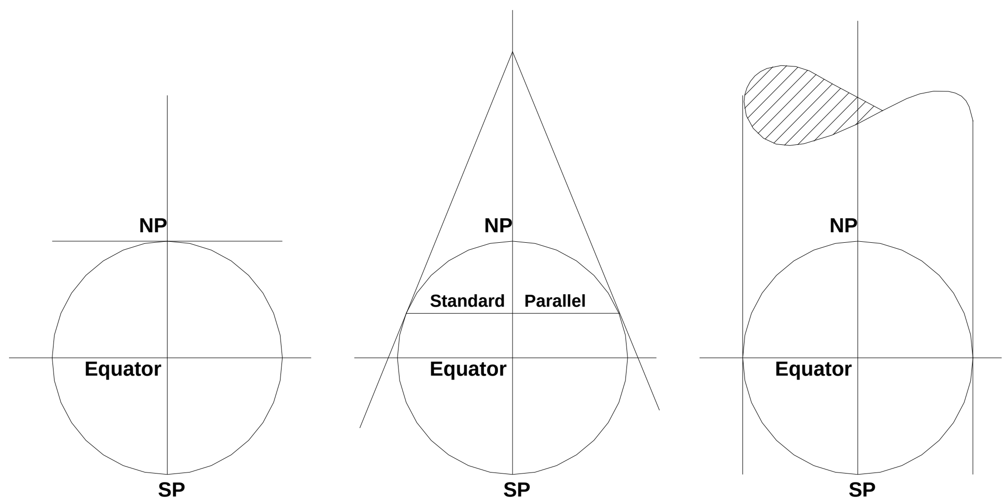
http://www.globalcogo.com/CR002.pdf

Local coordinates - many spatial data users.

- **Portrays relative relationships.**
- **x & y or north & east flat-earth values**
- **Is origin well defined and monumented?**
- **Elevation is ‘up’**
- **Vertical origin is geoid – approximated by mean sea level.**



(a) Gnomonic (b) Stereographic (c) Orthographic



(a) Tangent Plane (b) Lambert Conic Conformal (c) Mercator

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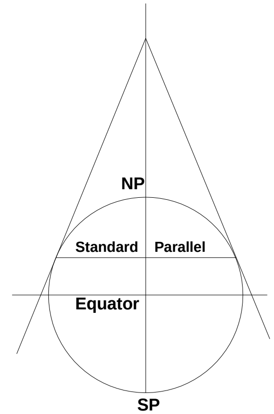
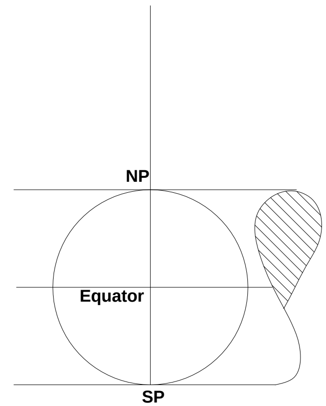
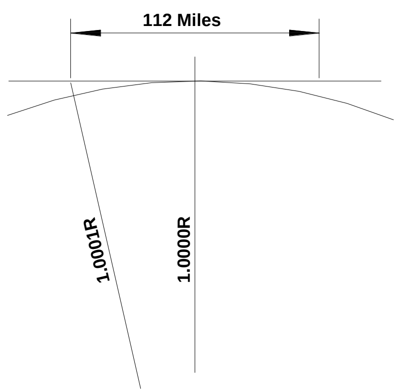
Specifically, for state plane coordinate systems:



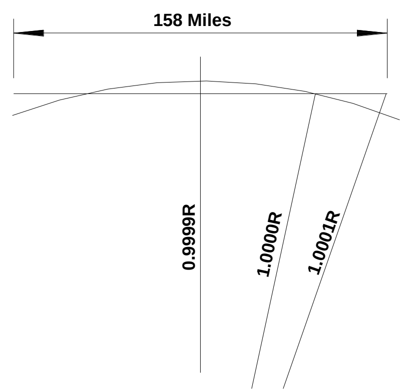
Lambert Conic Conformal



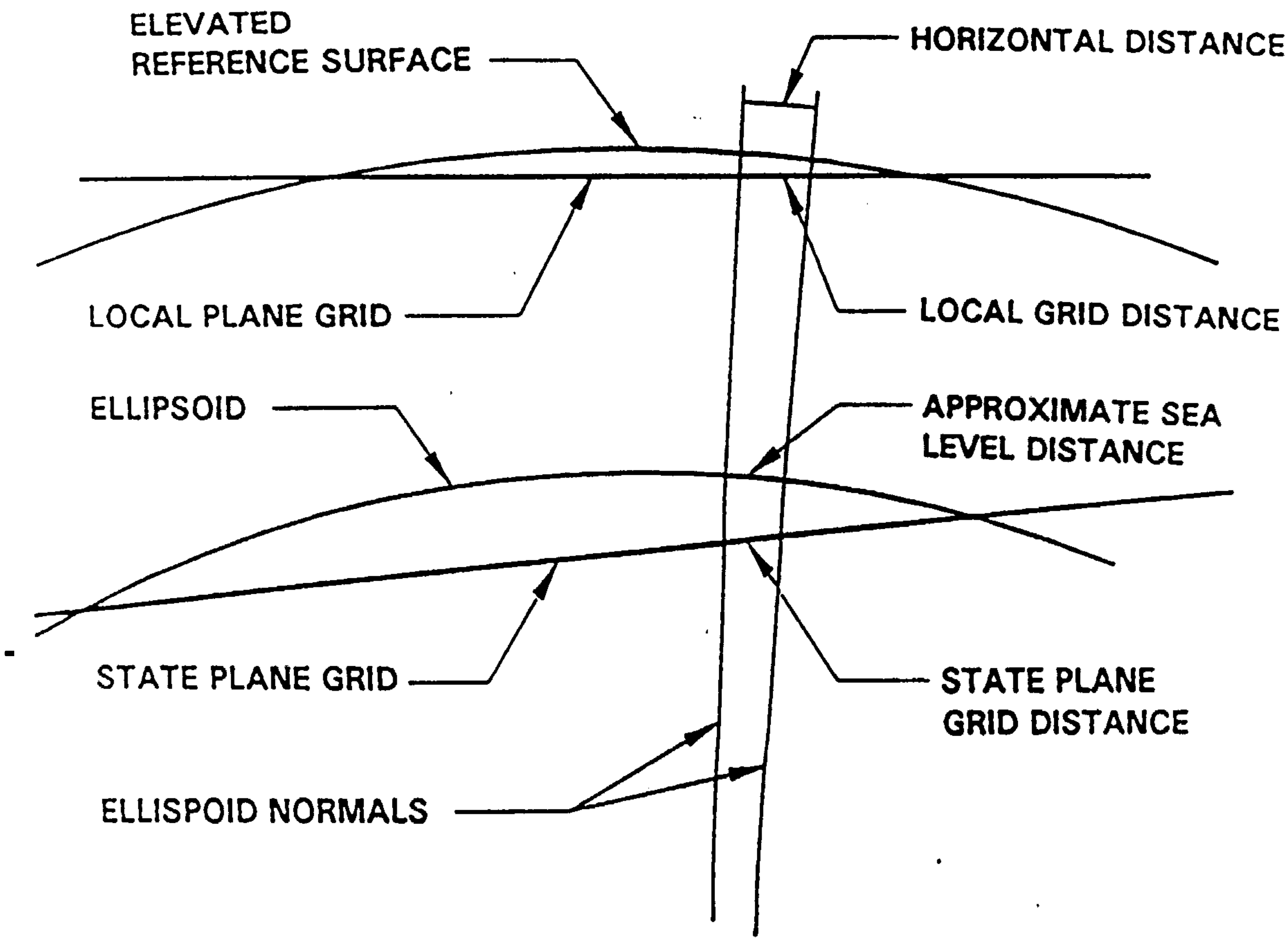
Transverse Mercator

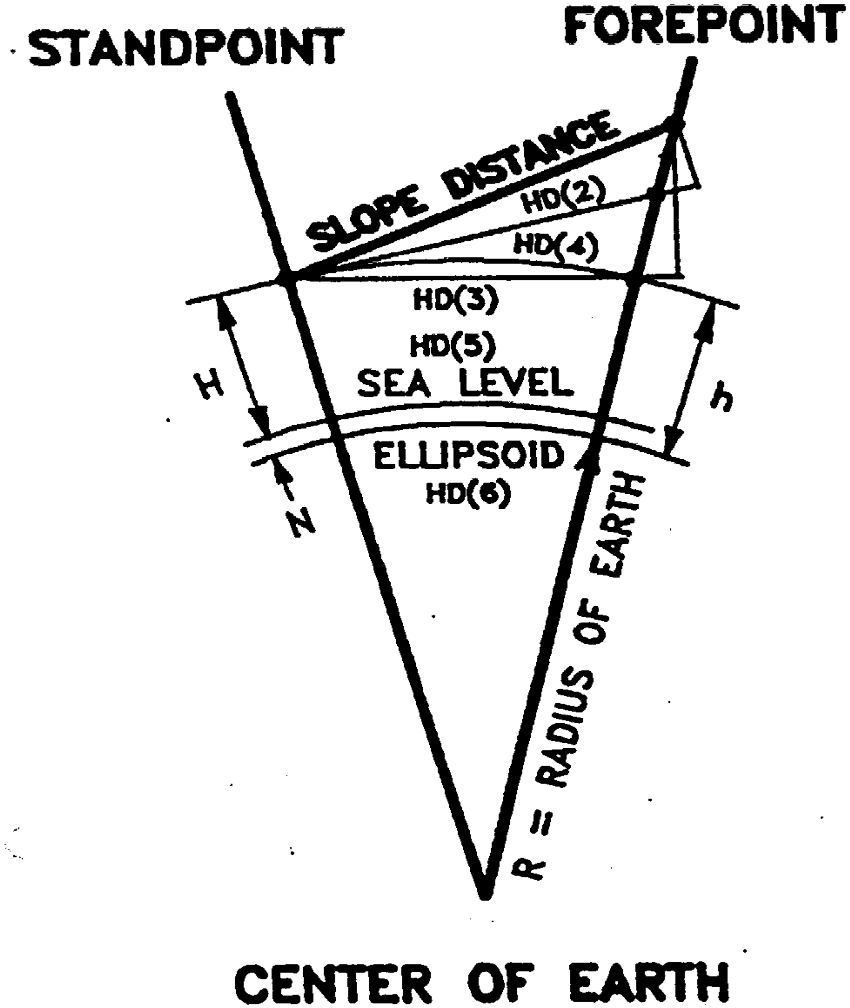


Tangent Projection
(Distances are only stretched)



Secant Projection
(Distances are both stretched and compressed)

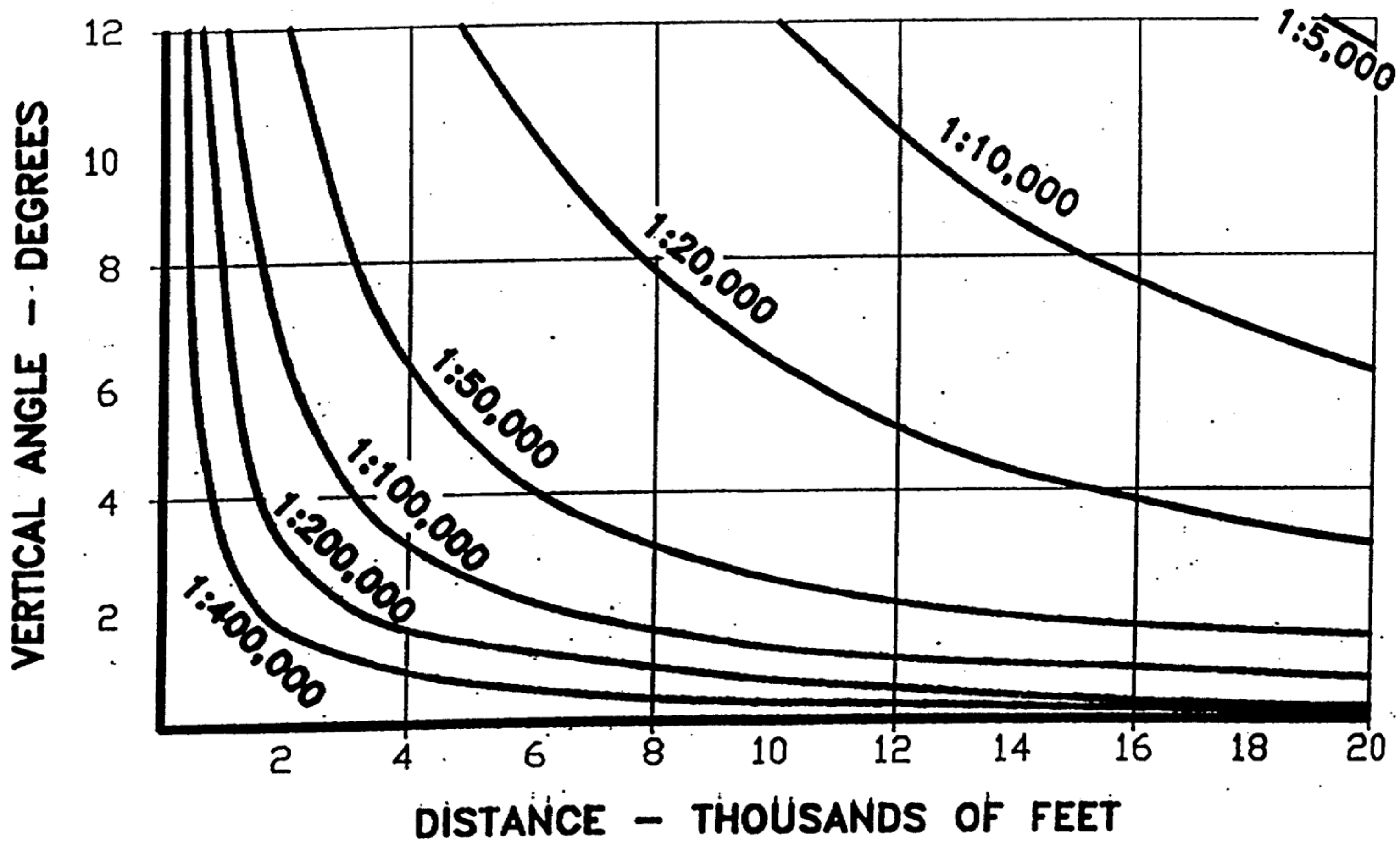

ELEVATED
REFERENCE SURFACE
HORIZONTAL DISTANCE
LOCAL PLANE GRID
LOCAL GRID DISTANCE
ELLIPSOID
APPROXIMATE SEA
LEVEL DISTANCE
STATE PLANE GRID
STATE PLANE
GRID DISTANCE
ELLISPOID NORMALS


STANDPOINT
FOREPOINT
SLOPE DISTANCE
HD(2)
HD(4)
HD(3)
HD(5)
SEA LEVEL
H
h
ELLIPSOID
HD(6)
N
R = RADIUS OF EARTH
CENTER OF EARTH

What is horizontal distance? Several choices are:

1. Right triangle component of slope distance – used the most.
2. Tangent plane distance between plumb lines.
3. Chord with same elevation at both ends.
4. Arc distance between plumb lines.
5. Sea level distance between plumb lines.
6. Ellipsoidal distance between plumb lines.
7. Grid inverse distance between coordinate points – worst choice because then a foot is not a foot!

In most cases, plane surveying uses definition #1



See: Burkholder, E.F. (1991); "Computation of Horizontal/Level Distances" Journal of Surveying Engineering, Volume 117, No. 3, pp 104-116.

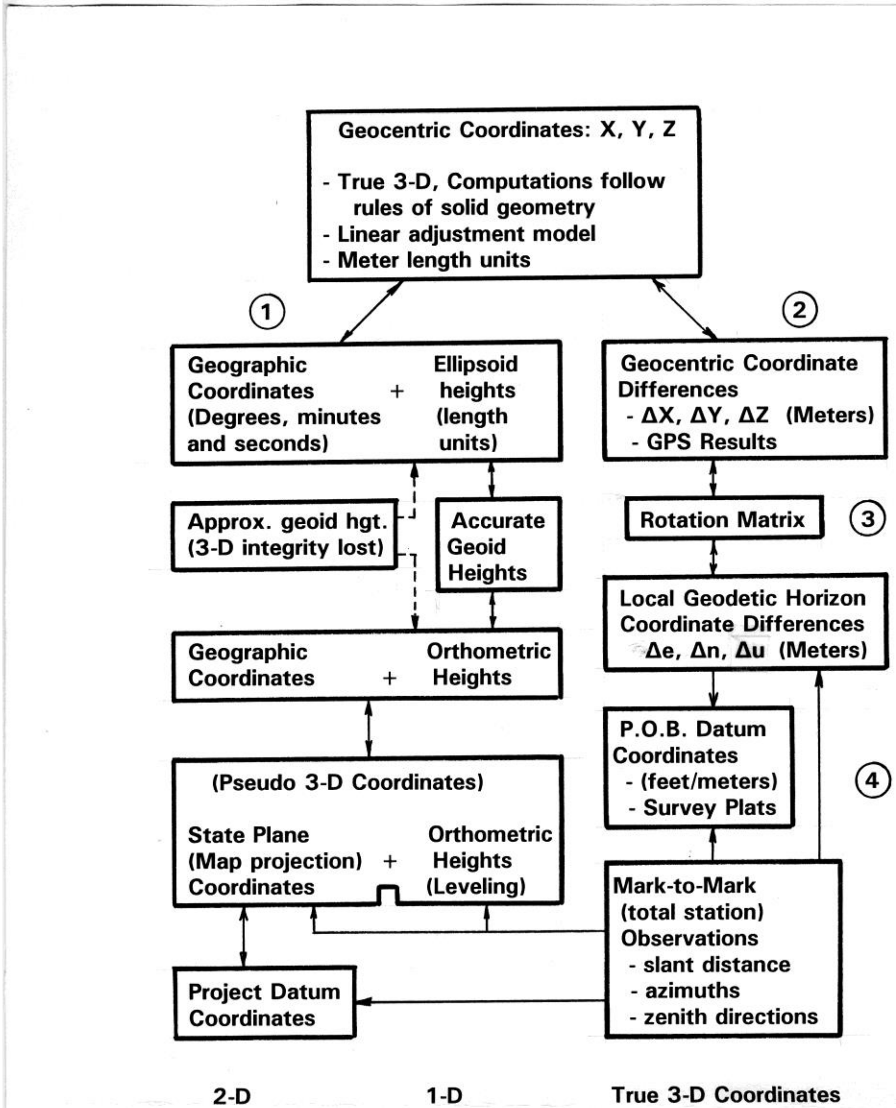
Points to remember:

- **Map projections are strictly 2 dimensional.**
- **Conformal map projections are mathematical.**
- **Distances on the map & on the ellipsoid are different.**
- **Distance on ellipsoid is not same as distance on the ground.**
- **When is a foot not a foot? What about weights/measures?**
- **How much distortion is allowed/permitted?**
- **A tolerance of 1:10,000 was adopted in 1930's.**
- **Corrections are needed to preserve data integrity.**
- **Conformal projections preserve angles.**

http://www.globalcogo.com/GM012.pdf

Geodetic & geographic (ellipsoidal/spherical)

- Computations on ellipsoid or sphere.
- Latitude and longitude curvilinear values.
- Position in degrees-minutes-seconds.
- Equations are cumbersome and complex.
- Third dimension is ellipsoid height or altitude. Elevation has a separate origin.


Geocentric Coordinates: X, Y, Z
- True 3-D, Computations follow rules of solid geometry
- Linear adjustment model
- Meter length units
1
2
Geographic Coordinates (Degrees, minutes and seconds) + Ellipsoid heights (length units)
Geocentric Coordinate Differences
- ΔX, ΔY, ΔZ (Meters)
- GPS Results
Approx. geoid hgt. (3-D integrity lost)
Accurate Geoid Heights
Rotation Matrix
3
Local Geodetic Horizon Coordinate Differences
Δe, Δn, Δu (Meters)
Geographic Coordinates + Orthometric Heights
P.O.B. Datum Coordinates
- (feet/meters)
- Survey Plats
4
(Pseudo 3-D Coordinates)
State Plane (Map projection) Coordinates + Orthometric Heights (Leveling)
Mark-to-Mark (total station) Observations
- slant distance
- azimuths
- zenith directions
Project Datum Coordinates
2-D
1-D
True 3-D Coordinates

Coordinate systems/geometry

- **Coordinate systems:**
  1. **Geocentric, Earth-centered Earth-fixed**
  2. **Geodetic, traditional latitude/longitude.**
  3. **Map projection – state plane coordinates.**
  4. **Local – assumed or modified map projection (LDP)**
- **Basic elements and conventions**

  - Meter is unit of length (display other as required).

  - Use right-handed rectangular as possible.

  - Rules of solid geometry (vector algebra & matrices).

Points to remember (Again):

- Map projections are strictly 2 dimensional.
- Map projections are mathematical, not graphical.
- Distances on the map & ellipsoid are different.
- Distance on ellipsoid is not same as at ground.
- When is a foot not a foot?
- How much distortion is allowed/permitted?
- A tolerance of 1:10,000 was adopted in 1930's.
- Corrections are needed to preserve data integrity.
- Conformal projections preserve angles.

Questions?

Time for a break – be back in 10 minutes.

- The first part looked at Datums, Coordinate Systems, and generic Map Projections.
- The next part will focus specifically on Low Distortion Projections (LDP’s) and on

  The 3-D Global Spatial Data Model (GSDM).

Looking specifically at Low Distortion Projections (LDP):

- Questions to audience:
  - What is a LDP and why are you interested in a LDP?
  - How will a LDP be beneficial in your work?
- Step back and look at philosophical issues:
  - Map projections are the prerogative of Cartographers.
  - Cartography is study/practice of making maps – includes:
    - i. Deciding what is to be mapped.
    - ii. Representing terrain of objects/features.
    - iii. Exclude irrelevant information.
    - iv. Goal is to reduce complexity – generalization.
    - v. Map design decides what to show.

Contrasting roles of cartographer and surveyor

- **Cartographers are concerned with map model/projection.**
  (Primarily as discussed on previous/next slide.)
- **Surveyors/GIS people are concerned with:**
  - Geometrical integrity of information on the map.
  - Deciding what information is to be conveyed.
  - Appearance of the map (colors).
  - Completeness of information on map.
  - Permanence of the information.
  - Storage and retrieval of maps.

Examples of concerns for Cartographers, Surveyors/GIS, etc.:

- **Topology is concerned with what lies next to what.**
- **Thematic maps highlight a particular message for reader.**
- **Discrete global grids use pixels - of varying sizes (raster).**
- **Geometrical integrity (vector) is essential for:**
  - **Surveyors/engineers/photogrammetrists/others**
- **What is the impact of the digital revolution?**
  - **For Cartographers?**
  - **For Surveyors/GIS et. al.?**
- **Spatial data are digital and 3-D (4-D if you count time).**
- **This consequence leads to re-examination of many issues.**

Some of those geometry related issues include:

- **Measurements are made in 3-D; relative distances.**
- **Reference frame is defined by others – absolute coordinates.**
- **Where is starting place – Point of Beginning (P.O.B.)?**
  - **What is stability/permanence of P.O.B?**
  - **What does it take for selection/use of common P.O.B.?**
- **How can we best provide answers/data to client/public?**
  - **Does client/public know what they want?**
  - **Surveyors provide survey plat/description to landowner.**
  - **And if client/public requests unrealistic results (format)?**
  - **What proprietary considerations are to be made?**

Examples of projections commonly encountered:

- **State plane coordinate systems.**
  - **- Lambert Conic Conformal for states “long” east/west.**
  - **- Transverse Mercator for states “long” north/south.**
  - **- Oblique Mercator, central axis at some non-north azimuth**
- **Universal Transverse Mercator (UTM)**
  - **- 6 degrees wide, 60 zones around the world**
  - **- Grid scale factor 0.9996 (1:2,500) at center of zone.**
  - **- Stop short of poles, defined for 80° South to 84° North.**
- **GPS Localization – can be considered a LDP**

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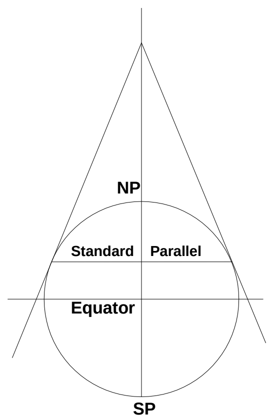
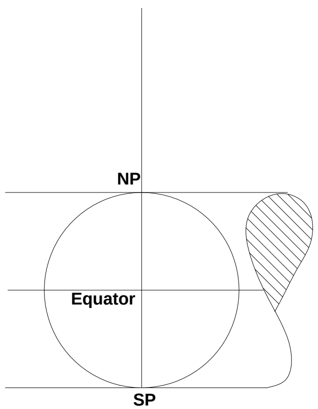
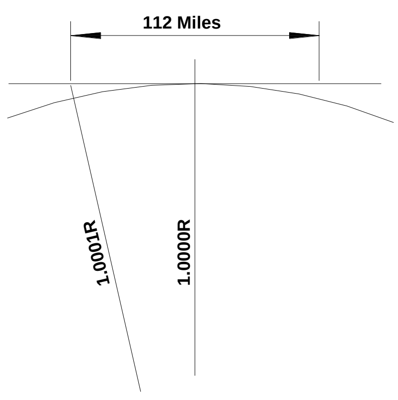
Specifically, for state plane coordinate systems:



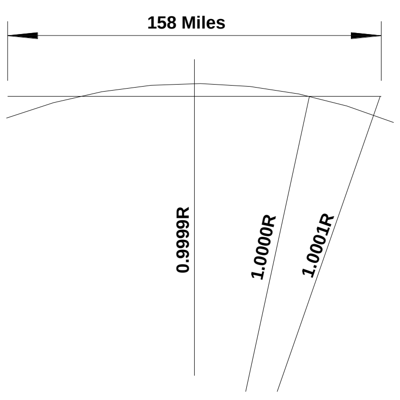
Lambert Conic Conformal



Transverse Mercator



Tangent Projection
(Distances are only stretched)



Secant Projection
(Distances are both stretched and compressed)

Elements needed to define a map projection:

- Ellipsoid, semi-major axis and 1/f or semi-minor axis.
- Select projection type:
  - Tangent/secant
  - Lambert, Mercator, Transverse Mercator, Oblique
- Fix projection to ellipsoid:
  - Central Meridian
  - Latitude of origin
  - Orientation of mapping grid, Meridian or oblique.
  - Level of maximum (plus/minus) distortion allowed.
  - Units for distances
  - False northing/easting to avoid negative coordinates

Low Distortion Projections (see slides 11 and 17 again):

- Driving force is to make grid/ground distances similar
  (Recognize difficulty of flattening the earth.)
- Examples of ways to accomplish that goal (and objections):
  - Raise ellipsoid to mean elevation of area to be mapped.
    (Some view that practice as using non-standard ellipsoid.)
  - Divide existing SPC by combined factor.
    (Works well for limited area but coordinates are not SPC.)
  - Impose 'scaling' on mapping equations.
    (Can work well, but departs from formal definitions)
- Few users take "ownership" of process but rely on software vendors to provide a system that "works."

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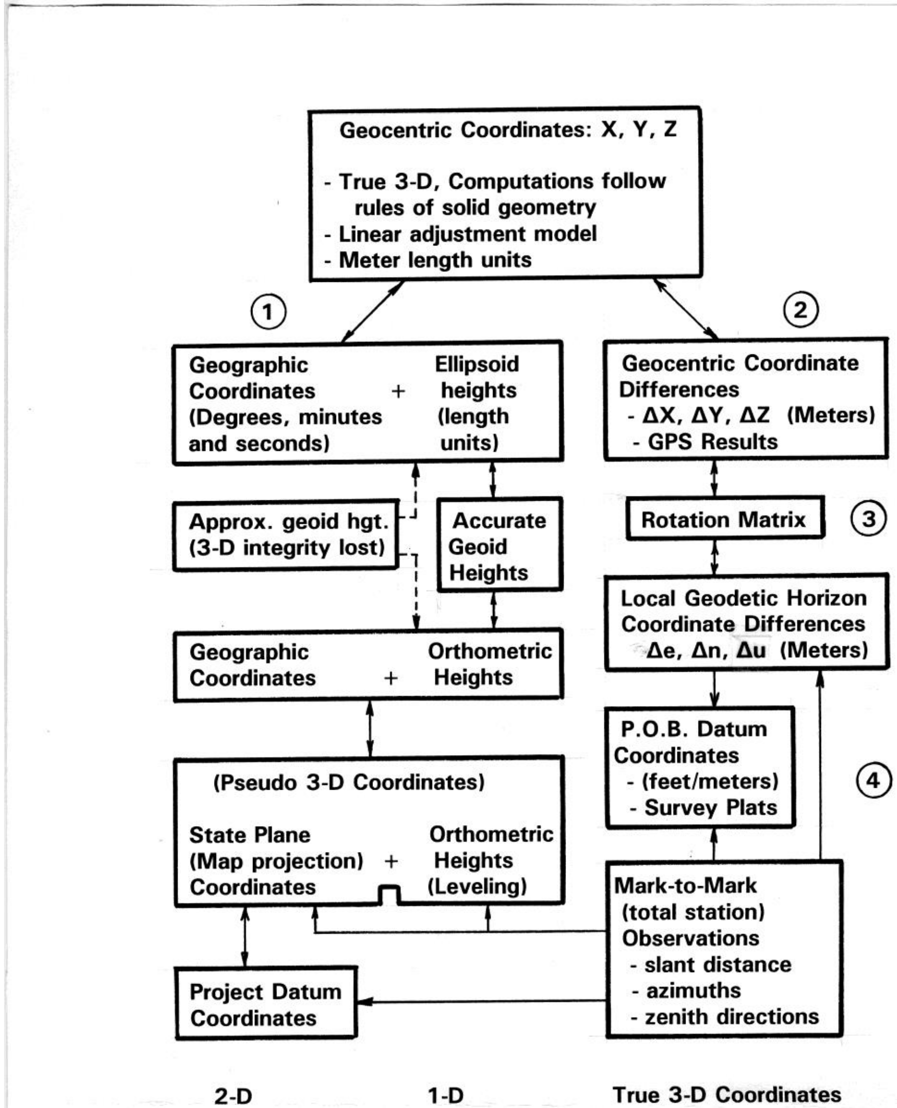


History of development of LDPs:

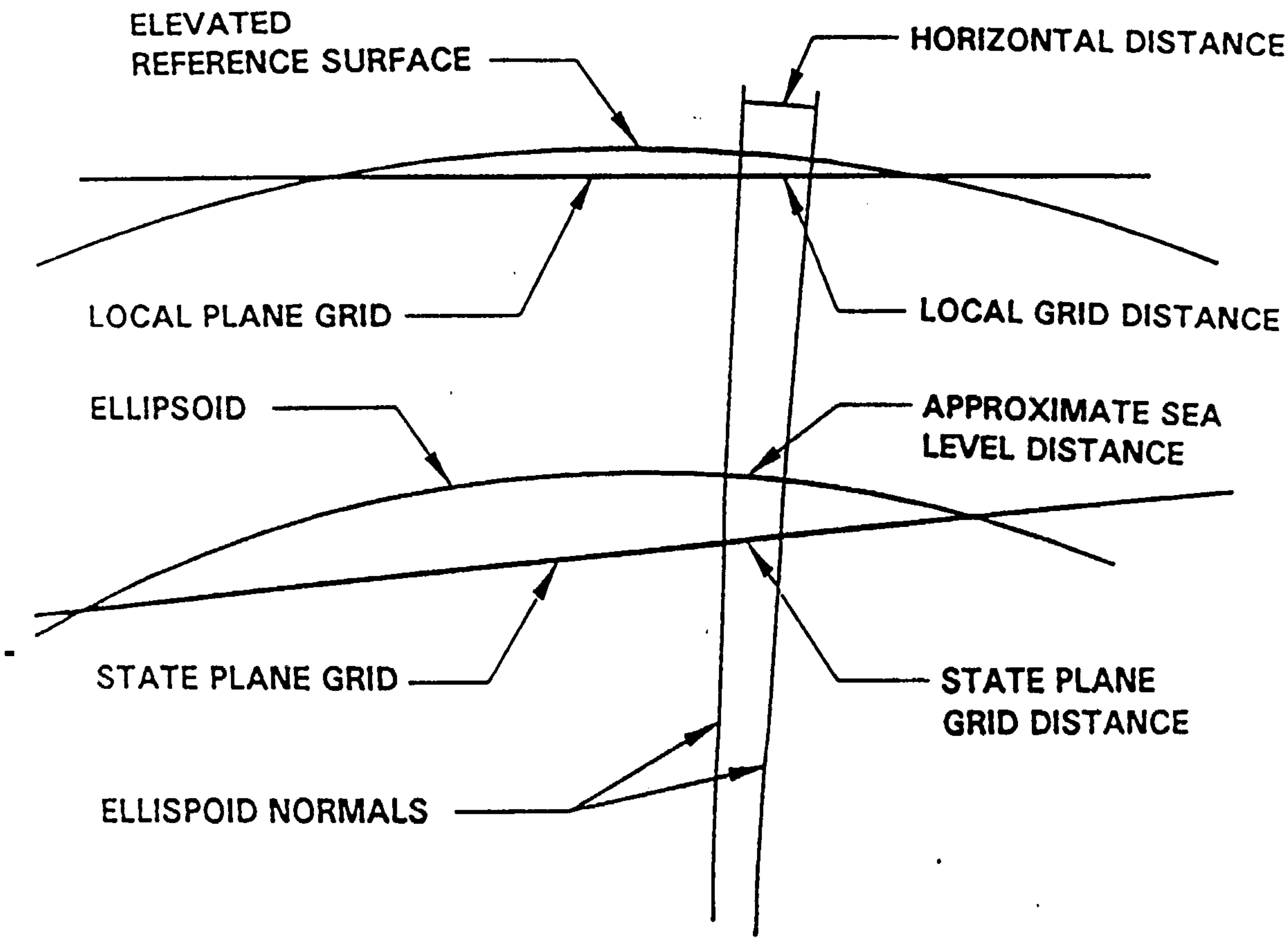
For informal summary of LDPs see

www.globalcogo.com/History-LDP.pdf

- Michigan SPC 1964
  - System designed at 800 feet to avoid elevation reduction
  - Reference surface returned to ellipsoid for NAD83.
- County Coordinate Systems:
  - Wisconsin
  - Minnesota
- State of Oregon – DOT
  - NGS involved in design/adoption
  - Probably best example of successful use of LDP.

Designing a Low Distortion Projection:

- **Parameters.**
  - Use standard GRS80 ellipsoid
  - Determine geographic location of area to be covered.
  - Determine the average elevation of area.
  - Choose projection type:
    - i. Lambert conic conformal, one or two standard parallels.
    - ii. Transverse Mercator,
    - iii. Oblique Mercator,
- **Computation of zone constants.**
  - Make sure you are using “reliable” equations.
  - Verify your solutions by testing.


ELEVATED
REFERENCE SURFACE
HORIZONTAL DISTANCE
LOCAL PLANE GRID
LOCAL GRID DISTANCE
ELLIPSOID
APPROXIMATE SEA
LEVEL DISTANCE
STATE PLANE GRID
STATE PLANE
GRID DISTANCE
ELLISPOID NORMALS

Design a LDP to cover Las Cruces at 1,200.00 m :

- Transverse Mercator with maximum distortion 1:100,000.
  - Use GRS80 ellipsoid + ellipsoid height at Las Cruces
    - a = 6,378,137.000 m + <u>1,200.0 m</u> = 6,379,337.000 m
    - 1/f = 298.25722210088
  - Pick Central Meridian (CM) as 106° 45’ 00.”0 W
  - Pick latitude of origin as 32° 15’ 00.”0 N
  - Choose central scale factor as 0.99999000
  - Choose false easting on CM 50,000.0 m
  - Choose northing of origin to be 0.000 m
  - Use Localcor.exe program – available from EFB.

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USER: Earl F. Burkholder DATE: April 15, 2015

TRANSVERSE MERCATOR PROJECTION TRANSFORMATIONS

PROJECTION NAME: Las Cruces Seminar April 2015

REFERENCE ELLIPSOID: GEODETIC REFERENCE SYSTEM 1980

a = 6378137.0000 METERS

1/f = 298.2572221008827

REFERENCE ELLIPSOID HEIGHT FOR PROJECTION = 1,200.0000 METERS

MODIFIED ELLIPSOID FOR: Las Cruces Seminar April 2015

a = 6379337.0000 METERS

1/f = 298.2572221008827

ZONE PARAMETERS:

CENTRAL MERIDIAN (W)	106 45 0.000000
LATITUDE OF FALSE ORIGIN	32 15 0.000000
FALSE NORTHING AT FALSE ORIGIN	0.0000 METERS
FALSE EASTING ON CENTRAL MERIDIAN	50,000.0000 METERS
SCALE FACTOR ON CENTRAL MERIDIAN	0.99990000000

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ZONE CONSTANTS:

RECTIFYING SPHERE RADIUS 6,368,647.1349 METERS

RECTIFYING LATITUDE CONSTANTS:

U(0) =	-0.005048250776	V(0) =	0.005022893948
U(2) =	0.000021259204	V(2) =	0.000029370625
U(4) =	-0.000000111423	V(4) =	0.000000235059
U(6) =	0.000000000626	V(6) =	0.000000002181

RECTIFYING LATITUDE OF FALSE ORIGIN 32 7 11.489747

GRID MERIDIAN ARC TO FALSE ORIGIN 3570210.5712 METERS

TRANSFORMATIONS:

NAME OF STATION: Reilly **FORWARD**

LATITUDE:	32 16 55.930010	NORTHING	3,571.6578 METERS
LONGITUDE:	106 45 15.160350	EASTING	49,603.2227 METERS
CONVERGENCE:	0 0 -8.10	SCALE FACTOR:	0.999990001940

NAME OF STATION: CrucesAir **FORWARD**

LATITUDE:	32 16 54.632690	NORTHING	3,544.8009 METERS
LONGITUDE:	106 55 22.247630	EASTING	33,714.4327 METERS
CONVERGENCE:	0 -5 32.33	SCALE FACTOR:	0.999993268031

Example – two NGS published HARN points:

Reilly	CrucesAir
ø = 32° 16’ 55.”93001 N	ø = 32° 16’ 54.”63269 N
λ = 106° 45’ 15.”16035 W	λ = 106° 55’ 22.”24763 W

State Plane Coordinates:

Reilly	CrucesAir
E = 452,506.490 m	E = 436,621.577 m
N = 142,268.771 m	N = 142,315.959 m

LDP Coordinates:

Reilly	CrucesAir
E(LDP) = 49,603.223 m	E(LDP) = 33,714.433 m
N(LDP) = 3,571.658 m	N(LDP) = 3,544.801 m

For comparison of inverse distances, see

www.globalcogo.com/SPCvsLDP-LC.pdf

Summary of Distances – from link on the previous slide:

- Distances from Reilly to CrucesAir

- State plane grid distance	15,884.983 m
- Ground distance (mean elevation)	15,889.066 m
- LDP distance (raw)	15,888.813 m
- LDP distance corrected for CF	15,889.061 m

- Observations:
  - LDP distance agrees with ground distance 1:62,800
  - "Correct" distance from LDP still possible.
  - This procedure has nothing to do with vertical.

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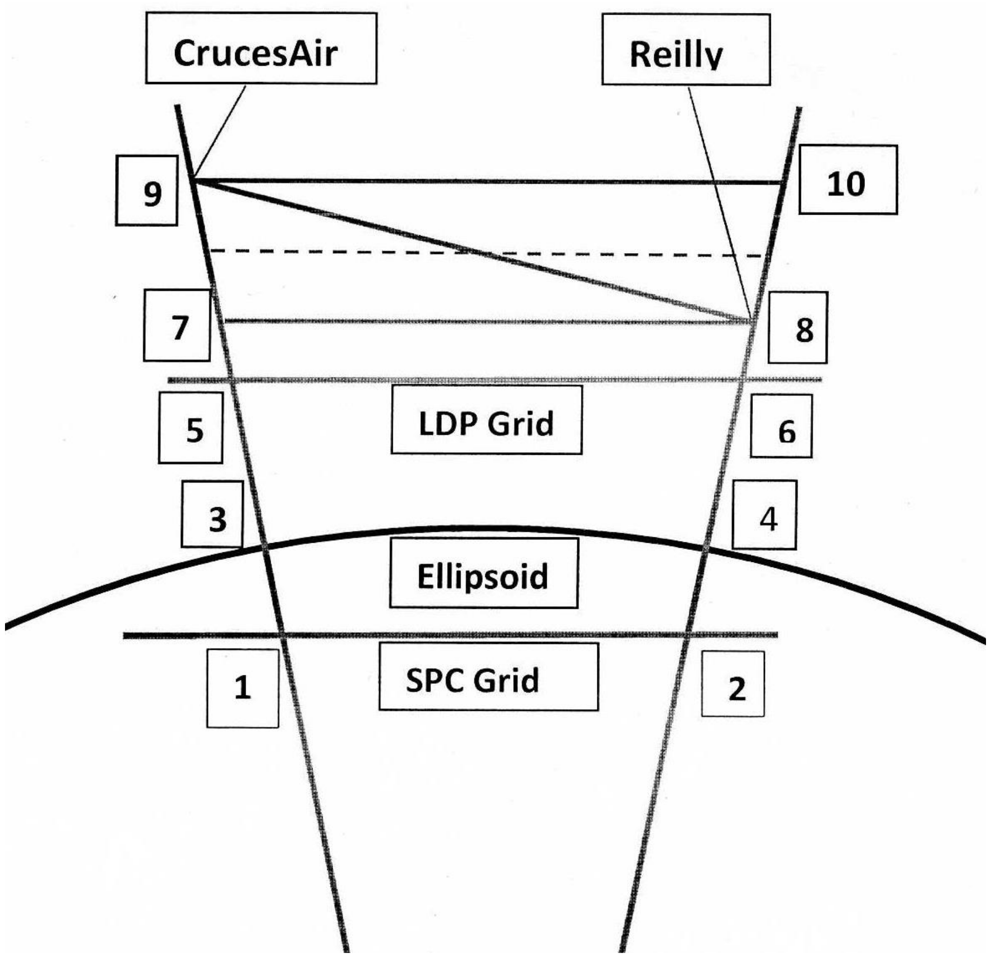

With regard to the diagram on the previous slide:

- Points 1 & 2 State plane coordinate inverse.
- Points 3 & 4 Ellipsoid distance (arc & chord "same").*
- Points 5 & 6 Horizontal distance from LDP.
- Points 7 & 8 Actual horizontal distance at Reilly.**
- Points 9 & 10 Actual horizontal distance at CrucesAir.**
- Dotted line is "mean" horizontal distance.

Real question is "what definition of horizontal is to be used?"

* In this case, arc is longer than the chord by 0.004 m.

** HD(3) has same ellipsoid height at both ends (see slide 12).

Notes:

- GSDM is used to compute X/Y/Z values at points 3/4,7/8, 9/10.
  - Input latitude/longitude/ellipsoid height – from NGS
  - For points on ellipsoid, height is 0.00 m.
  - Input ellipsoid height at Reilly for Point 7 at CrucesAir.
  - Input ellipsoid height at CrucesAir for Point 8 at Reilly.
- 3-D distance inverse uses 3-D Pythagorean Theorem.

$$\boldsymbol{Dist_{1\ to\ 2}} = \sqrt{(\boldsymbol{X_2} - \boldsymbol{X_1})^2 + (\boldsymbol{Y_2} - \boldsymbol{Y_1})^2 + (\boldsymbol{Z_2} - \boldsymbol{Z_1})^2}$$

Equations for computing X/Y/Z:

- **Need ellipsoid parameters:**

  - GRS80 ellipsoid, a = 6,378,137.00 e^2 = 0.006694380023

- **Need latitude, longitude, and ellipsoid height, (ø, λ, h)**
- **First, compute length of ellipsoid normal (N).**

$$N = \frac{a}{\sqrt{1-e^2 sin^2 \emptyset}}$$

- **Then compute ECEF coordinates, X/Y/Z (use east longitude)**

$$X = (N + h) cos\emptyset \; cos\lambda$$

$$Y = (N + h) cos\emptyset \; sin\,\lambda$$

$$Z = \left[N\left(1 - e^2\right) + h\right] sin\,\emptyset$$

HD(1) horizontal distance can also be computed from plane surveying latitudes/departures for each line – Δe & Δn.

This procedure uses a rotation matrix to convert the ECEF coordinate differences to local tangent plane differences. The values obtained are from "here" to "there" and the azimuth is with respect to the true meridian through point "here". Use latitude/longitude at "here" and use east longitude (or west longitude as a negative value). Point 1 is "here" Point 2 is "there".

$$\Delta e = -\Delta X \sin\lambda + \Delta Y \cos\lambda$$

$$\Delta n = -\Delta X \sin\varphi \cos\lambda - \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi$$

$$\Delta u = \Delta X \cos\varphi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi$$

$$HD(1) = \sqrt{\Delta e^2 + \Delta n^2}$$

Now we have a different set of comparisons to make but without the approximations for grid scale factor or computing the elevation factor. And, we can now carry vertical as well.

- **Using the 3-D Pythagorean Theorem:**
  - **- Ellipsoid distance is 15,885.949 + 0.004 m = 15,885.953 m**
  - **- HD(3) at Reilly is = 15,888.863 m**
  - **- HD(3) at CrucesAir is = 15,889.261m**
  - **- Mean distance is = 15,889.062 m**
- **Using the rotation matrix:**
  - **- HD(1) Reilly to CrucesAir is = 15,889.248 m**
  - **- HD(1) CrucesAir to Reilly is = 15,888.851 m**
  - **- Mean distance is = 15,889.050 m**

Some conclusions/points: The user is responsible for correct use!

1. **Surveyors are responsible for measurements.**
2. **GIS persons use measurements and specify how.**
3. **It is impossible to portray a curved earth on a flat map to scale without distortion.**
4. **Traditional computations are performed on ellipsoid or grid.**
5. **GSDM performs computations in 3-D space without distortion.**
6. **A LDP can be used effectively (if documented and followed).**
7. **The GSDM accommodates 3-D – LDP only 2-D**
8. **Not covered in this seminar – but the GSDM handles error propagation and standard deviations with aplomb.**
9. **Extensive documentation on-line at www.globalcogo.com**

Questions?

FYI –

- **Power point file at www.globalcogo.com/LC-Sem.pptx**
- **PDF file at www.globalcogo.com/LC-Sem.pdf**

Don’t forget Evaluations!

Thank you for coming!