Contrasting a Low Distortion Projection (LDP) With the Global Spatial Data Model (GSDM)

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SUMMARY

Portraying Earth's curved surface on a flat map while preserving geometrical integrity has challenged cartographers for many years. Numerous map projections have been developed to address specific problems and, for the most part, the utility of those projections remains valid for the purposes intended. However, the digital revolution and wide-spread use of 3-D digital geospatial data, both locally and on a world-wide scale, provide motivation for re-visiting the challenge of "flattening the Earth." The traditional approach for surveying, mapping, and engineering applications has been to design a conformal map projection that preserves angular relationships between lines on the map and corresponding lines on the Earth. It is well known that a conformal projection distorts distances on the curved Earth as they are projected to the flat map. Long distances are typically distorted more than short distances. A low distortion projection (LDP) minimizes the distance distortion by restricting the area covered by a given projection and by using an algorithm that provides 2-D rectangular near-ground-level coordinates for each point. The distinct advantage of a LDP is that the computed distance between a pair of points closely matches the actual horizontal ground-level distance between the same two points. On the other hand, the global spatial data model (GSDM) is based upon the 3-D geocentric Earth-centered Earth-fixed (ECEF) coordinates of each point and provides a true "user view" of the world without distorting either directions or distances. When using the GSDM, the origin (standpoint) is selected by the user and the forepoint can be any other ECEF point in a 3-D data base or point cloud. The local ground-level distance and the true azimuth from standpoint to forepoint are computed from the plane surveying components (Δn & Δe) obtained from the ECEF coordinate differences between the standpoint and the forepoint. This paper describes the objectives of each model and contrasts the two models while highlighting the advantages of the GSDM over a LDP. Examples are accessible via web links given in the paper.

Contrasting a Low Distortion Projection (LDP) With the 3-D Global Spatial Data Model (GSDM)

1. INTRODUCTION

Use of geospatial data (or simply spatial data) is widespread and has evolved dramatically since the advent of the digital revolution about 50 years ago. The characteristics of 3-D digital spatial data (4-D if monitoring changes over time) are now being exploited world-wide by disparate disciplines in an increasing number of applications. The shift from using analog map data to using digital spatial data motivates a re-evaluation of the underlying spatial data models, their underlying assumptions, their geometrical properties, and the veracity with which they accommodate real world experience. The analog/digital shift has not been instantaneous, in part, because existing analog procedures have been automated incrementally on an ad hoc basis and because many applications still treat horizontal and vertical spatial data separately. Although digitization and automation have significantly increased productivity, interoperability and the use of digital spatial data can be enhanced further by examining the fundamental characteristics of 3-D digital spatial data, by re-evaluating the underlying geometrical models, and by adopting procedures that are consistent with a global "user view" of the world.

Historically, conformal map projections have been used to "flatten the Earth" for engineering, surveying, and mapping applications, making it possible to compute positions on the Earth using plane Euclidean geometry concepts and tools. Although a conformal projection preserves the angular relationship between lines on the Earth and corresponding lines on the map, the flat-Earth distance between points on a conformal map is a distorted version of the corresponding curved-Earth distance. A Low Distortion Projection (LDP) is typically based on a conformal projection that covers only a small portion of the Earth's surface and, the design parameters of a LDP are chosen such that the transformed latitude/longitude position of each point is expressed in local plane coordinates that closely represent the actual horizontal ground-level distance between points on the Earth. On a conventional conformal projection, the distance distortion represents differences in distances between the curved ellipsoid distance and the map distance. However, on a LDP, an average ground elevation is incorporated into the algorithm such that the computed map distance between points closely matches the actual ground-level distance. Having the map distance closely approximate the horizontal distance is a huge benefit but the disadvantage is that a LDP covers only a small part of the Earth's surface, requiring multiple zones to cover larger areas. Keeping track of and working in different LDP zones can be confusing and a nuisance.

Globally, humans occupy three-dimensional space defined, for the most part, by the local gravity vector (horizontal is perpendicular to the plumb line at a point) and the Earth's spin axis (all meridians converge at the poles). The most appropriate model of real world space is one that is simple while best representing the features being modeled. Locally, plane Euclidean geometry portrays a 3-D flat-Earth view that ignores Earth's curvature. That "error" is corrected by using elevation above sea level for the third dimension with the consequence that local 3-D rectangular coordinate computations are valid only to the extent one can assume a flat-Earth. In the larger picture, modern measurement systems collect and store 3-D digital data. Those observations are processed to obtain spatial data components in a global coordinate system. The geocentric rectangular Earth-centered Earth-fixed (ECEF) coordinate system devised by the U.S. DOD for the NAVSTAR system of GPS satellites is a simple global rectangular coordinate system that models all 3-D space within the birdcage of orbiting satellites. The ECEF origin is at Earth's center of mass, the X/Y axes lie in the plane of the Equator, and the Z axis coincides with Earth's spin axis. The location of any point within the birdcage of orbiting satellites is described by a triplet of X/Y/Z metric coordinates and rules of solid geometry are applicable throughout. The global spatial data model (GSDM) is based upon the ECEF system and is formally defined in Burkholder (1997a).

The traditional global geodetic coordinate system of latitude/longitude/ellipsoid height is also used to describe 3-D locations. Several drawbacks to using geodetic coordinates for routine spatial data computation are that latitude and longitude are expressed in angular sexagesimal units of degrees, minutes, and seconds (making geodetic computations more cumbersome) and that elevation is referenced to the geoid (sea level) while ellipsoid height is referenced to the mathematical ellipsoid. That means that the origins for horizontal and vertical spatial data (datums) are different - latitude/longitude for horizontal and the geoid for vertical. The difference between ellipsoid height and elevation is known as geoid height which must be known or reliably estimated in order to relate ellipsoid heights to elevations or vice versa. If the geoid height is known, latitude/longitude coordinates and elevations can be transformed to true reliable 3-D rectangular ECEF coordinates using solid geometry equations.

The issue of grid/ground distance differences came to the fore in the 1980s as the use of GPS by the surveying community became more commonplace. Although the LDP name was adopted later, the grid/ground distance difference was often handled by what was called "project datum" or "surface" coordinates. Burkholder (1993a) discussed using the ECEF coordinate system for surveying computations and included a summary of 1991 DOT responses in Appendix II about how each state handled the grid/ground distance difference. A case for standardizing computational procedures is supported by reading the individual responses from the various states as listed in Appendix III. Burkholder (1993b) summarizes the distortion issues and included algorithms for developing local coordinate systems. However, with further study it became apparent that the bothersome grid/ground distance difference could be eliminated by using the GSDM and that the 3-D GSDM offers additional benefits not available when using a 2-D map projection based model.

Hindsight being what it is one can say "we are where we are because of where we came from" and that current practice is the result of applying new technology to previous methods. Using a LDP may be a convenient improvement over previous practice but the LDP is an incremental feature added to an existing 2-D model. When beginning with the assumption of a single origin for 3-D spatial data and exploiting the characteristics of 3-D digital spatial data, it is possible to achieve the same objective of a LDP while enjoying other valuable benefits. In addition to providing the benefits of a LDP, the GSDM more closely duplicates the human experience without distorting physical measurements, enhances spatial data interoperability world-wide, portrays a "user view" of the entire world from any standpoint selected by the user, and provides an efficient mechanism for establishing, tracking, and exploiting the concepts of spatial data accuracy. The GSDM is already defined and in place.

An incidental point of clarification is that the NAVSTAR global positioning system (GPS) put into place by the United States was the first in a family of satellite positioning options that have come to include the Russian global navigation satellite system (GLONASS), the

European GALILEO satellite system, and others. The proper description/abbreviation when referring to generic satellite positioning is global navigation satellite system (GNSS). In some cases, the acronyms GPS and GNSS are used interchangeably.

2. DESIGN CONSIDERATIONS

Circumstances often dictate the preferred way to solve an engineering problem. In this case, two separate spatial data models provide a similar solution for the grid/ground distance problem. But, the two approaches described herein are distinctly different. First, the LDP solution is described as an incremental approach in which the end goal (grid distance closely matching ground distance) is achieved by modifying an existing map projection solution. Conversely, the GSDM is described as a solution obtained by starting from the assumption of a single origin for 3-D data and building a solution based upon proven solid geometry concepts and equations. The LDP solution is logical in that it is an extension of a prior map projection solution. However, the GSDM provides the same (and more) benefits with a solution that avoids unintended consequences of a map projection solution. A fundamental difference between the two spatial data models is that the LDP is a 2-D solution while the GSDM is a 3-D solution.

Assumptions/features/design criteria associated with a LDP include:

- Each point to be mapped is defined by its latitude/longitude position on a named datum.
- A conformal map projection which preserves angular relationships is used to "flatten the Earth," obtaining north/east plane coordinates in place of latitude/longitude coordinates.
- On a conformal projection, the distance on the ellipsoid is unavoidably distorted when projected to the mapping grid. Distances may be either compressed or stretched.
- A tolerance for distance distortion is used as a map projection design parameter. An approximate distortion tolerance of 1:10,000 was used in the US for most of the state plane coordinate zones. For LDPs, the distortion tolerance is typically much smaller.
- The projection parameters and transformation algorithms for a LDP are designed to accommodate an average ground elevation in the area to be covered by the LDP zone.
- All meridians on the mapping grid are parallel with the central meridian. Convergence, the angle between true north and grid north, is a consideration in some applications.
- All map projection and LDP derived plane coordinates are strictly 2-D. There is no mathematical third dimension. Elevations must be computed separately.

Assumptions/features/design criteria associated with the GSDM include:

- The origin for 3-D geospatial data is taken to be Earth's center of mass.
- The position of any point within the birdcage of orbiting GNSS satellites is defined by a triplet of metric rectangular X/Y/Z coordinates in the ECEF coordinate system.
- Given a standard ellipsoid, such as the GRS80, the latitude/longitude/ellipsoid height of any X/Y/Z point can be computed with mathematical exactness. Note - it is NOT appropriate to mix ECEF coordinates from different datums, e.g., WGS84 and NAD83.
- The user selects any desired point - real or virtual - to be the origin (standpoint).
- The "users view" of the world is determined by selection of any forepoint in the data base or point cloud. The 3-D vector from standpoint (Point 1) to forepoint (Point 2) is defined by ECEF coordinate differences. $\Delta X = X_2 - X_1$, $\Delta Y = Y_2 - Y_1$, $\Delta Z = Z_2 - Z_1$.

- The $\Delta X/\Delta Y/\Delta Z$ components of the 3-D vector are rotated to the local perspective ($\Delta e/\Delta n/\Delta u$) based upon the latitude/longitude of the standpoint.
- The local tangent plane horizontal distance and the true azimuth from standpoint to forepoint are computed from the Δe and Δn components – same as in plane surveying.

3. COMPARISON OF A LDP WITH THE GSDM

The LDP and the GSDM both include plane Euclidean geometry computations that are fundamental to plane (flat-Earth) surveying computations. Using well-defined plane coordinates relieves a spatial data user from the burden of performing geodetic computations involving latitude and longitude positions on the ellipsoid.

<u>The specific objective of a LDP</u> is to provide reliable local plane coordinates for a point such that the plane coordinate inverse between a pair of local points will closely match the actual ground-level horizontal distance between the same two points. That objective presumes the curved-Earth latitude/longitude positions are the basis for determining the plane coordinates using a defined map projection in conjunction with an average elevation for the area to be covered by the LDP. Several important considerations are:

- The traditional map projection converts ellipsoid distance to grid distance and does not accommodate elevation as a design consideration.

- LDPs incorporate an "average" elevation of the area to be covered in the transformation algorithm such that the coordinates obtained are connected to the latitude/longitude positions on the ellipsoid and to the approximate ground elevation of the point. The algorithms, while tested and proven by geodesists and cartographers, are generally sufficiently complex as to be of little interest to most end users.

- LDPs are derived from equations of a map projection which models two-dimensional relationships. Other than the average design elevation, there is no mathematical definition for the third dimension. That is not a problem given that elevations (orthometric heights referenced to the geoid) are handled separately.

- LDP transformation equations are bidirectional in that mathematical exactness is preserved whether converting latitude/longitude to plane coordinates or plane coordinates to latitude/longitude.

- The transformation equations for a LDP are specific to a given area and, although built upon standard conformal mapping transformation equations, there is not a unique method for including the design elevation. Developing, testing, and publishing those equations is the responsibility of the organization hosting or building the LDP.

<u>The objective of the GSDM</u> is to provide an undistorted view of the real world from any location selected by the user (user view). Part of that user view is obtaining the ground-level horizontal distance between points. In this respect the GSDM and a LDP are similar – they both provide the ground-level horizontal distance between points. However, the GSDM also provides other advantages that are not available when using a LDP. Rather than resorting to a 2-D map projection and selected design elevations for particular zones, the GSDM uses the

ECEF coordinates to determine rectangular components of any vector selected by the user. Those rectangular ECEF components are rotated to local east/north/up components based upon the latitude/ longitude position of the origin (standpoint). That enables the user to select any point as the standpoint and to determine the local 3-D flat-Earth components to any named forepoint. The Δn & Δe components are identical to plane surveying latitudes and departures. The Δu component is the perpendicular distance to the forepoint from the tangent plane through the standpoint. Several important considerations are:

- Geocentric X/Y/Z coordinates are required for both the standpoint (Point 1) and the forepoint (Point 2).

- Geocentric coordinate differences are computed, standpoint to forepoint.

- Those geocentric differences are rotated to the local reference frame – either in a matrix operation or component by component using separate equations as shown below.

- The local tangent plane horizontal distance, Point 1 to Point 2, is the same horizontal distance routinely used in plane surveying and defined as HD(1) in Burkholder (1991).

- The true azimuth from Point 1 to Point 2 is computed simply as $\alpha = \arctan(\Delta e/\Delta n)$ with due respect to quadrant and is called the 3-D azimuth in Burkholder (1997b).

A complete listing of equations for 3-D coordinate geometry and error propagation computations is given in Burkholder (2008). Starting with the ECEF coordinates of the two points, equations for computing the local flat-Earth components (Δn, Δe, & Δu) of the line from Point 1 to Point 2 are:

$$\Delta X = X_2 - X_1 \qquad (1)$$

$$\Delta Y = Y_2 - Y_1 \qquad (2)$$

$$\Delta Z = Z_2 - ZY_1 \qquad (3)$$

$$\Delta e = -\Delta X\, sin\lambda + \Delta Y cos\lambda \qquad (4)$$

$$\Delta n = -\Delta X\, sin\varphi\, cos\lambda - \Delta Y sin\varphi\, sin\lambda + \Delta Z cos\varphi \qquad (5)$$

$$\Delta u = \Delta X\, cos\varphi\, cos\lambda + \Delta Y \cos\varphi\, sin\lambda + \Delta Z\, sin\varphi \qquad (6)$$

Where: φ = latitude of standpoint
λ = east longitude of standpoint (or negative west longitude)

$$HD(1) = \sqrt{\Delta e^2 + \Delta n^2} \qquad (7)$$

$$3D\ (true) azimuth = \tan^{-1}\left(\frac{\Delta e}{\Delta n}\right) \qquad (8)$$

It is also often necessary to compute the latitude and longitude of the standpoint from the ECEF geocentric coordinates. That is a bit more involved but it can be done using the following equations - or see Burkholder (2008).

$$\lambda = \tan^{-1}\left(\frac{Y}{X}\right) \quad \text{with due regard to quadrant.} \tag{9}$$

$$\varphi = \tan^{-1}\left[\left(\frac{Z}{\sqrt{X^2+Y^2}}\right)\left(1+\frac{e^2 N sin\varphi}{Z}\right)\right] \tag{10}$$

$$N = \frac{a}{\sqrt{1-e^2 sin^2\varphi}} \tag{11}$$

Where: a = ellipsoid semi-major axis = 6,378,137.000 m
e^2 = ellipsoid eccentricity squared = 0.006694380023

Notes:

- The parameters given above are for the GRS 1980 ellipsoid. Others can also be used.
- Equations (10) and (11) as given must be iterated for a solution. A non-iterative procedure is given in Burkholder (1993a or 2008).
- The GSDM equations are already in place and equally applicable world-wide. No projection parameters, zone constants, or elevations are needed to determine Δe or Δn.

4. CONTRASTING AND HIGHLIGHTING THE ADVANTAGES OF THE GSDM OVER A LDP

The similarities of the LDP and the GSDM are that each "solves" the grid/ground distance difference when performing local spatial data computations on the curved Earth. This section includes several contrasts and highlights the advantages of the GSDM over a LDP

- The equations and procedures already exist for the GSDM. The fundamental equations, concepts, and geometrical relationships are well-known and already being used.
- The GSDM portrays the world as it is without distorting any geometrical elements.
- The GSDM accommodates 3-D digital spatial data all over the world (and including near space within the birdcage of orbiting GNSS satellites) using one standard set of solid geometry equations.
- Each triplet of X/Y/Z rectangular metric ECEF coordinates is unique and valid anywhere within birdcage of orbiting satellites.
- There is no point-to-point distance limitation imposed when using the GSDM.
- The spatial data user does not need to be concerned with zone parameters, projection constants, or zone boundaries.
- The geometrical integrity established by modern measurement systems is fully supported with no loss of rigor.
- Local users have the local flat-Earth components of each/any vector readily available and can "get the job done" without performing geodetic computations on the ellipsoid.
- Some 3-D measurement systems provide X/Y/Z coordinates, others provide $\Delta X/\Delta Y/\Delta Z$ baselines. The GSDM accommodates both absolute positioning and relative positioning.
- The GSDM accommodates global interoperability. One could say that the LDP is the antithesis of interoperability.

- The GSDM is an "umbrella" 3-D system and a subordinate 2-D LDP can be fully implemented under the umbrella of the GSDM.

5. SOFTWARE AND EXAMPLE ILLUSTRATING FEATURES OF THE GSDM

All equations associated with the GSDM are generic and in the public domain. Users are encouraged write their own software solutions and to enjoy the benefits of using the GSDM. However, the term "BURKORD™ has been trademarked and covers: (1) the name of a software package that performs 3-D coordinate geometry and error propagation computations and (2) the design of a 3-D data base as used by the BURKORD™ software. Anyone offering a commercial product or service whose value relies upon or is enhanced by reference to or use of the BURKORD™ trademark is expected to pay an appropriate licensing fee to Global COGO. Prototype 3-D software for both DOS and Windows operating systems is available gratis at http://www.globalcogo.com/WBK3D.html.

The GSDM has been used successfully on various projects and features of the GSDM are described in articles posted at http://www.globalcogo.com/refbyefb.html. The link below leads to one of those articles that describes a class project using GPS equipment and 3-D data to accomplish a routine section breakdown on the USPLSS system. The article shows specific steps utilized in developing a 2-D plat based upon a 3-D GPS survey. Note that the 2-D survey plat shows ground-level horizontal distances and that azimuths are "grid" with respect to the true meridian through the standpoint (the SW Corner of Section 31). The name of the article is *From 3-D GPS Data to a 2-D Plat – a "No Distortion" Solution.*

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(Or, a copy is available from the author upon request.)

7. BIOGRAPHICAL NOTES

Earl F. Burkholder is retired from teaching in the Surveying Engineering Program at New Mexico State University (NMSU). His education includes a BSCE from the University of Michigan, a MSCE from Purdue University, and sabbatical study at the University of Maine. His career included 5 years working for a large international engineering company, teaching at the Oregon Institute of Technology for 13 years, being self-employed for 5 years, and teaching at NMSU for 12 years. Professional activities include serving two non-consecutive 4-year terms as Editor of the ASCE Journal of Surveying Engineering, serving on the ABET Applied Science Accreditation Commission (Chair 2000-2001), and President of the New Mexico Professional Surveyors during 2009. He is currently Chair of the ASCE Geomatics Division Executive Committee. He is a member of ASCE, ASPRS, NMPS, AGU, and AAGS. He wrote a book, “The 3-D Global Spatial Data Model: Foundation of the Spatial Data Infrastructure” which was published by CRC Press (Francis & Taylor) in April 2008. Other articles he has written are posted at http://www.globalcogo.com.

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