# An Algorithm For Accurate Area Computation

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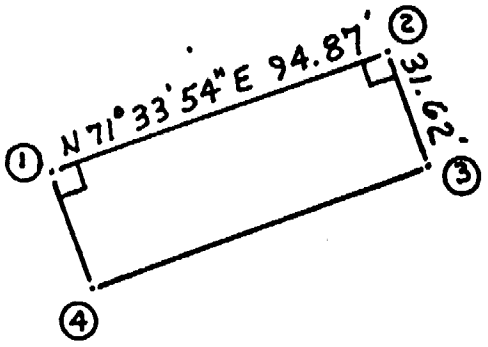
If the following well known formula for computing area by coordinates is programmed into a 10-digit calculator and large (state plane) coordinate values are entered into the program, it is possible to get a bad answer. The equation is:

$$2A = X_1Y_2 + X_2Y_3 + X_3Y_4 + \ldots X_{n-1}Y_n + X_nY_1 - (Y_1X_2 + Y_2X_3 + Y_3X_4 + \ldots Y_{n-1}X_n + Y_nX_1)$$

The problem can be illustrated with the following example:

Compute the area of the rectangle below from the length and width and compare that with the area computed from the coordinates of the corners.




| Point | X | Y |
|---|---|---|
| 1 | 2,160,107.36 | 507,032.16 |
| 2 | 2,160,197.36 | 507,062.16 |
| 3 | 2,160,207.36 | 507,032.16 |
| 4 | 2,160,117.36 | 507,002.16 |

Positive products are;

$X_1Y_2 = 1.095308704 * 10^{12}$ ft$^2$
$X_2Y_3 = 1.095289533$ "
$X_3Y_4 = 1.095229798$ "
$X_4Y_1 = 1.095248971$ "
$Sum_1 = 4.381077006 * 10^{12}$ ft$^2$

Negative products are;

$Y_1X_2 = 1.095289533 * 10^{12}$ ft$^2$
$Y_2X_3 = 1.095359410$ "
$Y_3X_4 = 1.095248971$ "
$Y_4X_1 = 1.095179097$ "
$Sum_2 = 4.381077011 * 10^{12}$ ft$^2$

The area is one half the difference of the sums of the products.

$Sum_2 = 4,381,077,011,000$ ft$^2$
$Sum_1 = 4,381,077,006,000$ ft$^2$
Difference = 5,000 ft$^2$

$$\text{Area} = \frac{\text{Diff.}}{2} = 2{,}500 \text{ ft}^2$$

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Now compute the area of the rectangle as:

Area = $94.87 * 31.62 = 2{,}999.789$ ft$^2$

= (to 4 s.f.) $3{,}000$ ft$^2$

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The discrepancy is due to the loss of significant figures in the subtraction of the sums of the products.

The problem is a common one (though overlooked by some programmers) and can be solved several ways. The usual approach is to require the user to subtract an appropriately large constant from all X and/or Y coordinate values. This approach places an extra responsibility on the user which he/she may not be willing or competent to accept.

This author is convinced that the following approach and algorithm is a better solution. Besides, it yields an extremely efficient method for computing area from coordinates.

First, the coordinates of the first point of the figure are subtracted from the coordinates of each successive point. This means that