Algorithm for Least Squares Adjustment of
9 Stations and 16 Baselines in the Wisconsin GPS CORS Network

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I. The algorithm listed here is generic but documents the specific computational process used to adjust a network of 9 GPS CORS Stations in Southeastern Wisconsin.

II. The purpose of these adjustments was not to duplicate the X/Y/Z coordinate values obtained and published by WisDOT but to obtain the covariance matrix of the adjustment to be used in computation of network and local accuracies. (Incidentally, the computed coordinate values compare quite favorably with the WisDOT published values.)

III. The solution matrix algorithm used is (Mikhail notation – Observations and Least Squares):

1. $\mathbf{W} = (\sigma_0^2)\mathbf{Q}^{-1}$ assume *a prior* reference variance, $\sigma_0^2 = 1.0$.
2. $\mathbf{V} + \mathbf{B\Delta} = \mathbf{F}$ least squares model for indirect observations.
3. $\mathbf{\Delta} = (\mathbf{B}^t \mathbf{W}\, \mathbf{B})^{-1} \mathbf{B}^t \mathbf{W}\, \mathbf{F}$ for computational efficiency, substitute $\mathbf{N} = (\mathbf{B}^t \mathbf{W}\, \mathbf{B})$
4. $\mathbf{V} = \mathbf{F} - \mathbf{B\Delta}$
5. Sigma naught (hat) squared = *posteriori* reference variance = $\mathbf{V}^t \mathbf{W}\, \mathbf{V} / r$
6. Covariance matrix of computed result = $\mathbf{\Sigma}_{\Delta\Delta} = (\mathbf{V}^t \mathbf{W}\, \mathbf{V} / r)\, \mathbf{N}^{-1}$

where:

- $\mathbf{W}$ = Weight matrix computed as inverse of co-factor matrix Q.
- $\mathbf{Q}$ = Matrix of covariances of observations – baselines (and stations).
- $\mathbf{V}$ = Vector of residuals.
- $\mathbf{B}$ = Coefficient matrix of observations – 1's and 0's for linear solution.
- $\mathbf{\Delta}$ = Vector of unknown parameters that is computed.
- $\mathbf{F}$ = Vector of constants obtained when writing the observation equations.
- r = Redundancy – number of observations minus number of unknowns.
- $\mathbf{\Sigma}_{\Delta\Delta}$ = Covariance matrix of the computed parameters – goal of the adjustment.

IV. RINEX data were downloaded from 9 WisDOT CORS sites such that 16 independent (non-trivial) vectors could be computed. Those RINEX data were used in various vendor baseline processing packages along with the NGS precise ephemeris for each. Those baselines and their covariances were used in a least squares adjustment (as described above) to obtain the covariance matrix of the network.

V. The computed X/Y/Z coordinates ($\mathbf{\Delta}$) and the network covariance matrix ($\mathbf{\Sigma}_{\Delta\Delta}$) were used to develop a compact BURKORD™ data file from which network and local accuracies were computed.

VI. The WisDOT CORS coordinates for stations FOLA and KEHA were used as "control." In reality, those two stations were allowed to "float" in the adjustment according to tolerance values assigned to each component prior to the adjustment. Tolerance values of 0.002 meters, 0.010 meters, 1.00 meters, and 5.00 meters were used.