

## Mathematical Models

A mathematical model is an abstract representation of the real world using geometrical figures and numbers. It gives relevance and meaning to the concepts of spatial data and location. A triangle, rectangle, or other geometrical figure drawn on a sheet of paper to represent a tract of land, is a simple mathematical model. Numerical dimensions representing the lengths of the sides or the size of an angle may also be included and are part of the model. When the graphical elements are properly oriented and the lines shown with proportional lengths, the drawing can be said to be a map and the scale of the map is stated as a ratio of units on the map to units on the ground such as 1 inch on the map equals 1 mile on the ground or a unitless ratio such as 1:100,000 which would have a metric equivalent of 1 cm on the map equals 100,000 cm (1 kilometer) on the ground.

**Measurements:** Numbers shown on a model may represent quantities which were measured and are therefore not exact. Such dimensions contain some uncertainty based upon the circumstance of the measurement. For example, a distance measured by GPS may be more accurate than the same distance measured by EDM, which may be more accurate than a stadia measurement which is probably more accurate than can be determined by pacing. Similarly, other measured quantities such as angles are not exact and some may be more accurate than others.

**Errorless Dimensions:** Other dimensions shown on a model may be design dimensions and may be considered errorless until some attempt is made to create the physical object. The width of a five-foot sidewalk shown on a set of plans is without error until the forms are built and the concrete is poured. After the concrete sets, then the width of the side walk can be measured and reported. (Of course, the person building the form is largely responsible for the final width of the sidewalk.) Another possible example is a street right-of-way which might be considered errorless because statute dictates a private owner may not legally encroach upon publicly held land. Other quantities which might be considered errorless are record measurements from a prior survey or the published location of higher order control monuments upon which a current survey is based.

**Derived Quantities:** A mathematical model may also depict derived quantities such as area or indirect measurements which were not measured but which were computed from other known and/or measured quantities. The accuracy of such derived quantities is dependent upon the accuracy of the underlying measurements and the appropriateness of the model. Error propagation is the mechanism by which the uncertainty of computed quantities is determined. In mathematical terms, rigorous error propagation is accomplished using concepts and rules of variance/covariance.

When formally defined and used, every mathematical model has two components - functional and stochastic. In practice, the functional model has been used extensively while use of the stochastic model is becoming more commonplace.