APPENDIX C

The Michigan Scale Factor

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THE MICHIGAN SCALE FACTOR

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BIOGRAPHICAL SKETCH

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ABSTRACT

The Michigan Coordinate System, which utilizes three Lambert Conic Conformal Projections, was formally adopted by the Michigan Legislature in 1964. The projections are based on the Clarke Spheroid of 1866, modified to place the reference surface at an elevation of approximately 800 feet above sea level. This was done in an effort to eliminate the need for the sea level reduction on most surveys since most of the land surface in Michigan is not far from the 800 foot elevation. When the sea level reduction is eliminated, the grid distance becomes the product of the horizontal ground distance and the scale factor. However, if one computes the scale factor for a point in Michigan according to the formula found in C&GS Publication 62-4, "State Plane Coordinates by Automatic Data Processing", or if one uses a calculator or a computer programmed to use the same formula, the result is not the same as is found in the "Plane Coordinate Projection Tables" for the State of Michigan. The published formula gives a scale factor which is valid on the sea level reference surface while the Michigan projection tables give the correct scale factor for the reference surface at the 800 foot elevation. The formula can be easily modified to give the correct scale factor; however, either scale factor will work if used in conjunction with the proper sea level factor. A similar problem is encountered in the Michigan Lambert zones when one attempts to use the constant "L_5" from Publication 62-4 for "k_0" in the formulas for computing scale factors from state plane coordinates as given by Professor Ralph Moore Berry in 1972. Again, correct determination of the Michigan scale factor is assured by using the correct constants in the published formulas.

DEFINITIONS

Sea level factor - that factor by which a short horizontal ground distance is multiplied to determine the corresponding distance on a reference surface. In this paper two reference surfaces are considered, the surface of the Clarke Spheroid of 1866 and the Michigan Spheroid. The difference between them is approximately 800 feet in elevation.

Scale factor - that factor by which a distance on a reference surface (spheroid) is multiplied to determine the corresponding distance on the projection surface.

Grid factor - the product of the sea level factor and the scale factor. The grid factor is constant for a given spheroid, zone, elevation and location.

These definitions are intended to be consistent with, although not as inclusive as, the definitions for the same terms as found in, "Definitions of Surveying and Associated Terms", [7].

INTRODUCTION

The geometry of the distance reduction for both the sea level reference surface and the 800 foot elevation reference surface is shown in Figure C1 where the horizontal ground distance, D_1, is reduced to D_2 and D_2' by;

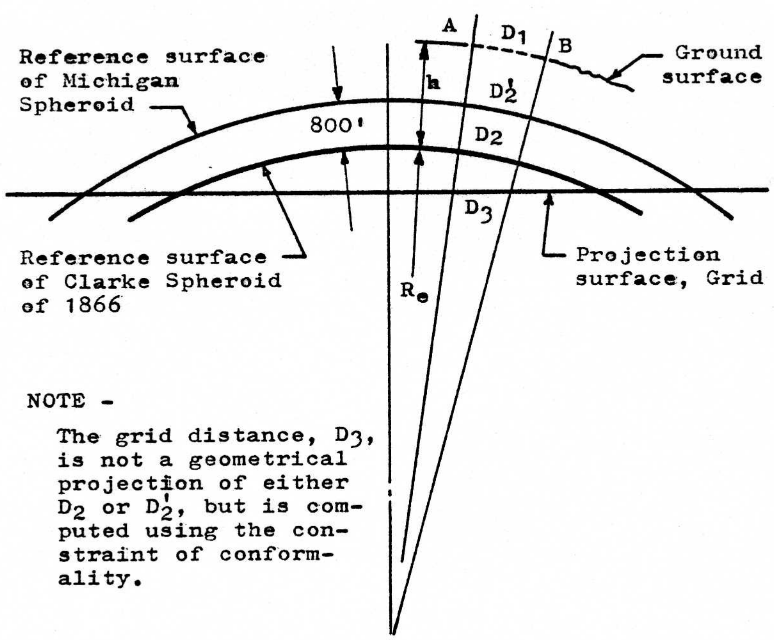

Figure No. C1 Distance Reduction from Ground to Grid

$$D_2 = D_1 * R_e/(R_e + h) \tag{1}$$

$$D_2' = D_1 * (R_e + 800)/(R_e + h). \tag{2}$$

The reduction of the distance on the reference surface to its corresponding grid distance, D_3, on the projection surface is given by;

$$D_3 = D_2 * SF_s = D_2' * SF_e \quad \text{where,} \tag{3}$$

- D_1 = the horizontal ground distance between the plumb lines at points "A" and "B".
- D_2 = the horizontal ground distance reduced to the surface of the Clarke Spheroid of 1866.
- D_2' = the horizontal ground distance reduced to Michigan Spheroid.
- R_e = the radius of curvature of the spheroid at a given latitude, often taken to be 20,906,000 feet, but see equations (10)&(13).
- h = the height of the horizontal ground distance above the Clarke Spheroid of 1866.
- D_3 = the grid distance on the projection surface.
- SF_s = the scale factor for a point on the sea level reference surface, Clarke's Spheroid.
- SF_e = the scale factor for a point on the elevated reference surface, the Mich. Spheroid.

In equation (1) the sea level factor is unity when $h = 0$, but in equation (2) the sea level factor is unity when $h = 800$ feet. The distance on the reference surface is either longer or shorter than the corresponding horizontal ground distance depending on whether the ground elevation is below or above the reference surface.

Substituting equations (1) and (2) into equation (3) gives;

$$D_3 = D_1 * \left(\frac{R_e}{R_e + h}\right) * SF_s = D_1 * \left(\frac{R_e + 800}{R_e + h}\right) * SF_e \tag{4}$$

from which the grid factor is determined by;

$$\frac{D_3}{D_1} = \left(\frac{R_e}{R_e + h} \right) * SF_s = \left(\frac{R_e + 800}{R_e + h} \right) * SF_e \tag{5}$$

and the ratio of the scale factors is given by;

$$SF_s/SF_e = (R_e + 800)/R_e. \tag{6}$$

However, in determining the size of the Michigan Spheroid, the ratio in equation (6) was held to be 1.0000382 exactly (page 1 of [10]). Hence equation (6) can be rewritten as;

$$SF_s = 1.00003820 * SF_e. \tag{7}$$

The purpose of this paper is to:

1. show that the scale factors obtained in Michigan by using the formula in Publication 62-4 (page 4 of [5]) differ from the scale factors listed in the projection tables by a factor of 1.00003820.

2. analyze the reason for the discrepancy.
3. demonstrate proper use of the Michigan scale factor.
4. show how scale factors in Michigan can be computed correctly from state plane coordinates using the formulas given by Berry [4].

BACKGROUND

The surface of the Earth is approximated by rotating an ellipse about its minor axis. The ellipse shown in Figure C2 represents a meridian section of the Earth. The size and shape of the Michigan Spheroid are specified by the length of the semi-major axis, a = 20,926,631.530789 American Survey feet, and the eccentricity, e = 0.08227 18542 23003 8 (page 1 of [10]). The location of a point on a meridian is specified by its latitude, ϕ, or by its co-latitude, P = $90^{\circ} - \phi$. Other quantities which are derived from these are;

- b = the semi-minor axis of the ellipse.
- M = the radius of curvature in the meridian section.
- N = the radius of curvature in the prime vertical, perpendicular to the meridian section.
- R_e = the geometrical mean radius of curvature for the surface of the spheroid at a given latitude.

$$b^2 = a^2 * (1 - e^2)$$

$$M = a * (1 - e^2)/(1 - e^2 \sin^2\phi)^{3/2} \quad (8)$$

$$N = a/(1 - e^2 \sin^2\phi)^{\frac{1}{2}} \quad (9)$$

$$R_e = (M*N)^{\frac{1}{2}} = (a * (1-e^2)^{\frac{1}{2}})/(1-e^2 \sin^2\phi) \quad (10)$$

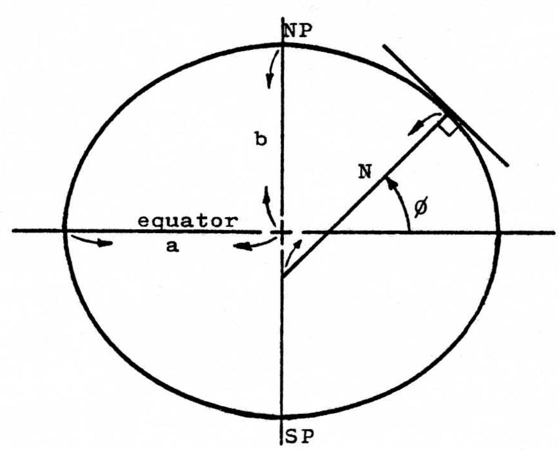

Figure No. C2 Elements in the Meridian Section Ellipse

The North American Datum of 1927 (NAD 1927) is the national reference for the horizontal control network which defines the coordinates used in the state plane coordinate systems. The NAD 1927 is, in turn, computed on the Clarke Spheroid of 1866. However, in an effort to simplify computations by eliminating the need for the sea level reduction on most surveys, the three Michigan Lambert projections were defined on a spheroid having a reference surface approximately 800 feet above the surface of the Clarke Spheroid of 1866. This was achieved by multiplying the semi-major axis of the Clarke Spheroid by 1.0000382 (exact) and holding the eccentricity unchanged [9] [10].

Table C1 shows a comparison of the ellipsoidal parameters and derived values for "b" and for "R_e" at the central parallel, ϕ_o, of each of the three Michigan zones. Although the values are listed for both spheroids to show the difference, the values of the Michigan Spheroid should be used in <u>all</u> state plane coordinate computations in Michigan.

TABLE C1 COMPARISON OF SPHEROIDS (Part 1)

	Clarke, 1866	Michigan	Ratio	Difference
a	20,925,832.16'	20,926,631.53'	1.00003820	799.37'
e	0.082271854223	0.082271854223	1.00000000	none
b	20,854,892.01'	20,855,688.67'	1.00003820	796.66'
South Zone, ϕ_o = 42° 53' 06".055446				
R_e	20,920,471.45'	20,921,270.62'	1.00003820	799.17'
Central Zone, ϕ_o = 44° 56' 36".092428				
R_e	20,925,571.01'	20,926,370.37'	1.00003820	799.36'
North Zone, ϕ_o = 46° 17' 07".101225				
R_e	20,928,899.07'	20,929,698.56'	1.00003820	799.49'

ANALYSIS OF SCALE FACTOR FORMULA

The formula for the scale factor in a Lambert Zone as given by Claire in "State Plane Coordinates by Automatic Data Processing" (page 4 of [5]) is;

$$k = \frac{L6 * R_m * (1 - 0.0067686580 * \sin^2\phi)^{\frac{1}{2}}}{20,925,832.16 * \cos\phi} \qquad (11)$$

which can be restated as;

$$k = \frac{L6 * R_m * (1 - e^2 * \sin^2\phi)^{\frac{1}{2}}}{a * \cos\phi}, \text{ where} \qquad (12)$$

L_6 = the projection constant, ℓ, computed from the basic equations for the Lambert projections with two standard parallels (page 11 of [12]).

R_m = the map radius of a given parallel. This value is tabulated for each minute of latitude in the projection tables.

ϕ = the latitude of a given point.

e^2 = 0.0067686580, the square of the eccentricity of both spheroids, Clarke 1866 and Michigan.

a = 20,925,832.16 feet, the semi-major axis of the Clarke Spheroid of 1866.

When the value of "a" of the Clarke Spheroid of 1866 is used in equation (12) instead of the value of "a" for the Michigan Spheroid, an erroneous scale factor is obtained. Since the ratio of the two values of "a" is 1.0000382, the resulting scale factors are of the same ratio. The correct scale factor is applicable to the reference surface at the 800 foot elevation and agrees with the scale factors in the projection tables [10]. By equation (7) the erroneous value of the scale factor is applicable to the sea level reference surface.

CHOOSING THE SEA LEVEL FACTOR

The sea level factor is determined from the elevation of a point and the radius of curvature of the spheroid at the same point. As mentioned earlier the value of R_e is often taken to be 20,906,000 feet, although it could be computed using equation (10). However, since the ratio in equation (6) was held to be 1.0000382 exactly, a value of R_e for the entire state is obtained by solving equation (6) for R_e.

$$R_e = 800/(1.0000382 - 1) = 20,942,400 \text{ feet} \tag{13}$$

The values of R_e at the central parallel of each of the three Michigan zones are listed in Table 1. These values of R_e could also be used.

The grid factor is defined as the product of the scale factor and the sea level factor, but since the grid factor is also the ratio of the grid distance to the ground distance, it remains constant for a given location and elevation. Thus, it is a matter of choice whether one uses the correct scale factor at the 800 foot elevation with the elevated sea level factor, SLF_e,

$$SLF_e = \frac{R_e + 800}{R_e + h} = \frac{20,943,200}{20,942,400 + h} \tag{14}$$

or if one uses the erroneous scale factor as computed by the formula in Publication 62-4 with the conventional sea level factor, SLF_s,

$$SLF_s = \frac{R_e}{R_e + h} = \frac{20,942,400}{20,942,400 + h}. \tag{15}$$

Note that equation (14) reduces the horizontal ground distance to the 800 foot elevation and that equation (15) reduces the horizontal ground distance to sea level. Thus, when the ground elevation is acceptably near the 800 foot elevation, equation (14) becomes unity and the grid factor equals the correct scale factor.

EXAMPLE OF GRID FACTOR COMPUTATION

The following is an example of determining the grid factor in Michigan at a latitude of 45° and at an elevation of 1200 feet above mean sea level. The scale factor formula given in Publication 62-4 and restated in equation (12) is;

$$k = \frac{L_6 * R_m * (1 - e^2 * \sin^2 \phi)^{1/2}}{a * \cos \phi}, \text{ where}$$

L_6	=	0.7064074100 (central zone)	page 7 of [10]
R_m	=	20,981,064.925	page 20 of [10]
e^2	=	0.006768657997	Table 1
a	=	20,926,631.53 (Michigan)	Table 1
a	=	20,925,832.16 (Clarke, 1866)	Table 1

Using the Michigan Spheroid value of "a", k is;

$$\frac{(0.7064074100)(20981064.925)(1-0.006768657997*\sin^2 45°)^{1/2}}{20,926,631.53 * \cos 45°}$$

$$k = \frac{14,796,078.60}{14,797,363.06} = 0.9999131966 \text{ (correct).} \quad (16)$$

Using the Clarke Spheroid of 1866 value of "a", k is;

$$\frac{(0.7064074100)(20981064.925)(1-0.006768657997*\sin^2 45°)^{1/2}}{20,925,832.16 * \cos 45°}$$

$$k = \frac{14,796,078.60}{14,796,797.83} = 0.9999513933 \text{ (erroneous).} (17)$$

The elevated sea level factor by equation (14) is;

$$SLF_e = \frac{R_e + 800}{R_e + h} = \frac{20,943,200}{20,943,600} = 0.9999809011 \quad (18)$$

and the conventional sea level factor by equation (15) is;

$$SLF_s = \frac{R_e}{R_e + h} = \frac{20,942,400}{20,943,600} = 0.9999427033 \quad (19)$$

The grid factor, being the product of the scale factor and the appropriate sea level factor, can be obtained by using either scale factor. The correct scale factor at the 800 foot reference surface (16) times the elevated sea level factor (18) is;

$$GF = (0.9999131966)(0.9999809011) = 0.9998940994 \quad (20)$$

and the erroneous scale factor (17) times the conventional sea level factor is;

$$GF = (0.9999513933)(0.9999427033) = 0.9998940994 \quad (21)$$

Although the same grid factor was obtained separately using both scale factors, it will be shown in the next section that there is only one "correct" scale factor. It is recommended that the correct scale factor be used in all cases.

FURTHER ANALYSIS OF THE SCALE FACTOR FORMULA

The analysis of the scale factor formula is not complete until each term in the equation is examined and until it is assured that all spheroidal differences have been taken into account. Investigating equation (12) term by term, L_6 is derived by Thomas (page 117 of [12]) and shown by Berry [3] to be;

$$L_6 = \frac{\ln (N_s * \sin P_s \;/\; N_n * \sin P_n)}{\ln \tan(Z_s/2) - \ln \tan(Z_n/2)}, \quad \text{where} \qquad (22)$$

N_s = the radius of curvature in the prime vertical at the south standard parallel.

N_n = the radius of curvature in the prime vertical at the north standard parallel.

P_s = the colatitude of the south standard parallel.

P_n = the colatitude of the north standard parallel.

Z_s = the conformal colatitude of the south standard parallel.

Z_n = the conformal colatitude of the north standard parallel.

The conformal colatitude which appears in the denominator of equation (22) is derived by Thomas (page 87 of [12]) as;

$$\tan(Z/2) = \tan(P/2) * \left[\frac{1 + e * \cos P}{1 - e * \cos P}\right]^{e/2}. \qquad (23)$$

When the eccentricity, e, is held constant the conformal colatitude is independent of the semi-major axis, a, of the spheroid.

Considering the numerator of equation (22) and recalling from equation (9) that,

$$N = a/(1 - e^2 * \sin^2\phi)^{1/2} \qquad (24)$$

the numerator of equation (22) becomes;

$$\ln\left[\frac{a * \sin P_s}{(1 - e^2 * \sin^2\phi_s)^{1/2}} \Big/ \frac{a * \sin P_n}{(1 - e^2 * \sin^2\phi_n)^{1/2}}\right]$$

$$= \ln\left[\frac{\sin P_s}{\sin P_n} * \frac{(1 - e^2 * \sin^2\phi_n)^{1/2}}{(1 - e^2 * \sin^2\phi_s)^{1/2}}\right] \qquad (25)$$

which is also independent of the semi-major axis, "a", when the eccentricity is held constant. Since the numerator and the denominator of equation (22) are both independent of the semi-major axis of the spheroid, the term, L_6, is also independent of the semi-major axis when the eccentricity is held constant.

Next, consider the map radius, R_m, which according to Adams and Claire (page 6 of [2]) is;

$$R_m = K * (\tan Z/2)^{L_6} \qquad (26)$$

where K is the map radius of the equator as shown by Berry [3] and is computed by;

$$K = \frac{N_s * \sin P_s}{L6 * (\tan Z_s/2)^{L6}} = \frac{N_n * \sin P_n}{L6 * (\tan Z_n/2)^{L6}}. \quad (27)$$

Substituting for "N" from equation (9),

$$K = \frac{a * \sin P_s}{(1 - e^2 * \cos^2 P_s)^{1/2} * L_6 * (\tan Z_s/2)^{L6}}. \quad (28)$$

Substituting equation (28) into equation (26),

$$R_m = \frac{a * \sin P_s * (\tan Z/2)^{L6}}{(1 - e^2 * \cos^2 P_s)^{1/2} * L_6 * (\tan Z_s/2)^{L6}}. \quad (29)$$

Substituting equation (29) into equation (12), the scale factor becomes;

$$k = \frac{L6 * a * \sin P_s * (\tan Z/2)^{L6} * (1 - e^2 \sin^2 \phi)^{\frac{1}{2}}}{L6 * a * \cos \phi * (\tan Z_s/2)^{L6} * (1 - e^2 \cos^2 P_s)^{\frac{1}{2}}}$$

$$= \frac{\sin P_s * (\tan Z/2)^{L6} * (1 - e^2 \sin^2 \phi)^{1/2}}{\cos \phi * (\tan Z_s/2)^{L6} * (1 - e^2 \cos^2 P_s)^{1/2}} \quad (30)$$

which is also independent of the semi-major axis for a given eccentricity. The scale factor depends only on the latitude of a given point once the eccentricity of the spheroid is determined and the location of the standard parallels is selected. Therefore, there is only one "correct" scale factor for a given latitude in a given Michigan Lambert zone. Although equation (30) is not very efficient for routine computation of the scale factor, the results from equation (30) are consistent with the scale factors published in the projection tables [10].

DETERMINATION OF THE MAPPING RADIUS, R_m

The value of the mapping radius can be computed directly using equation (26). However, eleven significant figures are required to get R_m correct to three decimal places of feet. Since the tenth significant figure of some calculators is not to be trusted, it is even more important to use another method to compute R_m. Rather than "looking it up in the tables", Adams and Claire [2] developed a way to compute the distance from the central parallel which is a much smaller number. This method is used by Claire in Publication 62-4 as;

$$R_m = L_3 + s\, L_5 \left[1 + \left(\frac{s}{10^8}\right)^2 \left[L_9 - \left(\frac{s}{10^8}\right) L_{10} + \left(\frac{s}{10^8}\right)^2 L_{11}\right]\right] \quad (31)$$

where,

$$s = 101.2794065\left[60 * (L_7 - \phi') + L_8 - \phi'' + \left[1052.893882 - (4.483344 - 0.023520 \cos^2\phi) \cos^2\phi\right] \sin\phi \cos\phi\right],$$

the length of the meridian arc from ϕ_o to ϕ and, (32)

L_3 = the map radius of the central parallel, ϕ_o.

L_5 = the scale factor along the central parallel.

L_7 = the degrees and minutes portion of the rectifying latitude for the central parallel (minutes).

L_8 = the seconds portion of the rectifying latitude for the central parallel.

L_9 =, L_{10} =, L_{11} = } coefficients for the series expansion of the change in the map radius between ϕ and ϕ_o as given in equation (403) of Thomas [12].

ϕ' = the degrees and minutes portion of ϕ (minutes).

ϕ'' = remainder of ϕ expressed in seconds.

Figure C3 shows some of the elements listed above on a developed Lambert conic conformal projection and helps to illustrate the method used by Claire. Instead of computing R_m directly, he starts with R_o, the mapping radius of the central parallel of the zone and adds algebraically (north is minus) the distance along the central meridian to the parallel of latitude which goes through a given point.

The concept of a rectifying sphere is used in equation (32) to determine the meridian arc distance between ϕ and ϕ_o. A rectifying sphere is a sphere which has the same circumference as the ellipse of a meridian section of a sphereoid On a sphere the latitude increases linearly with the arc

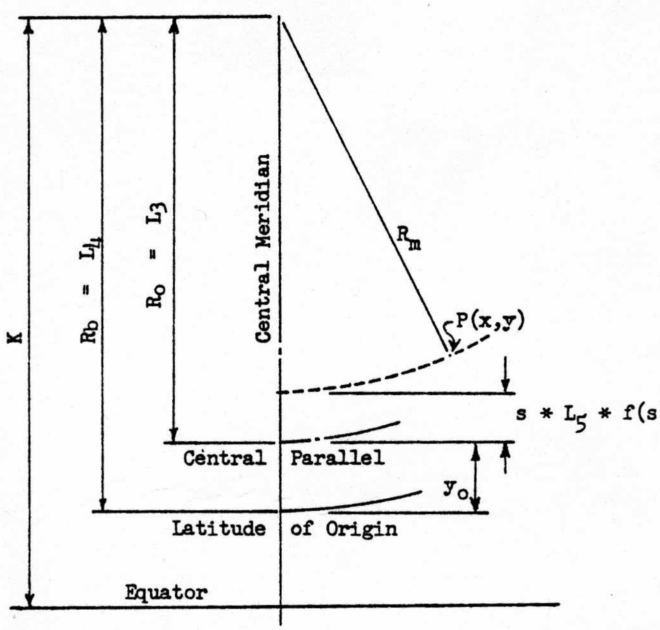

Figure No. C3 Developed Lambert Conic Conformal Projection distance. However, on an ellipse the relationship is more complex. Adams (Appendix of [1]) derived the relationship between the geodetic latitude, ϕ, and the rectifying latitude, ω, and Claire (page 42 of [5]) restates it as;

$$\omega'' = \phi'' - (1052''.893882 - (4''.483344 - 0''.023520*\cos 2\phi)*\cos^2\phi)*\sin\phi*\cos\phi \quad (33)$$

The numerical values in equation (33) were determined using the eccentricity of the Clarke Spheroid of 1866. However, since the Michigan Spheroid has the same eccentricity, the rectifying latitude for a given geodetic latitude is the same on either spheroid.

By equation (32) the meridian distance, s, is the product of the length per second of arc times the number of seconds of arc of rectifying latitude which corresponds to the interval of geodetic latitude, $\phi_o - \phi$. The seconds of rectifying latitude is given by equation (33) and Claire (page 43 of [5]) gives the quantity 101.2794065 as ". . . the length, in feet, of one second of arc on a sphere whose circumference equals the meridional arc of the Clarke 1866 ellipsoid." A different length factor is required for use with the Michigan Spheroid which has a longer meridional arc.

Since an ellipse is symmetrical to both axes, the length of an ellipitical quadrant is equal to the length of a quadrant of its rectifying sphere. The length factor is computed by dividing the ellipitical quadrant arc by 32,400 seconds per quadrant. Adams (page 122 of [1]) gives the length of a meridian arc of an ellipse as;

$$M = a\,(1 - e^2) \int_0^{\phi} \frac{d\phi}{(1 - e^2 \sin^2\phi)^{3/2}} \quad (34)$$

which, when evaluated for limits of 0° to 90° with e^2 = 0.006768657997 and using the formula given by Clark (page 405 of [6]) and coefficient for e^8 as given by Jordan (page 67 of [8]), gives;

$$s, (0^\circ - 90^\circ) = 1.568134898 * a. \quad (35)$$

Since equation (35) is linear in "a", the resulting length factor will be of the same ratio as the values for the semi-major axis of the two spheroids. Table C2 gives a comparison of the values for the meridian quadrant and the length factor for both spheroids.

TABLE C2 COMPARISON OF SPHEROIDS (Part 2)

	Clarke, 1866	Michigan	Ratio	Difference
a, (Table 1)	20,925,832.16'	20,926,631.53'	1.00003820	799.37'
s, (0° - 90°)	32,814,527.69'	32,815,781.20'	1.00003820	1253.51'
length factor	101.2794065'/"	101.2832753'/"	1.00003820	.0038688'/"

Since the distance "s" as given by equation (32) uses the length factor for the Clarke Spheroid of 1866, the value of "s" will be too small by a factor of 1.00003820 when used in any of the three Michigan Lambert zones. Claire circumvents the problem by using the "sea level" value of the scale factor of the central parallel, L_5, in equation (31) which is too large by a factor of 1.00003820 (see equation (7)). Thus, the product of "$s*L_5$" in equation (31) is unchanged and the same formula can be used for the Michigan zones as is used for Lambert zones in other states. Claire's value of L_5 is to be used with the length factor for the Clarke Spheroid of 1866, but the "correct" scale factor for the central parallel of the zone is to be used with the length factor for the Michigan Spheroid. The correct scale factor for the central parallel of each Michigan zone and Claire's value of L_5 are listed in Table C3.

TABLE C3 COMPARISON OF CENTRAL PARALLEL SCALE FACTORS

	Correct Scale Factor @ ϕ_o	Claire's L_5	Ratio
South Zone	0.99990 68822	0.99994 50783	1.00003820
Central Zone	0.99991 27095	0.99995 09058	1.00003820
North Zone	0.99990 28379	0.99994 10344	1.00003820

Formulas for the constants, L_9, L_{10}, and L_{11} in equation (31) are derived by Thomas [12] and restated by Claire [5],

$$L_9 = \frac{1}{6 * R_o * N_o} * 10^{16} \qquad (36)$$

$$L_{10} = \frac{(5 * R_o - 4 * N_o) * \tan \phi_o}{24 * R_o^2 * N_o^2} * 10^{24} \qquad (37)$$

$$L_{11} = \frac{(5 + 3 * \tan^2\phi_o)}{120 * R_o * N_o^3} * 10^{32}, \quad \text{where} \qquad (38)$$

R_o = M = the radius of curvature in the meridian section at the central parallel.

N_o = N = the radius of curvature in the prime vertical at the central parallel.

Although these constants should be computed using the value of "a" for the Michigan Spheroid, it turns out that the difference in R_m caused by using Claire's constants, which are computed on the Clarke Spheroid of 1866, is less than 0.01 foot at a distance of 500,000 feet from the central parallel. As one gets closer to the central parallel, the difference becomes even smaller. The values of L_9, L_{10}, and L_{11} for each Michigan zone are listed in Table C4 for both spheroids. Claire's values from Publication 62-4 are also listed. Since the difference is so small, either set of constants can be used; however, use of the correct constants is recommended.

Since the method of computing R_m used by Claire gives the same value as one obtains by using equation (26) little harm would have been done if the value of L_5 would have been noted as not being the scale factor of the central parallel for the three Michigan Lambert zones. However, as noted in the next section, proper use of L_5 is crucial in determining the correct scale factor from state plane coordinates.

TABLE C4 COMPARISON OF CONSTANTS L_9, L_{10}, AND L_{11}

	Clarke, 1866	Michigan	Claire 62-4
South Zone			
L_9	3.808078	3.80779	3.80808
L_{10}	4.157064	4.15659	4.15706
L_{11}	32.89009	32.8851	33
Central Zone			
L_9	3.806222	3.80593	3.80622
L_{10}	4.468752	4.46824	4.46875
L_{11}	34.60000	34.5947	35
North Zone			
L_9	3.805012	3.80472	3.80501
L_{10}	4.684299	4.68376	4.68430
L_{11}	35.85450	35.8490	36

COMPUTATION OF SCALE FACTOR FROM STATE PLANE COORDINATES

An engineering approach to computing scale factors was presented by Professor Ralph M. Berry in 1972 [4]. His formulas are tailor made for computer processing and the results are generally reliable to seven or eight significant figures. The equation given by Berry for the scale factor is;

$$k = k_o * (1 + K * q^2) \quad \text{with } q = s/10^6 \quad \text{where,} \qquad (39)$$

k_o = the scale factor of the central parallel of a Lambert zone, generally Claire's value of L_5.

K = an empirical constant for a given zone. These constants are tabulated for all zones in the appendix of [4].

q = a function of the distance from the central parallel.

s = the distance from the central parallel.

The correct value of the scale factor of the central parallel as listed in Table 3 must be used to compute a correct scale factor in Michigan. If Claire's value of L_5 for Michigan Lambert zones is used, the resulting scale factor is too large by a factor of 1.00003820. From equation (7) one can see that the erroneous scale factor is really a "sea level" scale factor. Although the correct grid factor could still be obtained using equation (15), use of the correct scale factor is recommended. If the ground elevation is far enough from 800 feet MSL to make the sea level reduction significant, the elevated sea level factor should be used with the correct scale factor to compute the correct grid factor.

CONCLUSION

Although inferences could be made as to the merits of the use of project datums, zone datums, and sea level reductions (especially when incorporating the 1983 NAD), that is sufficient for another paper and beyond the scope of this paper on the Michigan scale factor.

The Michigan scale factor is applicable to a reference surface 800 feet above sea level and should be used for distances which are near (or have been reduced to) that reference surface. The Michigan scale factor can be determined correctly by;

1. Scaling the latitude of a point from an appropriate topographic map and using the latitude as an argument to select the corresponding scale factor from the projection tables.
2. Using the formula listed in Publication 62-4, modified to use the Michigan Spheroid value of "a" rather than the value of "a" for the Clarke Spheroid of 1866.
3. Computing the scale factor from state plane coordinates according to the procedure given by Professor Ralph M. Berry [4]. However, the correct value of the scale factor for the central parallel of the zone must be used for k_o (see Table 3).

The author has encountered several cases where the wrong scale factor has been used due to confusion caused by the discrepancies discussed in this paper. It is also disconcerting to discover that commercial computer programs are available which give a sea level scale factor in Michigan. However, the point is made that it is our responsibility as professionals to understand and to verify the answers obtained from a "black box". It is hoped that this paper will increase our understanding of the Michigan scale factor and give it a better chance of being used correctly.

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APPENDIX C: REFERENCES

1. Adams, Oscar S; 1949, "Latitude Developments Connected With Geodesy and Cartography," U.S. Coast & Geodetic Survey Special Publication No. 67

2. Adams, Oscar S. and Charles N. Claire; 1935, "Manual of Plane Coordinate Computation," U.S. Coast & Geodetic Survey Special Publication No. 193

3. Berry, Ralph Moore; 1971, "The Michigan Coordinate System," Text of a Geodetic Control Seminar sponsored by the Michigan Society of Registered Land Surveyors and held at the University of Michigan in December, 1971

4. Berry, Ralph Moore; 1972, "Simple Algorithms for the Calculation of Scale Factors for Plane Coordinate Systems," Proceedings of the American Congress on Surveying & Mapping, 32nd Annual Meeting, Washington D.C., March, 1972

5. Claire, Charles N; 1968, "State Plane Coordinates by Automatic Data Processing," U.S. Coast & Geodetic Survey Publication 62-4

6. Clark, David; 1963, Plane and Geodetic Surveying - Volume II, Fifth Edition, Constable & Company Ltd., London

7. "Definitions of Surveying and Associated Terms," 1972, Prepared by a Joint Committee of A.C.S.M. and A.S.C.E.

8. Jordan-Eggert; 1962, Jordan's Handbook of Geodesy, Volume III, First Half, translated by Martha W. Carta, Corps of Engineers, U.S. Army, Army Map Service, Washington D.C.

9. Michigan Compiled Laws, Sections 54.231 to 54.239

10. "Plane Coordinate Projection Tables, Michigan (Lambert)," U.S. Coast & Geodetic Survey Publication No. 65-3, 1968

11. Richardus, Peter; and Ron K. Adler; 1972, Map Projections for Geodesists, Cartographers, and Geographers, North Holland Publishing Co., Amsterdam - London, and American Elsevier Co, Inc., New York

12. Thomas, Paul D; 1952, "Conformal Projections in Geodesy and Cartography," U.S. Coast & Geodetic Survey Special Publication No. 251