Comparison of Distances on Nebraska Page Baseline

Earl F. Burkholder – May 28, 2015

I. For those who want to know, this item has its origin in a thread posted on Surveyor Connect – see http://surveyorconnect.com/index.php?mode=thread&id=318897

II. Sincere thanks again to Jerry Penry for the original posting. It is turning out to be a very productive post for me because I am preparing a Second Edition to the book, "The 3-D Global Spatial Data Model" published by CRC Press in 2008. I look forward to including this example (for status, see www.globalcogo.com/SecEd.html).

III. Yes, we're using public information, except that I need to ask what is the origin of that 1900 distance of 8,251.7569 meters? I used the NGS data sheets for stations "Page Northeast Base" and "Page Southwest Base" – PID's NN0646 and NN0174.

IV. In the Surveyor Connect thread, Bill93 compliments my efforts (thank you) with the exception that it is too much work. I will have to disagree. I'm retired, geometry is my hobby, and my goal is to provide the tools needed perform and understand the computations. Yes, it is a lot of work, but it is satisfying to see the pieces coming together so nicely. Let's say, my goal is teaching to fish instead of handing out fish.

V. Upon sleeping on what I posted earlier and in response to John Hamilton's (and other) comments, I have refined a previous post. I appreciate all the helpful suggestions.

VI. For example, the question came up as to whether the elevation we use is surface mark elevation or underground mark elevation. In an effort to show it makes no difference, I assigned a standard deviation of 1.0 meters to the elevation used.

VII. Since the BURKORD™ software accommodates them; I arbitrarily assigned standard deviations of 0.001 meters to the north/south and east/west directions of the published values for the stations. Those standard deviations do not change the geometrical answers (distances etc.) but illustrate the value of using the stochastic portion of the global spatial data model (GSDM).

VIII. I've attached a pdf file to this posting which shows all the computations using a gratis version of the BURKORD™ software which handles both 3-D geometry and error propagation. BURKORD™ 9 handles 500 points, is DOS-based, and menu-driven. Yes, several years ago I worked with a computer expert to write a Windows version - WBK. Get a gratis 50 point version of WBK at http://www.globalcogo.com/WBK3D.html .

IX. The WBK program is an excellent prototype (admittedly, it still has some install bugs). But we are working to develop a commercial version of BURKORD™ that should be available by the time the Second Edition of the 3-D book is published. We'll see.

X. In working through this example, I've relied on several definitions of horizontal distance as described in item #4 at www.globalcogo.com/refbyefb.html. (Note ASCE holds the copyright of that article and does not permit me to post it. But, if you ask me via email (eburk@globalcogo.com), I am permitted to send you a copy.)

XI. Very briefly, HD(1) is the right-triangle component of a slope distance as used in plane surveying. The HD(3) horizontal distance lies between plumb lines and has the same elevation at both ends. As one would expect, that makes the definition of horizontal distance very elevation dependent. But, using ECEF coordinates, HD(3) is very easy to compute and, as a chord, it is the same as the mark-to-mark distance.

XII. The equations for computing geocentric X/Y/Z coordinates are readily available from various sources, but are restated here for the reader's convenience.

- Use GRS80 ellipsoid where a = 6,378,137.000 meters and e^2 = 0.006694380023.
- Use the latitude/longitude from the NGS data sheet. As a reminder, use longitude east or use longitude west as a negative value.
- Use ellipsoid height (h) obtained from orthometric height (H) and geoid height (N) as h = H + N. But, be careful the value N is negative in the continental USA.
- In equations below, ø = latitude and λ = longitude.

Equations:

$$N = \frac{a}{\sqrt{1-e^2\ sin^2\emptyset}} \quad (1)$$

$$X = (N + h)\ cos\varphi \cos\lambda \quad (2)$$

$$Y = (N + h)\ cos\varphi \sin\lambda \quad (3)$$

$$Z = [N(1 - e^2) + h\ ]sin\emptyset \quad (4)$$

XIII. The geocentric ECEF coordinates were computed for:

1. Point 101 – Page Southwest Base
2. Point 102 – Page Northeast Base
3. Point 103 – latitude/longitude for Page SW on the ellipsoid, i.e., h = 0.0 m.
4. Point 104 – latitude/longitude for Page NE on the ellipsoid, i.e., h = 0.0 m.
5. Point 105 – latitude/longitude for Page SW, but at ellipsoid height of Page NE.
6. Point 106 – latitude/longitude for Page NE, but at ellipsoid height of Page SW.

XIV. HD(3) is a chord distance and – for points at the same elevation – is computed as:

$$Dist = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2} \quad (5)$$

XV. In this case, the ellipsoid arc distance is longer than the chord by 0.0006 m.

XVI. The "horizontal distance" was computed at the ellipsoid height of each end-point using equation (5). The "mean horizontal distance" is the dotted line = 8,251.7619 m.

XVII. The reported distance measured in 1900 is 8,251.7569 m.

XVIII. The difference is 0.005 meters in a distance of 8,251.76 m or 1:1,650,000, **very close.**

XIX. As John Hamilton points out, the difference may be due to a number of reasons.

EFB 05/29/2015

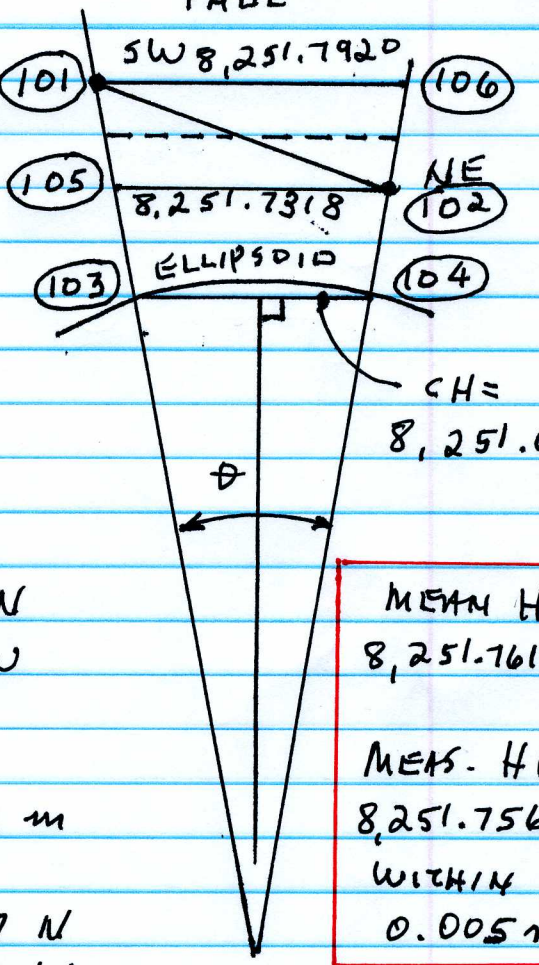
NEBRASKA BASELINE COMPARISON

"PAGE"

101 PAGE SW END
102 PAGE NE END

103 SW ON ELLIPSOID
104 NE ON ELLIPSOID

105 SW @ ELEV OF NE
106 NE @ ELEV OF SW



FROM NGS DATA SHEETS

SW ϕ = 42° 25' 25."39941 N
λ = 98° 26' 00.79833 W
H = 626.231 m
N = −24.71 m
h = H + N = 601.52 m

NE ϕ = 42° 28' 53.47527 N
λ = 98° 22' 13.99939 W
H = 580.1 m
N = −24.04 m
h = H + N = 555.06 m

MEAN HD = 8,251.7619 m

MEAS. HD = 8,251.7569 m WITHIN 0.005 m !

a = 6,378,137.000 m
e^2 = 0.00669438 0023

GEOMETRICAL MEAN RADIUS = $\dfrac{a\sqrt{1-e^2}}{1-e^2 \sin^2 \phi}$

ϕ_{mean} = 42° 27' 09.43734 R = 6,376,199.397 m

ARC of CHORD $\theta = 2 * \sin^{-1}\left(\dfrac{CH}{2R}\right)$ $L = R\theta$

ARC, $L = 6,376,199.397 * 2 * \left[\sin^{-1}\left(\dfrac{8,251.0132}{2 * 6,376,199.397}\right)\right]$

= 8,251.0138 m

'BURKORD(TM)' COMPUTES 3-D COORDINATE GEOMETRY POSITIONS FOR SPATIAL
DATA UTILIZING GPS VECTORS, LOCAL COORDINATE DIFFERENCES AND
3-D SURVEYING MEASUREMENTS.

USE OF BURKORD(TM) LICENSED TO:
Your Name Goes Here
Your company name goes here
Line 2 of address
Anytown, Anystate or Country/Zip

USER: Earl F. Burkholder
DATE: May 28, 2015

PROGRAM: BURKORD(TM) - VERSION 9A.04, JAN 2004 S/N 9AA03001
DATA FILE: NEBaseLn.dat
OUTPUT FILE: NEBaseLn.005

CLIENT/AGENCY: Jerry Penry and Surveyor Connect Readers
JOB/PROJECT: Computation of "Page" Baseline Distance in Nebraska

* *

AN EXPANDED LISTING OF POINTS 1 TO 106

Note – Points 1-6 have no standard deviations.
Points 101 to 106 are same but have assumed standard deviations: 0.001 m on N/E and 1.0 m on vertical.
Assuming standard deviations does not change geometrical computations – only the standard deviations.

1	Page SW Base			X	Y	Z	E	N	U		
LAT (N+S-)	42 25 25.399410	X:	-691633.4273	X	0.000E+00		E	0.000E+00			
LON (E+W-)	-98 26 0.798331	Y:	-4664826.6782	Y	0.000E+00	0.000E+00	N	0.000E+00	0.000E+00		
EL HGT	601.5200 M	Z:	4280869.6679	Z	0.000E+00	0.000E+00	0.000E+00	U	0.000E+00	0.000E+00	0.000E+00
2	Page NE Base			X	Y	Z	E	N	U		
LAT (N+S-)	42 28 53.475270	X:	-685867.8717	X	0.000E+00		E	0.000E+00			
LON (E+W-)	-98 22 13.999392	Y:	-4661262.5517	Y	0.000E+00	0.000E+00	N	0.000E+00	0.000E+00		
EL HGT	555.0600 M	Z:	4285575.9513	Z	0.000E+00	0.000E+00	0.000E+00	U	0.000E+00	0.000E+00	0.000E+00
3	SW on ellipsoid			X	Y	Z	E	N	U		
LAT (N+S-)	42 25 25.399411	X:	-691568.3051	X	0.000E+00		E	0.000E+00			
LON (E+W-)	-98 26 0.798329	Y:	-4664387.4520	Y	0.000E+00	0.000E+00	N	0.000E+00	0.000E+00		
EL HGT	0.0000 M	Z:	4280463.8777	Z	0.000E+00	0.000E+00	0.000E+00	U	0.000E+00	0.000E+00	0.000E+00
4	NE on ellipsoid			X	Y	Z	E	N	U		
LAT (N+S-)	42 28 53.475269	X:	-685808.2801	X	0.000E+00		E	0.000E+00			
LON (E+W-)	-98 22 13.999390	Y:	-4660857.5584	Y	0.000E+00	0.000E+00	N	0.000E+00	0.000E+00		
EL HGT	0.0000 M	Z:	4285201.0902	Z	0.000E+00	0.000E+00	0.000E+00	U	0.000E+00	0.000E+00	0.000E+00

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    5     SW at elev NE                          X        Y        Z           E        N        U
LAT (N+S-)   42 25 25.399410 X:  -691628.3974  X 0.000E+00                   E 0.000E+00
LON (E+W-)  -98 26  0.798331 Y: -4664792.7534  Y 0.000E+00 0.000E+00         N 0.000E+00 0.000E+00
EL HGT           555.0600 M Z:  4280838.3256  Z 0.000E+00 0.000E+00 0.000E+00  U 0.000E+00 0.000E+00 0.000E+00

    6     NE at elev SW                          X        Y        Z           E        N        U
LAT (N+S-)   42 28 53.475272 X:  -685872.8596  X 0.000E+00                   E 0.000E+00
LON (E+W-)  -98 22 13.999389 Y: -4661296.4507  Y 0.000E+00 0.000E+00         N 0.000E+00 0.000E+00
EL HGT           601.5200 M Z:  4285607.3282  Z 0.000E+00 0.000E+00 0.000E+00  U 0.000E+00 0.000E+00 0.000E+00

  101     Page SW Base                           X        Y        Z           E        N        U
LAT (N+S-)   42 25 25.399410 X:  -691633.4273  X 0.117E-01                   E 0.100E-05
LON (E+W-)  -98 26  0.798331 Y: -4664826.6782  Y 0.791E-01 0.533E+00         N-0.668E-11 0.100E-05
EL HGT           601.5200 M Z:  4280869.6679  Z-0.730E-01-0.493E+00 0.455E+00  U 0.361E-11-0.327E-10 0.100E+01

  102     Page NE Base                           X        Y        Z           E        N        U
LAT (N+S-)   42 28 53.475270 X:  -685867.8717  X 0.115E-01                   E 0.100E-05
LON (E+W-)  -98 22 13.999392 Y: -4661262.5517  Y 0.783E-01 0.532E+00         N 0.474E-11 0.100E-05
EL HGT           555.0600 M Z:  4285575.9513  Z-0.725E-01-0.493E+00 0.456E+00  U 0.727E-11 0.874E-11 0.100E+01

  103     SW on ellipsoid                        X        Y        Z           E        N        U
LAT (N+S-)   42 25 25.399411 X:  -691568.3051  X 0.988E-06                   E 0.100E-05
LON (E+W-)  -98 26  0.798329 Y: -4664387.4520  Y-0.791E-07 0.467E-06         N 0.000E+00 0.100E-05
EL HGT             0.0000 M Z:  4280463.8777  Z 0.730E-07 0.493E-06 0.545E-06  U 0.000E+00 0.000E+00 0.000E+00

  104     NE on ellipsoid                        X        Y        Z           E        N        U
LAT (N+S-)   42 28 53.475269 X:  -685808.2801  X 0.988E-06                   E 0.100E-05
LON (E+W-)  -98 22 13.999390 Y: -4660857.5584  Y-0.783E-07 0.468E-06         N 0.000E+00 0.100E-05
EL HGT             0.0000 M Z:  4285201.0902  Z 0.725E-07 0.493E-06 0.544E-06  U 0.000E+00 0.000E+00 0.000E+00

  105     SW at elev NE                          X        Y        Z           E        N        U
LAT (N+S-)   42 25 25.399410 X:  -691628.3974  X 0.117E-01                   E 0.100E-05
LON (E+W-)  -98 26  0.798331 Y: -4664792.7534  Y 0.791E-01 0.533E+00         N-0.668E-11 0.100E-05
EL HGT           555.0600 M Z:  4280838.3256  Z-0.730E-01-0.493E+00 0.455E+00  U 0.000E+00-0.310E-10 0.100E+01

  106     NE at elev SW                          X        Y        Z           E        N        U
LAT (N+S-)   42 28 53.475272 X:  -685872.8596  X 0.115E-01                   E 0.100E-05
LON (E+W-)  -98 22 13.999389 Y: -4661296.4507  Y 0.783E-01 0.532E+00         N 0.474E-11 0.100E-05
EL HGT           601.5200 M Z:  4285607.3282  Z-0.725E-01-0.493E+00 0.456E+00  U-0.348E-11 0.212E-11 0.100E+01
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 INVERSES BETWEEN POINTS

   101     Page SW Base
 X =   -691633.4273  LAT (N+S-)   42 25 25.399410    +/- 0.100E-02 METERS    N
 Y =  -4664826.6782  LON (E+W-)  -98 26  0.798331    +/- 0.100E-02 METERS    E   STANDARD DEVIATIONS
 Z =   4280869.6679  EL HGT           601.5200 M    +/- 0.100E+01 METERS    U

  DELTA X/Y/Z WITH SIGMAS      5765.5556M +/- 0.152E+00M     3564.1265M +/- 0.103E+01M     4706.2834M +/- 0.955E+00M
  DELTA E/N/U WITH SIGMAS      5180.4872M +/- 0.163E-02M     6422.8967M +/- 0.174E-02M      -51.8011M +/- 0.141E+01M
  LOCAL PLANE INV: DIST =      8251.7301M +/- 0.192E-02M  N AZI. =  38 53 18.43 +/-   0.0 SEC

   102     Page NE Base
 X =   -685867.8717  LAT (N+S-)   42 28 53.475270    +/- 0.100E-02 METERS    N
 Y =  -4661262.5517  LON (E+W-)  -98 22 13.999392    +/- 0.100E-02 METERS    E   STANDARD DEVIATIONS
 Z =   4285575.9513  EL HGT           555.0600 M    +/- 0.100E+01 METERS    U

  DELTA X/Y/Z WITH SIGMAS     -5765.5556M +/- 0.152E+00M    -3564.1265M +/- 0.103E+01M    -4706.2834M +/- 0.955E+00M
  DELTA E/N/U WITH SIGMAS     -5185.2904M +/- 0.163E-02M    -6419.0969M +/- 0.174E-02M       41.1189M +/- 0.141E+01M
  LOCAL PLANE INV: DIST =      8251.7902M +/- 0.192E-02M  N AZI. = 218 55 51.51 +/-   0.0 SEC

   101     Page SW Base
 X =   -691633.4273  LAT (N+S-)   42 25 25.399410    +/- 0.100E-02 METERS    N
 Y =  -4664826.6782  LON (E+W-)  -98 26  0.798331    +/- 0.100E-02 METERS    E   STANDARD DEVIATIONS
 Z =   4280869.6679  EL HGT           601.5200 M    +/- 0.100E+01 METERS    U


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   103     SW on ellipsoid
 X =   -691568.3051  LAT (N+S-)   42 25 25.399411    +/- 0.100E-02 METERS    N
 Y =  -4664387.4520  LON (E+W-)  -98 26  0.798329    +/- 0.100E-02 METERS    E   STANDARD DEVIATIONS
 Z =   4280463.8777  EL HGT             0.0000 M    +/- 0.522E-08 METERS    U

  DELTA X/Y/Z WITH SIGMAS      5760.0250M +/- 0.141E-02M     3529.8936M +/- 0.967E-03M     4737.2125M +/- 0.104E-02M
  DELTA E/N/U WITH SIGMAS      5180.0371M +/- 0.141E-02M     6422.3365M +/- 0.141E-02M       -5.3406M +/- 0.129E-05M
  LOCAL PLANE INV: DIST =      8251.0115M +/- 0.141E-02M  N AZI. =  38 53 18.46 +/-   0.0 SEC

   104     NE on ellipsoid
 X =   -685808.2801  LAT (N+S-)   42 28 53.475269    +/- 0.100E-02 METERS    N
 Y =  -4660857.5584  LON (E+W-)  -98 22 13.999390    +/- 0.100E-02 METERS    E   STANDARD DEVIATIONS
 Z =   4285201.0902  EL HGT             0.0000 M    +/- 0.575E-08 METERS    U

  DELTA X/Y/Z WITH SIGMAS     -5760.0250M +/- 0.141E-02M    -3529.8936M +/- 0.967E-03M    -4737.2125M +/- 0.104E-02M
  DELTA E/N/U WITH SIGMAS     -5184.8021M +/- 0.141E-02M    -6418.4903M +/- 0.141E-02M       -5.3406M +/- 0.129E-05M
  LOCAL PLANE INV: DIST =      8251.0115M +/- 0.141E-02M  N AZI. = 218 55 51.54 +/-   0.0 SEC
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  103     SW on ellipsoid
 X =   -691568.3051  LAT (N+S-)   42 25 25.399411    +/- 0.100E-02 METERS    N
 Y =  -4664387.4520  LON (E+W-)  -98 26  0.798329    +/- 0.100E-02 METERS    E   STANDARD DEVIATIONS
 Z =   4280463.8777  EL HGT             0.0000 M    +/- 0.522E-08 METERS    U

*****************************************

  105     SW at elev NE
 X =   -691628.3974  LAT (N+S-)   42 25 25.399410    +/- 0.100E-02 METERS    N
 Y =  -4664792.7534  LON (E+W-)  -98 26  0.798331    +/- 0.100E-02 METERS    E   STANDARD DEVIATIONS
 Z =   4280838.3256  EL HGT           555.0600 M    +/- 0.100E+01 METERS    U

 DELTA X/Y/Z WITH SIGMAS      5760.5257M +/- 0.152E+00M     3530.2017M +/- 0.103E+01M     4737.6257M +/- 0.955E+00M
 DELTA E/N/U WITH SIGMAS      5180.4872M +/- 0.163E-02M     6422.8967M +/- 0.174E-02M       -5.3411M +/- 0.141E+01M
 LOCAL PLANE INV: DIST =      8251.7301M +/- 0.192E-02M  N AZI. =  38 53 18.43 +/-   0.0 SEC

  102     Page NE Base
 X =   -685867.8717  LAT (N+S-)   42 28 53.475270    +/- 0.100E-02 METERS    N
 Y =  -4661262.5517  LON (E+W-)  -98 22 13.999392    +/- 0.100E-02 METERS    E   STANDARD DEVIATIONS
 Z =   4285575.9513  EL HGT           555.0600 M    +/- 0.100E+01 METERS    U

 DELTA X/Y/Z WITH SIGMAS     -5760.5257M +/- 0.152E+00M    -3530.2017M +/- 0.103E+01M    -4737.6257M +/- 0.955E+00M
 DELTA E/N/U WITH SIGMAS     -5185.2526M +/- 0.163E-02M    -6419.0501M +/- 0.174E-02M       -5.3411M +/- 0.141E+01M
 LOCAL PLANE INV: DIST =      8251.7301M +/- 0.192E-02M  N AZI. = 218 55 51.51 +/-   0.0 SEC

  105     SW at elev NE
 X =   -691628.3974  LAT (N+S-)   42 25 25.399410    +/- 0.100E-02 METERS    N
 Y =  -4664792.7534  LON (E+W-)  -98 26  0.798331    +/- 0.100E-02 METERS    E   STANDARD DEVIATIONS
 Z =   4280838.3256  EL HGT           555.0600 M    +/- 0.100E+01 METERS    U

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  106     NE at elev SW
 X =   -685872.8596  LAT (N+S-)   42 28 53.475272    +/- 0.100E-02 METERS    N
 Y =  -4661296.4507  LON (E+W-)  -98 22 13.999389    +/- 0.100E-02 METERS    E   STANDARD DEVIATIONS
 Z =   4285607.3282  EL HGT           601.5200 M    +/- 0.100E+01 METERS    U

 DELTA X/Y/Z WITH SIGMAS     -5760.5677M +/- 0.152E+00M    -3530.2275M +/- 0.103E+01M    -4737.6603M +/- 0.955E+00M
 DELTA E/N/U WITH SIGMAS     -5185.2904M +/- 0.163E-02M    -6419.0970M +/- 0.174E-02M       -5.3411M +/- 0.141E+01M
 LOCAL PLANE INV: DIST =      8251.7903M +/- 0.192E-02M  N AZI. = 218 55 51.51 +/-   0.0 SEC

  101     Page SW Base
 X =   -691633.4273  LAT (N+S-)   42 25 25.399410    +/- 0.100E-02 METERS    N
 Y =  -4664826.6782  LON (E+W-)  -98 26  0.798331    +/- 0.100E-02 METERS    E   STANDARD DEVIATIONS
 Z =   4280869.6679  EL HGT           601.5200 M    +/- 0.100E+01 METERS    U
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  DELTA X/Y/Z WITH SIGMAS      5760.5677M +/- 0.152E+00M     3530.2275M +/- 0.103E+01M     4737.6603M +/- 0.955E+00M
  DELTA E/N/U WITH SIGMAS      5180.5249M +/- 0.163E-02M     6422.9436M +/- 0.174E-02M       -5.3411M +/- 0.141E+01M
  LOCAL PLANE INV: DIST =      8251.7903M +/- 0.192E-02M  N AZI. =  38 53 18.42 +/-   0.0 SEC

   106     NE at elev SW
 X =   -685872.8596  LAT (N+S-)   42 28 53.475272    +/- 0.100E-02 METERS    N
 Y =  -4661296.4507  LON (E+W-)  -98 22 13.999389    +/- 0.100E-02 METERS    E   STANDARD DEVIATIONS
 Z =   4285607.3282  EL HGT           601.5200 M    +/- 0.100E+01 METERS    U

************************************  End of BURKORD™ computations.
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HD(3) and Mark-to-Mark Distances are identical when points are at same elevation.
HD(3) (Mark-to- Mark) is to be compared with the local tangent plane distances HD(1) given above.
HD(1) and HD(3) distances are defined and described in item #4 at www.globalcogo.com/refbyefb.html.

	HD(3)	HD(1)	Mark-to-Mark
101 to 102	not at	8,251.7301 m	8,251.8927 m
102 to 101	same elev.	8,251.7902 m	8,251.8927 m
103 to 104	8,251.0132 m	8,251.0115 m	8,251.0132 m
104 to 103	8,251.0132 m	8,251.0115 m	8,251.0132 m
105 to 102	8,251.7318 m	8,251.7301 m	8,251.7318 m
102 to 105	8,251.7318 m	8,251.7301 m	8,251.7318 m
101 to 106	8,251.7920 m	8,251.7903 m	8,251.7920 m
106 to 101	8,251.7920 m	8,251.7903 m	8,251.7920 m

Using standard arc-chord comparison, the ellipsoid arc distance is longer than the chord (103 to 104) by 0.0006 m. That means that the computed ellipsoid distance is 8,251.0138 meters which agrees exactly with distance as computed by others.

The mean horizontal distance of 105 to 102 and 101 to 106 is (8,251.7318 + 8,251.7920)/2 = 8,251.7619 meters. This also is the same value determined by others and agrees within 0.005 meters of the given 8,251.7569 meters. But, it is not to say that this is the elevation NGS used in various computations.

But, it does appear that many persons at NGS have done an excellent job of preserving geometrical integrity over the past 115 years in dealing with ellipsoids, datums, adjustments, and geoid models. Kudos to the folks at NGS!!

Lastly, the (strictly arbitrary) assumption of 0.001 meters uncertainty for horizontal components and 1.00 meters on vertical (except on the ellipsoid) provided a standard deviation of 0.0019 meters for the inversed distances for points not on the ellipsoid and 0.0014 meters for points on the ellipsoid. Conclusion – vertical is not "critical" on horizontal distance computation. But, that 0.005 meter difference in measured/computed came from somewhere. Ideas anyone?