Laying out a Parallel With the GSDM

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Example 12, Chapter 15, in the 2nd Edition of "The 3-D Global Spatial Data Model" describes how to lay out a parallel of latitude using the GSDM. The point being that the GSDM provides a tool to "go anywhere" traversing in 3-D.

Although the user is free to select any appropriate starting point, the Initial Point in New Mexico is used as a starting point in this example. The NGS data sheet link for "Initial Point" in New Mexico is . . .

https://www.ngs.noaa.gov/cgi-bin/ds_desig.prl

The INITIAL POINT defines the intersection of the New Mexico Principal Meridian and the Baseline

The presumption in this example is that a parallel of latitude is to be surveyed consistent with the instructions in the BLM Manual for laying out parallels as part of the USPLSS. That means the surveyor will traverse 80 chains per mile using horizontal distance at ground level, not on the ellipsoid and not using state plane coordinates.

A traverse in 3-D is stated simply as:

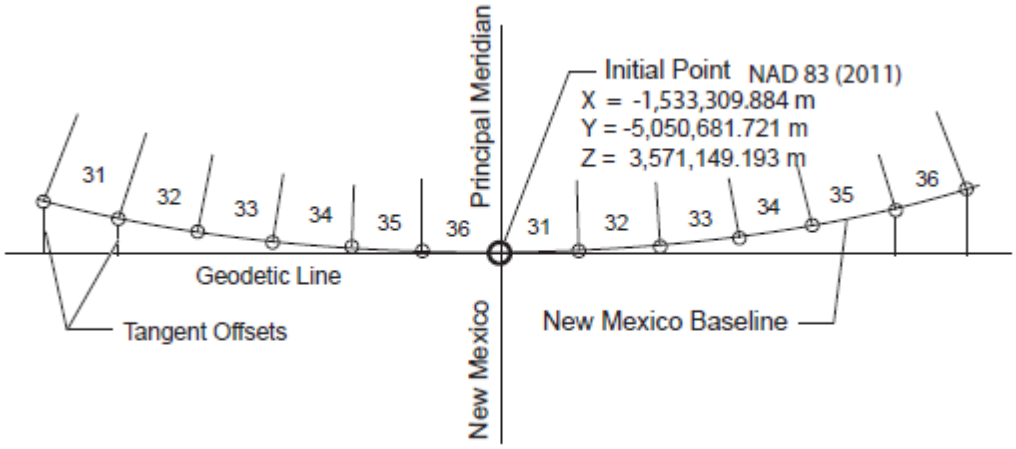
$$X_2 = X_1 + \Delta X$$
$$Y_2 = Y_1 + \Delta Y \quad (1)$$
$$Z_2 = Z_1 + \Delta Z$$

Where the geocentric components (ΔX/ΔY/ΔZ) are computed using one of the following standard GSDM methods. In the context of laying out a parallel, an ellipsoid height difference is taken to be equivalent to an elevation difference.

1. ΔX/ΔY/ΔZ are computed from horizontal distance, azimuth, and ellipsoid height difference.
2. ΔX/ΔY/ΔZ are computed from slope distance, azimuth, and zenith direction.
3. ΔX/ΔY/ΔZ are computed from Δφ/Δλ/Δ*h.*

For options 1 and 2, the local Δe/Δn/Δu components are converted to ΔX/ΔY/ΔZ using the rotation matrix at the standpoint. The Δφ/Δλ/Δ*h* components in option 3 are converted directly to ΔX/ΔY/ΔZ components for use in equation 1. With the geocentric X/Y/Z coordinates of all points, layout of a parallel uses the P.O.B. datum local differences with traditional radial stake-out procedures.

The accompanying diagram can be used to illustrate two BLM methods – the solar method and the tangent-offset method.



The diagram above presumes that the geodetic line is straight and that the parallel of latitude is curved. The tangent off-sets are the distances between the two lines.

Another (equally valid) option would be to show the parallel of latitude as being a straight line while the geodetic line curves to the south away from the parallel of latitude.

Solar method

Survey point locations can be precomputed at any preferred interval - 10, 20, 40, or 80 chains. This example uses 40 chains. But, since the GSDM uses meters exclusively, the traverse computations use a traverse distance of 40 chains = 804.674 m. Coordinates for the South 1/4 Corner of Section 31 are computed starting at the Initial Point and traversing due east a horizontal distance of 804.674 m. (If a slope distance is used, then a vertical angle or a zenith direction is also input.) The next step starts at the South 1/4 Corner of Section 31 and heads due east again to the SE Corner of Section 31. This “leap frog” process continues to the SE Corner of Section 36 (also the SE Corner of the Township) and is analogous to the solar method as prescribed by BLM.

Tangent method

The tangent method is the same as the solar method except that all points on the south line of the township are computed from the same Initial Point, i.e., increasing the horizontal distance by 804.674 m

each time. The computed points lie on the geodetic line which diverges from the true latitude of parallel by the off-sets associated with the BLM tangent off-set method. Tangent off-sets - found in the BLM Standard Field Tables - are tabulated according to latitude and distance every half-mile up to 6 miles.

Points on the true parallel

The method of computing and laying out points on the true parallel is "better" than either BLM method. In this case, the 3-D traverse computation uses $\Delta\varphi/\Delta\lambda Y/\Delta h$ to compute the X/Y/Z coordinates at each mile and half mile. The $\Delta\varphi$ and Δh values are both zero because the latitude does not change and because the computations are carried out at the same elevation (ellipsoid height) as the beginning point.

The question is finding the appropriate longitude difference at that latitude and elevation. Said longitude difference can be found using standard geodesy equations or. . . since the solar method and the tangent off-set methods are the same for the first half-mile, the geocentric X/Y/Z of the South 1/4 Corner are computed as given before. Then the latitude and longitude values of the South 1/4 Corner are computed from those X/Y/Z values. In that first half-mile, the latitude will change slightly but the longitude difference is very close to a longitude difference computed by theoretical methods. The longitude difference for that first half-mile section then used for each subsequent half-mile section of the township line.

Well and good, but no one "surveys" in terms of ECEF X/Y/Z coordinates. Instead the user chooses a P.O.B. (the Initial Point). Then local coordinate differences ($\Delta e/\Delta n/\Delta u$) from the P.O.B. to each point to be laid out are computed by applying the rotation matrix to the corresponding $\Delta X/\Delta Y/\Delta Z$ differences. Table 15.14 in the 2nd Edition (and repeated here) gives the local easting and northing differences from the P.O.B. to the SE Corner of each subsequent Section. Yes, the northing value is the tangent off-set as stipulated by the BLM. Incidentally, using those local eastings and northings, standard radial surveying techniques are an efficient way to lay out points on the parallel of latitude.

The following off-set values are taken from a BURKORD™ computer printout for the New Mexico Baseline (www.globalcogo.com/parallel-layout.pdf) and reflect the same values as shown in Table 15.14 of the 2nd Edition. Notes about the printout include:

1. An ellipsoid height of 1,450 meters was used for the entire computation.
2. The beginning NAD 83 (2011) latitude and longitude are as published by NGS for the Initial Point.
3. The BURKORD™ software is a DOS-based menu-driven prototype and available free from author.
4. The software will also accommodate input of standard deviations for the measurements.
5. The stochastic portion of the GSDM stores the covariance information computed for each point.
6. If standard deviations are input, the output will provide the standard deviations of the computed quantities. In this case, all input values are used as "exact."
7. No standard deviations were used in this example – all "sigma" values are zero.

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LISTING OF POINTS WITH RESPECT TO MASTER P.O.B:             11  Initial Point

  NUMBER     NORTH     SIGMA       EAST       SIGMA        UP       SIGMA     STATION
    31      0.1381  0.000E+00  1609.3479  0.000E+00  -0.2028 0.000E+00  SE Cor Sec 31
    32      0.5525  0.000E+00  3218.6957  0.000E+00  -0.8111 0.000E+00  SE Cor Sec 32
    33      1.2431  0.000E+00  4828.0431  0.000E+00  -1.8250 0.000E+00  SE Cor Sec 33
    34      2.2099  0.000E+00  6437.3902  0.000E+00  -3.2444 0.000E+00  SE Cor Sec 34
    35      3.4529  0.000E+00  8046.7366  0.000E+00  -5.0694 0.000E+00  SE Cor Sec 35
    36      4.9722  0.000E+00  9656.0823  0.000E+00  -7.2999 0.000E+00  SE Cor Sec 36
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