Geometrical Parameters of the Geodetic Reference System 1980

by Earl F. Burkholder

Abstract. The four defining parameters of the Geodetic Reference System 1980 were adopted by the XVII General Assembly of the International Union of Geodesy and Geophysics meeting in Canberra, Australia in December 1979. Since only one of the four defining parameters (a, the semimajor axis of the ellipsoid) is an element of the geometrical ellipsoid, a second geometrical parameter must be derived. The defining parameters adopted by the IUGG are listed and formulas are quoted for computing values of various geometrical elements to any accuracy desired. Finally, values of various geometrical elements of the Geodetic Reference System 1980 are listed to 16 significant figures.

Introduction

The parameters of the Geodetic Reference System of 1980 (GRS 1980) were adopted by the XVII General Assembly of the International Union of Geodesy and Geophysics meeting in Canberra, Australia in December 1979. The four defining parameters are elements of physical geodesy, one of them being "a," the semimajor axis of the ellipsoid. The second defining parameter normally required for geometrical geodesy, i.e., e^2 or 1/f, is not one of the four defining parameters, but must be computed from them. The purpose of this article is to list formulas for computing e^2 and various other geometrical elements of the reference ellipsoid and to give values for them to 16 significant figures.

Definitions and Formulas

The definitions and formulas in this section are as given by Moritz (1980). The defining parameters of the Geodetic Reference System 1980 (GRS 1980), which are held exact, are:

a	$= 6378137$ m	the equatorial radius of the earth.
GM	$= 3986005 \times 10^8 M^3/S^2$	the geocentric gravitational constant of the Earth, including the atmosphere.
J_2	$= 108263 \times 10^{-8}$	the dynamical form factor of the Earth, excluding permanent tidal deformation.
ω	$= 7292115 \times 10^{-11}$ rad/S	the angular velocity of the Earth.

The closed form computational formulas for computing the square of the eccentricity of the ellipsoid, e^2, are given as:

$$e^2 = 3J_2 + (4/15)(\omega^2 a^3/GM)(e^3/2_{q_0}) \quad \text{where} \tag{1}$$

$$2_{q_0} = (1 + 3/e'^2)\arctan(e') - 3/e', \quad \text{and} \tag{2}$$

$$e' = e/(1 - e^2)^{1/2} \tag{3}$$

Since equation (1) has "e" on both sides of the equals sign, it must be solved iteratively even though it is in closed form.

Constraints

Moritz (1980) also gives the value of e^2 (and other computed ellipsoid elements) to 12 significant figures which is accurate enough for most geometrical geodesy applications. However, if one is not satisfied (for whatever reason) with 12 significant figures and has access to a suitable computer, the ellipsoidal elements may be computed to any desired accuracy.

The limit to the number of significant figures attainable from equation (1) is deter-

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mined by the term 2_{q_0} which is defined in equation (2) and is the difference of two separate terms. Since the first five digits in each of the two terms in equation (2) are the same, the number of significant digits in the value 2_{q_0} is 4-5 less than the number of significant digits in the value of e′. Therefore, it takes a 16-17-digit computer to compute e^2 to 12 significant figures and the 20-digit computer used by this author will give a value of e^2 to 15-16 significant figures. Linear regression and numerical analysis of the residuals of the computed values were used to confirm the 16th significant figure which may vary slightly if e^2 is computed on a computer having greater significant figure capacity. The computational accuracy of all subsequent derived values is dependent on the accuracy of e^2, the significant digit capacity of the computer and on the number of terms included in any equation containing an infinite series.

Additional Formulas

The equations listed in this section are all in closed form except for the equation for the length of the meridian quadrant. The equation for the meridian quadrant is one given by Schmid (1971) and can be extended to any accuracy desired. The e^{14} term contributes less than 0.0000 0000 4 m to the meridian length and was omitted from the value given in the next section.

The equation for R_2 is a closed form equation derived from the formula for the surface area of the ellipsoid given by Jordan (1962).

$$b = a(1-e^2)^{1/2}, \quad \text{the semiminor axis.} \qquad (4)$$

$$c = a^2/b, \quad \text{the polar radius of curvature.} \qquad (5)$$

$$f = (a-b)/a \quad \text{the flattening.} \qquad (6)$$

$$E = (a^2-b^2)^{1/2}, \quad \text{the linear eccentricity.} \qquad (7)$$

$$Q = a(1-e^2)(\pi/2)(1 + (3/4)e^2 + (45/64)e^4 + (175/256)e^6 + (11025/16384)e^8 + (43659/65536)e^{10} + (693693/1048576)e^{12} + (2r+1)/2^{4r}\binom{2r}{r}^2 e^{2r} + \ldots), \qquad (8)$$

the length of the meridian from the equator to the pole.

$$R_1 = (a+a+b)/3, \quad \text{the arithmetic mean radius} \qquad (9)$$

$$R_2 = [a(1-e^2)^{1/2}/\sqrt{2}][1/(1-e)^2) + (1/2e)\ln\{(1+e)/(1-e)\}]^{1/2}, \qquad (10)$$

the radius of a sphere having the same surface area as the ellipsoid.

$$R_3 = \sqrt[3]{a^2b}, \qquad (11)$$

the radius of a sphere having the same volume as the reference ellipsoid.

Geometrical Geodesy Values, GRS 1980

a = 6,378,137 m (exact) } parameters

e^2 = 0.00669 43800 22903 416 } parameters (1)

e'^2 = 0.00673 94967 75481 622

e = 0.08181 91910 42831 85 (2)

e' = 0.08209 44381 51933 42 (3)

b = 6,356,752.31414 0347 m (4)

c = 6,399,593.62586 4032 m (5)

f = 0.00335 28106 81183 637 (6)

$1/f$ = 298.25722 21008 827

E = 521,854.00970 03544 m (7)

Q = 10,001,965.72922 984 m (8)

R_1 = 6,371,008.77138 0116 m (9)

R_2 = 6,371,007.18088 3514 m (10)

R_3 = 6,371,000.78997 4137 m (11)

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