

STANDARD DEVIATION - POSITIONAL TOLERANCE

What is the definition of "small" or "large" when dealing with random errors? The relative magnitude of a random error is defined mathematically by its standard deviation (which is also the square root of the variance). In words, the standard deviation of a data set is the square root of the sum of the squares of the difference between each observation and the mean of a set of observations divided by n-1. The standard deviation of the **mean** is the data set standard deviation divided by the square root of n, the number of data set elements.

Standard deviation is not a very precise quantity (seldom more than 2 significant digits) and can be determined by estimation or observations and calculation. Professional judgement is the basis of estimation and repeated measurements is the basis of calculation.

- **Estimation:** Standard deviation is estimated on the basis of experience and/or the least count of equipment being used. Would it be acceptable to say the standard deviation of careful plumb bob taping is about 0.01 feet in 100 feet or about 0.03 feet in 1000 feet? Tests (repeated measurements and calculations) could be conducted to verify the estimate.
- **Calculation:** A quantity such as a distance is measured a number of times using the same equipment under the same conditions. Each result is recorded and the standard deviation is computed using an equation as shown in the example below.

$$\sigma_i = \sqrt{\frac{\sum_{i=1}^n (\text{mean} - x_i)^2}{n - 1}}$$

σ_i = Greek letter (Sigma) for standard deviation.
 mean = Average of data set.
 x_i = Single measurement.
 n = Number of observations.

No.	Observation	Mean	Differ	Diff Squared
1.	999.98	999.994	-0.014	0.000196
2.	1,000.02	999.994	0.026	0.000676
3.	999.95	999.994	-0.044	0.001936
4.	1,000.03	999.994	0.036	0.001296
5.	999.99	999.994	-0.004	0.000016
Total	4,999.97		Sum	= 0.004120

Results: $\sigma_i = 0.032 \text{ ft.}$ $\sigma_{\text{mean}} = \sigma_i / \sqrt{n} = 0.014 \text{ ft.}$