STOCHASTIC MODEL
Application to the Global Spatial Data Model (GDSM)

The functional component of the GSDM is defined by the solid geometry equations which support the definition of each spatial position by its geocentric earth-centered earth-fixed rectangular X/Y/Z coordinates.

The stochastic component of the Global Spatial Data Model represents an application of the laws of variance/covariance propagation as described in Chapter 4 of Mikhail (1976) and make use of the following matrix formulation as applied to equations of the functional model:

$$\Sigma_{YY} = J_{YX}\,\Sigma_{XX}\,J^t_{YX} \qquad \text{where:} \qquad (1)$$

Σ_{YY} = Covariance matrix of computed result.
Σ_{XX} = Covariance matrix of variables used in computation.
J_{YX} = Jacobian matrix of partial derivatives of the result with respect to the variables.

In particular, the following symbols are used in the stochastic model:

$\sigma_X^2\ \sigma_Y^2\ \sigma_Z^2$	= Variances of geocentric coordinates for a point.
$\sigma_{XY}\ \sigma_{XZ}\ \sigma_{YZ}$	= Covariances of geocentric coordinates for a point.
$\sigma_e^2\ \sigma_n^2\ \sigma_u^2$	= Variances of a point in the local reference frame.
$\sigma_{en}\ \sigma_{eu}\ \sigma_{nu}$	= Covariances of a point in the local reference frame.
$\sigma_{\Delta X}^2\ \sigma_{\Delta Y}^2\ \sigma_{\Delta Z}^2$	= Variances of geocentric coordinate differences.
$\sigma_{\Delta X\Delta Y}\ \sigma_{\Delta X\Delta Z}\ \sigma_{\Delta Y\Delta Z}$	= Covariances of geocentric coordinate differences.
$\sigma_{\Delta e}^2\ \sigma_{\Delta n}^2\ \sigma_{\Delta u}^2$	= Variances of coordinate differences in local frame.
$\sigma_{\Delta e\Delta n}\ \sigma_{\Delta e\Delta u}\ \sigma_{\Delta n\Delta u}$	= Covariances of coordinate differences in local frame.
$\sigma_S^2\ \sigma_\alpha^2$	= Variances of local horizontal distance and azimuth.
$\sigma_{S\alpha}$	= Covariance of local horizontal distance with azimuth.
σ_z^2	= Variance of zenith direction.

The stochastic information for each point is stored as its geocentric covariance matrix.

A. The covariance matrix is symmetric 3 X 3. Six numbers are required to store upper (or lower) triangular values.

B. Units in the covariance matrix is meters squared.

C. Standard deviation is square root of diagonal elements.

$$\begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} \\ \sigma_{XY} & \sigma_Y^2 & \sigma_{YZ} \\ \sigma_{XZ} & \sigma_{YZ} & \sigma_Z^2 \end{bmatrix}$$