Standard Deviations of a 3-D Inverse Using the GSDM

Each point in a BURKORD™ data base is defined by its geocentric X/Y/Z coordinates and the stochastic information for each point (if it exists) is stored as its geocentric covariance matrix. Correlation between points may exist and may also be stored.

datum accuracy

$$\begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} \\ \sigma_{XY} & \sigma_Y^2 & \sigma_{YZ} \\ \sigma_{XZ} & \sigma_{YZ} & \sigma_Z^2 \end{bmatrix}$$

- The geocentric covariance matrix is symmetric 3 X 3. Six numbers are required to store upper or lower triangular values.
- Units in the covariance matrix is meters squared.
- Standard deviation is square root of diagonal elements.

Matrix formulation of the 3-D geocentric coordinate inverse and covariance propagation are:

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}}_{J_1} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ X_2 \\ Y_2 \\ Z_2 \end{bmatrix} ; \qquad \Sigma_\Delta = J_1 \, \Sigma_{1-2} \, J_1^T \qquad (1) \, \& \, (2)$$

The Jacobian matrix and the general matrix variance/covariance propagation as noted above are used to find the overall geocentric inverse covariance matrix as:

$$\Sigma_\Delta = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \sigma_{X_1}^2 & \sigma_{X_1Y_1} & \sigma_{X_1Z_1} \\ \sigma_{X_1Y_1} & \sigma_{Y_1}^2 & \sigma_{Y_1Z_1} \\ \sigma_{X_1Z_1} & \sigma_{Y_1Z_1} & \sigma_{Z_1}^2 \end{bmatrix} & \begin{bmatrix} \sigma_{X_1X_2} & \sigma_{X_1Y_2} & \sigma_{X_1Z_2} \\ \sigma_{Y_1X_2} & \sigma_{Y_1Y_2} & \sigma_{Y_1Z_2} \\ \sigma_{Z_1X_2} & \sigma_{Z_1Y_2} & \sigma_{Z_1Z_2} \end{bmatrix} \\ \begin{bmatrix} \sigma_{X_1X_2} & \sigma_{Y_1X_2} & \sigma_{Z_1X_2} \\ \sigma_{X_1Y_2} & \sigma_{Y_1Y_2} & \sigma_{Z_1Y_2} \\ \sigma_{X_1Z_2} & \sigma_{Y_1Z_2} & \sigma_{Z_1Z_2} \end{bmatrix} & \begin{bmatrix} \sigma_{X_2}^2 & \sigma_{X_2Y_2} & \sigma_{X_2Z_2} \\ \sigma_{Y_2X_2} & \sigma_{Y_2}^2 & \sigma_{Y_2Z_2} \\ \sigma_{Z_2X_2} & \sigma_{Z_2Y_2} & \sigma_{Z_2}^2 \end{bmatrix} \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

The matrix operation above can be used to compute:

1. **Local accuracy** if the full covariance matrix is employed. **(see Burkholder 1999)**
2. **Network accuracy** if the correlation between points 1 and 2 is taken to be zero.
3. **P.O.B. accuracy** if only the covariance matrix of point 2 is used.

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