Standard Deviations of a 3-D Inverse Using the GSDM

The matrix formulation for computing local coordinate differences from geocentric coordinate differences is:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta u \end{bmatrix} = \underbrace{\begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix}}_{J_2} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \tag{4}$$

$\phi =$ Geodetic latitude
$\lambda =$ Geodetic longitude
(at point 1)

The Jacobian matrix noted above is used with the general error propagation formulation to get the covariance matrix of local coordinate differences as:

$$\Sigma_{3D\text{-}INV} = \begin{bmatrix} \sigma^2_{\Delta e} & \sigma_{\Delta e \Delta n} & \sigma_{\Delta e \Delta u} \\ \sigma_{\Delta e \Delta n} & \sigma^2_{\Delta n} & \sigma_{\Delta n \Delta u} \\ \sigma_{\Delta e \Delta u} & \sigma_{\Delta n \Delta u} & \sigma^2_{\Delta u} \end{bmatrix} = J_2 \begin{bmatrix} \sigma^2_{\Delta X} & \sigma_{\Delta X \Delta Y} & \sigma_{\Delta X \Delta Z} \\ \sigma_{\Delta X \Delta Y} & \sigma^2_{\Delta Y} & \sigma_{\Delta Y \Delta Z} \\ \sigma_{\Delta X \Delta Z} & \sigma_{\Delta Y \Delta Z} & \sigma^2_{\Delta Z} \end{bmatrix} J_2^t \tag{5}$$

The functional model equations for a 2-D tangent plane inverse distance and azimuth are:

$$S = \sqrt{\Delta e^2 + \Delta n^2} \tag{6}$$

$$\alpha = \tan^{-1}\left(\frac{\Delta e}{\Delta n}\right) \tag{7}$$

And, the Jacobian matrix of those partial derivatives is:

$$J_3 = \begin{bmatrix} \dfrac{\partial S}{\partial \Delta e} & \dfrac{\partial S}{\partial \Delta n} & \dfrac{\partial S}{\partial \Delta u} \\ \dfrac{\partial \alpha}{\partial \Delta e} & \dfrac{\partial \alpha}{\partial \Delta n} & \dfrac{\partial \alpha}{\partial \Delta u} \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta e}{S} & \dfrac{\Delta n}{S} & 0 \\ \dfrac{\Delta n}{S^2} & \dfrac{\Delta e}{S^2} & 0 \end{bmatrix} \tag{8}$$

Finally, using the covariance propagation formulation, the 2-D results are:

$$\Sigma_{2D\text{-}INV} = \begin{bmatrix} \sigma^2_S & \sigma_{S\alpha} \\ \sigma_{S\alpha} & \sigma^2_\alpha \end{bmatrix} = J_3 \begin{bmatrix} \sigma^2_{\Delta e} & \sigma_{\Delta e \Delta n} & \sigma_{\Delta e \Delta u} \\ \sigma_{\Delta e \Delta n} & \sigma^2_{\Delta n} & \sigma_{\Delta n \Delta u} \\ \sigma_{\Delta e \Delta u} & \sigma_{\Delta n \Delta u} & \sigma^2_{\Delta u} \end{bmatrix} J_3^t \tag{9}$$

Take square roots to get standard deviations and convert radians to seconds using 206,264.8062471 seconds per radian.

