Using the GSDM to Compute Spherical Excess

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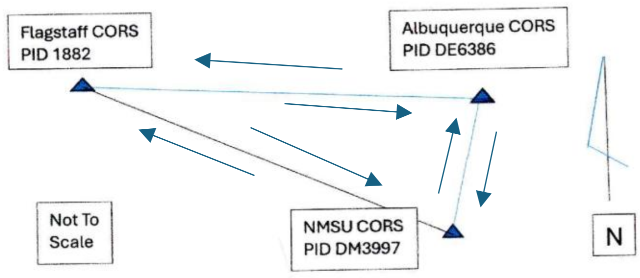

Figure 1 Schematic of CORS relative locations

NMSU CORS, PID = DM3997 (2011)

X =	-1,555,457.539 m	Latitude =	32° 16' 27."35268 N
Y =	-5,169,962.414 m	Longitude =	253° 15' 19."18972 E
Z =	3,386,819.082 m	Ellipsoid Height =	1,188.529 m

Albuquerque CORS, PID = DE6386 (2011)

X =	-1,488,635.936 m	Latitude =	32° 10' 24."85449 N
Y =	-5,003,947.553 m	Longitude =	253° 25' 57."58655 E
Z =	3,654,557.572 m	Ellipsoid Height =	1,620.630 m

Flagstaff CORS, PID = DL1882 (2011)

X =	-1,926,744.784 m	Latitude =	35° 10' 38."30162 N
Y =	-4,852,286.564 m	Longitude =	248° 20' 34."70915 E
Z =	3,655,166.113 m	Ellipsoid Height =	2,088.804 m

Equations for computing local components – trig functions used at standpoint

$$\begin{bmatrix}\Delta e\\ \Delta n\\ \Delta u\end{bmatrix}=\begin{bmatrix}-\sin\lambda & \cos\lambda & 0\\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi\\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi\end{bmatrix}\begin{bmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{bmatrix}=\begin{bmatrix}X_2-X_1\\ Y_2-Y_1\\ Z_2-Z_1\end{bmatrix}$$

Local tangent plane inverse equations

$HD(1) = \sqrt{(\Delta e^2 + \Delta n^2)}$ - not needed   Azimuth (adjusted for quadrant) = $\tan^{-1}(\Delta e/\Delta n)$

Inverse: NMSU to Albuquerque:

ΔX = 66,821.603 m   ΔY = 166,014.861 m   ΔZ = 267,738.490 m   HD(1) = 321,947.606 m
Δe = 16,158.171 m   Δn = 321,541.871 m   Δu = -7,726.479 m   Azi = 2° 52' 36."54

Inverse: Albuquerque to NMSU:

ΔX = -66,821.603 m   ΔY = -166,014.861 m   ΔZ = -267,738.490 m   HD(1) = 321,925.754 m
Δe = -16,709.713 m   Δn = -321,491.799 m   Δu = -8,588.847 m   Azi = 182° 58' 31."09

Inverse: Albuquerque to Flagstaff
ΔX = -438,108.848 m ΔY = 151,660.989 m ΔZ = 608.541 m HD(1) = 463328.376 m
Δe = -463,165.802 m Δn = 12,272.915 m Δu = -16,358.622 m Azi = 271° 31’ 04.”30

Inverse: Flagstaff to Albuquerque
ΔX = 438,108.848 m ΔY = -151660.989 m ΔZ = -608.541 m HD(1) = 463,294.416 m
Δe = 463,153.037 m Δn = -11,444.622 m Δu = -17,293.733 m Azi = 88° 35’ 04.”18

Inverse: Flagstaff to NMSU
ΔX = 371,287.245 m ΔY = -317,675.850 m ΔZ = -268,347.031 m HD(1) = 556,905.663 m
Δe = 462,316.139 m Δn = -310,495.901 m Δu = -25,266.101 m Azi = 123° 53’ 08.”46

Inverse: NMSU to Flagstaff
ΔX = -371,287.245 m ΔY = 317,675.850 m ΔZ = 268,347.031 m HD(1) = 556,984.129 m
Δe = -447,068.679 m Δn = 332,206.134 m Δu = -23,472.816 m Azi = 306° 36’ 54.”59

Interior angle at each station = difference in azimuth from given station to other stations
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Interior angle at NMSU CORS = 360° + 2° 52’ 36.”54 - 306° 36’ 54.”59 = 56° 15’ 41.”95

Interior angle at Albuquerque CORS = 271° 31’ 04.”30 - 182° 58’ 31.”09 = 88° 32’ 33.”21

Interior angle at Flagstaff CORS = + 123° 53’ 08.”46 - 88° 35’ 04.”18 = 35° 18’ 04.”28

Total interior angles	180° 06’ 19.”44
Plane triangle	180° 00’ 00.”00
Spherical excess =	**000° 06’ 19.”44**