Compute the 3-D Position of the Finial on Skeen Hall at NMSU

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This project was part of a SUR 410 Spring 2010 class project conducted by Kyle Spolar, Ford Prather, and Earl F. Burkholder. The control points for the project include points from the NMSU Supplemental GPS Network described in Example 1, Chapter 15 of the 2nd Edition "The 3-D Global Spatial Data Model." As shown in the accompanying diagram, horizontal and vertical angles to the top of the finial were observed from REILLY (BS-BROMILOW), EFB (BS-WAKEMAN), and WAKEMAN (BS-EFB). The class assignment was to compute the 3-D position of the top of the finial on Skeen Hall using geodesy equations, state plane coordinate equations, and specifically the GSDM for the computations.

The GSDM least squares solution is linear (requires no iteration) and was developed using the following matrix equations. Equation 1 computes the X/Y/Z coordinates for the finial and equation 2 gives the $\Delta X/\Delta Y/\Delta Z$ components from each of three control points to the top of the finial. The zenith directions and heights of instrument were measured directly in each case while the azimuth was computed from the backsight directions and measured horizontal angles.

$ \begin{bmatrix} X_f \\ Y_f \\ Z_f \end{bmatrix} = \begin{bmatrix} X_{control \ pt} \\ Y_{control \ pt} \\ Z_{control \ pt} \end{bmatrix} + \begin{bmatrix} \Delta \\ \Delta \\ \Delta \end{bmatrix} $	X _{control pt to finial} Y _{control pt to finial} Z _{control pt to finial}		(1)
$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} -\sin\lambda & -\sin\phi \\ \cos\lambda & -\sin\phi \\ 0 & \cos\phi \end{bmatrix}$	$ \begin{bmatrix} \cos\lambda & \cos\phi & \cos\lambda \\ \sin\lambda & \cos\phi & \sin\lambda \\ \phi & & \sin\phi \end{bmatrix} \begin{bmatrix} S \sin\alpha & \sinZ \\ S \cos\alpha & \sinZ \\ HI + S & \cosZ \end{bmatrix} $	where	(2)
$\Delta X, \Delta Y, and \Delta Z$ ø and λ	 geocentric components of each ve geodetic latitude and longitude of (East longitude is used at each ins the slope distance instrument state 	ector. instrument s trument stat	station. ion.)
α Z HI	 = geodetic azimuth instrument stati = Zenith direction instrument statio = Height of instrument at each statio 	on to station n to station F on.	FINIAL.

Using equation 2 three times, once from each control station, provides a set of 9 equations containing only 6 unknowns – three slope distances and the X/Y/Z coordinates of the finial. A matrix formulation of the solution can be accomplished using either method, adjustment of observations only (Av = f) or adjustment of indirect observations (v + $B\Delta$ = f). Admittedly, the evaluation of partial derivates, in either case, can be a challenge. As a learning exercise and as a confidence builder, it was gratifying to see that pencil/paper efforts gave the same results as "canned math software" for the partial derivatives.

Notes:

- 1. The method of adjustment-of-indirect-observations ($v + B\Delta$) was used with equal weights.
- 2. The zenith directions were corrected for refraction. The impact was insignificant.
- 3. Additional details are given in Example 3, Chapter 15, 2nd Edition of the 3-D book.

Program name/version: LSAdjGPSvectors: Version 2014-A

The user is: Program used on: The input file for program is: The name of this output file is: Output file annotated/renamed: The project is:	Earl F. Burkholder September 18, 2016 LSFinial.dat LSFinial.txt Figure 15.6.docx Location of Finial at NMSU
The name/dimensions of the F ve	ector are:
f 91	
-1,556,178.0081	
-5,169,236.6562	
3,387,552.6253	
-1,556,515.8421	
-5,169,057.5006	
3,387,657.6292	
-1,556,490.3463	
-5,169,360.0343	
3,387,212.7359	
The name/dimensions of the B m 1 0 0 0.97164224963 0 0 0 1 0 -0.21897071682 0 0 0 0 1 -0.08923656150 0 0 1 0 0 0 -0.58087030934 0 0 1 0 0 0.67997353083 0 0 1 0 0 0.44746584350 0 1 0 0 0 0 -0.20393014633 0 1 0 0 0 -0.42705545241 0 0 1 0 0 *0.88092913221	atrix are: B 96

The name/dimensions of the Q matrix are: Q 99 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1

W is the inverse of the Q matrix.

EQUAL WEIGHTS

Ninv

6.66275057E-1 -1.86767426E-1 -1.20049225E-1 -6.98990373E-1 5.67734232E-1 -4.96413369E-2 -1.86767426E-1 5.36798033E-1 2.01882825E-1 3.17029501E-1 -5.63831775E-1 3.68999481E-1 -1.20049225E-1 2.01882825E-1 5.76138539E-1 2.12263948E-1 -4.64810325E-1 5.69270729E-1 -6.98990373E-1 3.17029501E-1 2.12263948E-1 1.76753046E0 -7.1657529E-1 1.79833463E-1 5.67734232E-1 -5.63831775E-1 -4.64810325E-1 -7.1657529E-1 1.92115739E0 -5.34474265E-1 -4.96413369E-2 3.68999481E-1 5.69270729E-1 1.79833463E-1 -5.34474265E-1 1.64894704E0

Delta - the answers.

-1,556,406.814 m -5,169,185.096 m 3,387,573.640 m	X Y FINIAL Z
235.4829 m 187.6750 m 409.6740 m	Slope distance REILLY to FINIAL Slope distance WAKEMAN to FINIAL Slope distance EFB to FINIAL
Residuals.	
0.0007 m 0.0036 m -0.0009 m	Vector from REILLY
-0.0133 m -0.0187 m 0.0112 m	Vector from WAKEMAN
0.0126 m 0.0151 m -0.0102 m	Vector from EFB

Redundancy, R = 3

```
SigmaNaughtSquared = 3.85583294E-4
```

The covariance matrix of the computed parameters is

```
      2.56904531E-4
      -7.20143992E-5
      -4.62889755E-5
      -2.6951901E-4
      2.18908835E-4
      -1.91408702E-5

      -7.20143992E-5
      2.06980354E-4
      7.78426447E-5
      1.22241279E-4
      -2.17404113E-4
      1.42280035E-4

      -4.62889755E-5
      7.78426447E-5
      2.22149396E-4
      8.18454322E-5
      -1.79223096E-4
      2.19501283E-4

      -2.6951901E-4
      1.22241279E-4
      8.18454322E-5
      <u>6.81530217E-4</u>
      -2.7629946E-4
      6.93407792E-5

      2.18908835E-4
      -2.17404113E-4
      -1.79223096E-4
      -2.7629946E-4
      <u>7.40766193E-4</u>
      -2.06084348E-4

      -1.91408702E-5
      1.42280035E-4
      2.19501283E-4
      6.93407792E-5
      -2.06084348E-4
      <u>6.35806433E-4</u>
```

Standard deviations computed from the diagonal elements of the covariance matrix above are:

Coordinates of FINIAL	Slope Distance to FINIAL from		
Sigma X = 0.016 m	REILLY	Sigma	= 0.026 m
Sigma Y = 0.014 m	WAKEMAN	Sigma	= 0.027 m
Sigma Z = 0.015 m	EFB	Sigma	= 0.025 m



Diagram of Control Points Used to Survey the Skeen Hall Finial



Skeen Hall on Campus of New Mexico State University, Las Cruces, New Mexico