Using GPS Results in True 3-D Coordinate System

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"Using GPS Results in a Coordinate System Designed for Transportation & Engineering Projects" is the title of the paper as presented at the ASCE GPS '91 Specialty Conference, "Transportation Applications of GPS Positioning Strategy," Sept. 18-21, 1991, Sacramento, CA. The presentation version follows . . .

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Significant developments since the paper was first published include:

The geospatial community is currently (2022) enthralled by the concept of "digital twins." As a consequence of the digital revolution, the impact of digital twins is felt in various disciplines and is especially significant for the surveying/engineering community. The worldwide economic impact of geospatial digital twins is anticipated to exceed 1 trillion U.S. dollars by the year 2030.

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As the saying goes, "the devil is in the detail." The 3-D global spatial data model (GSDM) was formally defined in 1997 and is an outgrowth of the 1991 paper. In particular, Figure 6 of the ASCE paper (Figure 3 in the formal definition) shows the existence of pseudo 3-D and true 3-D as a consequence of reconciling a map projection distance with horizontal ground distance. Two important points here are:

1. As documented in Appendix III of the 1991 paper, 46 out of 50 state DOTs weighed in on the question of using GPS technology and resolving the grid/ground distance dilemma.
2. The digital twin concept can accommodate either pseudo 3-D or true 3-D. Traditional practice includes use of separate horizontal and vertical datums (pseudo 3-D) while emerging practice (e.g., driverless anything) enjoys the integrity provided by a single 3-D datum with the origin at Earth's center of mass (true 3-D). See . . .
   http://www.globalcogo.com/GSDM-and-DT.pdf.

Find additional information posted at http://www.globalcogo.com

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Using GPS Results in a Coordinate System Designed for Transportation & Engineering Projects[1]

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Abstract

GPS computations are performed in the geocentric coordinate system and transformed to geographic (latitude/longitude) or state plane coordinates for extended use in data base applications or engineering projects. State plane coordinates are very useful but, for some applications, the difference between grid and ground distance becomes particularly bothersome. State Department of Transportation (DOT) offices are confronted with the grid/ground distance dilemma on highway projects where centerline stationing must be ground based and state plane coordinates are used for project control. A questionnaire was sent to all 50 state DOT's asking for input as to how the problem is currently being addressed and asking for suggestions for solutions. Responses received from 46 of the 50 state DOTs are summarized in Appendix D. The proposal included in this paper is to use the geocentric coordinate system as the basis for all three dimensional control computations, including conventional terrestrial measurements. To eliminate the distance distortion problem, results can be outputted in a local coordinate system. Equations for transforming between geographic, geocentric and local coordinate systems are included in Appendix B.

Introduction

Until recently, practically all surveying computations have been performed using two dimensional (2-D) coordinates to express horizontal position and elevation for the third dimension. Due to the curved reference surface for elevation, horizontal and vertical coordinates can not be combined into a 3 dimensional (3-D) rectangular Cartesian coordinate system without sacrificing geometrical integrity over long distances. The earth-centered earth-fixed (ECEF) geocentric coordinate system used for global positioning system (GPS) surveying is a true 3-D rectangular Cartesian coordinate system which permits use of standard solid geometry computational rules throughout.

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[1] Presented at the ASCE GPS '91 Specialty Conference, "Transportation Applications of GPS Positioning Strategy," September 18-21, 1991, Sacramento, CA.

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Historically, curvilinear 2-D geographic (latitude/longitude) coordinates have been transformed to 2-D plane coordinates via a map projection and elevation has been used for a third dimension. This system should appropriately be called "pseudo 3-D" for two reasons: 1) the plane containing the 2-D coordinates is not the reference surface for the third dimension and 2) the reference surface for the third dimension is not a plane but a curved surface, variously taken to be any level surface, the geoid, or the ellipsoid.

GPS measurements are made in the ECEF system and results are typically quoted in terms of delta-X, delta-Y, and delta-Z between points. Normal practice is to use these delta values to compute geocentric coordinates which, in turn, are used to compute geographic, then state plane or local coordinates. 2-D results can be very impressive and quite useful but, without careful attention and accurate knowledge of the local geoid, true 3-D integrity is compromised.

Problems in using state plane coordinates are described by Burkholder (1991a) for those cases where it is desirable to minimize differences between grid distance on the projection and horizontal ground distance as measured in the field. Additional modeling considerations for accurate horizontal distance computations are described by Burkholder (1991b). These problems (and others) can be accommodated with the 3-D Geodetic Model proposed by Leick (1990). This paper describes use of the 3-D model.

The North American Datum of 1983 (NAD 83) is an ECEF datum but the final position of each control station was computed in a 2-D environment (Vincenty and Schwartz, 1989, page 89). However, GPS points within the high-precision state networks being established by the National Geodetic Survey (NGS) are being computed and adjusted in the true 3-D environment (Spofford, 1991).

DOT Questionnaire

Research for this paper included sending a copy of the paper, "Design of a Local Coordinate System for Surveying, Engineering, and LIS/GIS," (Burkholder 1991a) to each of the 50 state Department of Transportation (DOT) offices along with a questionnaire asking how the grid/ground distance difference is handled when using state plane coordinates. The questionnaire was designed to accommodate a "check-off" response but also provided an opportunity for comment and/or extended discussion. Appendix D contains a list of the questions sent and shows a numeric tabulation of the "check-off" responses. Appendix D also contains a listing of the comments (by section) received back from each state DOT.

The initial response from 28 DOTs was gratifying and indicated a high level of interest. A follow-up mailing secured responses from an additional 18 states for a total of 46 out of 50 state DOTs. Overall impressions from the DOT responses are:

1. Many states are using GPS for establishing geodetic control points for use by the DOT and even more use state plane coordinates for various applications.

2. Twelve state DOTs have not used GPS while only four responding claim no use of state plane coordinates.

3. Thirteen of 46 states responding indicated the grid/ground distance difference is not a problem.

4. Twelve states indicated the grid/ground distance difference is handled by using datum coordinates obtained by using a datum adjustment factor. Reference to the comment section in Appendix D shows significant variance in the way a datum adjustment factor is applied.

5. Suggestions for solving the grid/ground distance difference problem were quite varied. Representative responses were:

   a. Use project datum coordinates.
   b. Use average elevation and scale factors.
   c. Use computers to compute and apply corrections.
   d. Avoid problem by restricting activity to small area.

6. Suggestions for points to be made in this paper show a lot of insight. A summary of many excellent suggestions is:

   a. Computer resources make it possible for end users to make routine conversions.
   b. Current practice exceeds design intent of original state plane systems. A universally accepted system is needed to avoid on-going datum conversion costs.
   c. Stay with existing state plane system. Other local systems cause problems and increase costs.
   d. A datum adjustment factor is the best way to handle grid/ground distance differences.
   e. More education and training is needed in using the state plane coordinate systems.
   f. Use NAD 83 routinely and use GPS to build a "good" network.

Geocentric Coordinates:

As shown in Figure 1, the geocentric coordinate system is a right-handed three dimensional rectangular Cartesian system with three axes (X, Y, & Z) and an origin at earth's center of mass. The X/Y plane coincides with earth's equator with the X axis at 0° longitude. The Z axis is parallel to earth's spin axis corrected for polar wandering to the Conventional Terrestrial Pole. Meters is the unit of distance along each axis and can be either positive or negative.

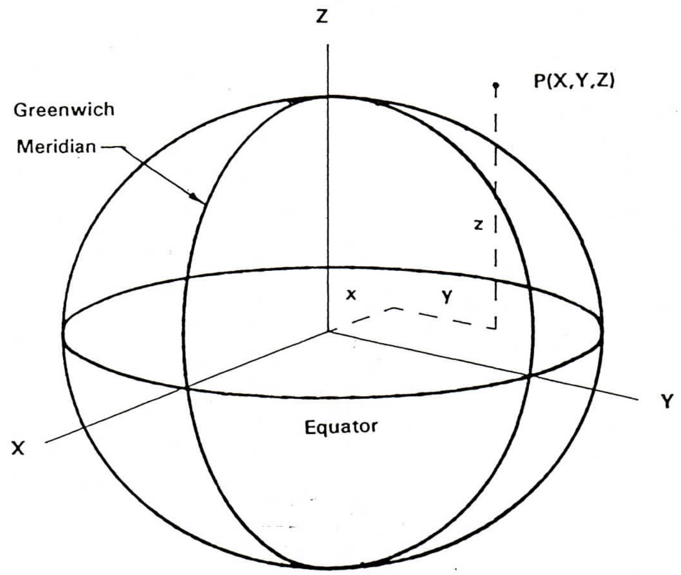

Figure 1. Geocentric Coordinate System

The distance between any two points within 50,000 km of the origin can be expressed within 0.1 mm with a 12 digit number (GPS satellite orbits are typically 27,000 km from earth's center). The position of any point within a 100,000 km radius can be uniquely described within 0.1 mm using a triplet of 12 digit numbers (X,Y,Z coordinates).

Geographic Coordinates:

Geographic coordinates fall into two categories; astronomical latitude and longitude which are referenced to the local plumb line (vertical) and geodetic latitude and longitude which are referenced to the local normal. Except for deflection-of-the-vertical and polar wandering, astronomical and geodetic are used interchangeably. In practice geodetic latitude/longitude are used in the computational model and astronomical latitude/longitude are associated with field surveying observations. When the difference is significant or affects subsequent computations, astronomical latitude/longitude are converted to geodetic values by computing and applying appropriate corrections.

Geographic coordinates are given in angular units. Typical units are degrees, minutes, and seconds, but most computers work with radians as the angular unit. Some users prefer to work with decimal degrees while others choose grads or mils. Conversions are common. Twelve digits of degrees, minutes, and seconds for an angle over 100 degrees gives 5 decimal places of seconds which represents 0.3 mm of arc on earth's surface and 1.3 mm of arc at GPS satellite height.

As illustrated in Figure 2, latitude is the angular distance either north (+) or south (-) of the equator which is in turn defined as 90° from the north pole. Longitude is defined as the angular distance eastward from an arbitrarily accepted reference meridian, the Greenwich Meridian being the world standard. In the western hemisphere it is common practice to use a west longitude but mathematical transformations utilizing longtitude are commonly formulated using east longitude. Mathematically, a negative west longitude is equivalent to an east longitude.

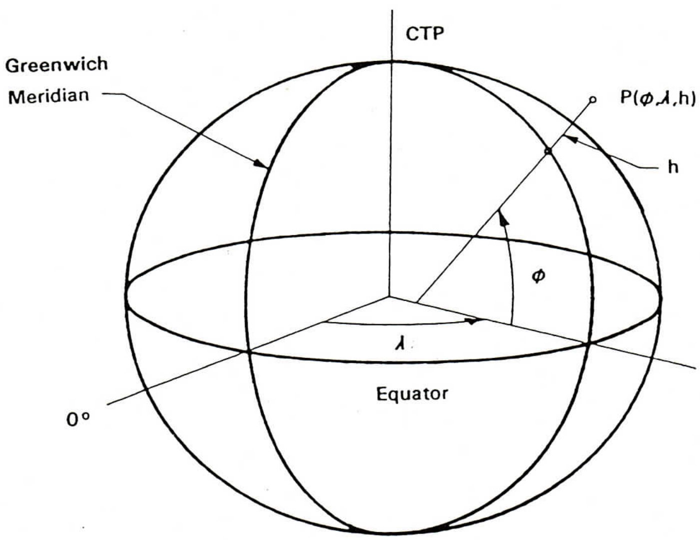

Figure 2, Geographical Coordinate System

Geographical coordinates are sometimes referred to as 3-D coordinates because they are global and describe any point on the earth's surface. Technically though, geographical coordinates are only 2-D because they are restricted to the earth's surface and another value (elevation or height) is required to describe how far a point is above or below the surface. Using a vertical distance along with latitude and longitude, it is possible to describe a 3-D position with geographical coordinates.

But, describing the true 3-D position of a point with geogrpahic coordinates is made difficult by ambiguity of the reference surface. The reference ellipsoid enjoys specific mathematical definition and can be used for vertical reference. However, it is a curved surface and elliposidal height loses geometrical meaning if used (as many are tempted to do) with state plane or project datum coordinates.

The natural vertical reference surface is a level surface which is at all points perpendicular to the local plumb line. It is used extensively to compare relative elevations and to determine which way water will run. As long as horizontal (2-D) and vertical (1-D) are not combined, the integrity of each can be preserved. But, true 3-D geometrical integrity is lost if horizontal geographic coordinates are combined with elevation unless the distance above mean sea level and the geoid height are both accurately known. Then 3-D integrity is assured by using ellipsoid height, not elevation.

State Plane (Map Projection) Coordinates:
Because angular units of geographical coordinates are cumbersome to work with, map projections have been devised to permit 2-D latitude/longitude coordinates to be expressed equivalently with 2-D plane coordinates having length units. Without going into a treatise on map projections, a conformal projection is used because it preserves horizontal angular relationships between the curved surface and the plane projection surface (be it plane, cone, or cylinder). However, horizontal distance is distorted in converting a distance measured at some elevation on the earth to the map projection surface (referred to as the grid).

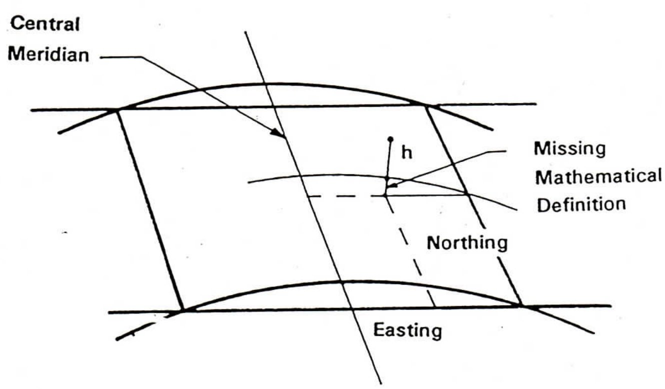

Figure 3, Map Projection Coordinates and Missing Link With Vertical

The disadvantage of working with angular units is avoided by using state plane coordinates but the implied accuracy of 3-D coordinates is missing because, as shown in Figure 3, there is no precise mathematical relationship vertically between the plane of the N/E coordinates (grid) and the ellipsoid. Unsuspecting users are victums of a imperfect model if accurate 3-D computations are attempted using height or elevation with state plane coordinates. Admittedly, for most purposes the error is inconsequential if applied only over a limited area. But the fact remains, the model is an approximation and can not be extended without consequence.

Local Geodetic Horizon Coordinates (Trimble, 1990):
The local geodetic horizon (LGH) is a plane, perpendicular to the local ellipsoid normal, passing through the station mark. Figure 4 shows that coordinates in the LGH relative to the station are regular plane surveying latitude and departure (delta north and delta east) referenced to the geodetic meridian. The third component is parallel to the ellipsoid normal through the station. It is called "up" in this paper because it is the perpendicular distance between the forepoint and the plane of the LGH. It is not the same as $h_2 - h_1$ because of ellipsoid curvature.

A word of caution: The LGH plane from point A to point B will be slightly different from the LGH plane from point B to point A, again because of ellipsoid curvature. The two planes will intersect midway between the two points.

Possible Solution

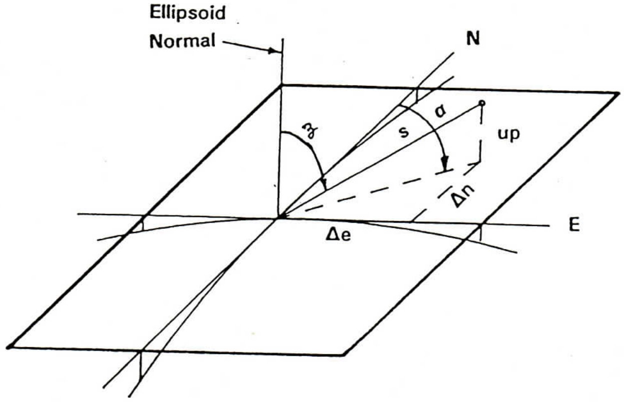

Figure 4, Local Geodetic Horizon (LGH) Coordinates

Greater efficiency in surveying computations can be achieved by using the geocentric coordinate system as the basis for all control computations. GPS results are already expressed in, and conventional terrestrial (total station mark-to-mark) observations are easily converted to, geocentric coordinate differences. The control network is then computed and adjusted using a linear model in a true 3-D environment. Geometrical integrity is preserved because observations are not altered by grid scale factors, elevation factors, and second term corrections.

Those who wish to continue using state plane coordinates may do so even if they use the 3-D Geodetic Model. Without loss of integrity, 3-D geocentric coordinates can be converted to geographic coordinates and height. The latitude and longitude can then be converted to 2-D state plane (or any other map projection) coordinates. In fact, the results of a 3-D network adjustment can be routinely converted and written to two files; a 3-D file and a state plane coordinate file. The 3-D Geodetic Model provides a better, easier way to perform surveying computations without taking away benefits of existing methods or procedures.

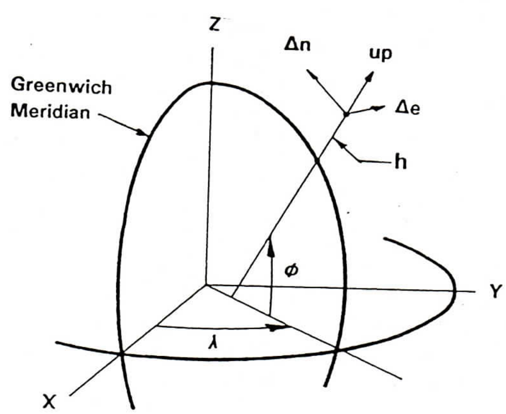

Figure 5, ECEF and LGH Coordinate Systems

Today's surveying and mapping professional has an impressive array of tools available to enhance productivity. To use these tools competently, the profesisonal needs to develop a greater awareness of what canned software is doing to the measurements and what assumptions are implicit in using it. Mathematics of the 3-D Geodetic Model are more straight forward and easier to understand than, say, the geodesic or conformal mapping. But, using the 3-D model does require practice to visualize rectangular coordinates in the ECEF system.

Figure 5 shows two rectangular coordinate systems. One is the geocentric ECEF system perpendicular to and parallel with earth's spin axis. The other is a LGH system, a local right-handed system, east, north and up (left-handed is north, east and up). Coordinate differences in one system are rotated to coordinate differences in the other with two successive rotations. The two rotations are combined into one rotation matrix (Leick, 1980, eq. 7.10) which conveniently transforms coordinate differences from one system to the other. Because the rotation matrix is orthogonal (Vanicek & Krakiwsky, 1986, p 38) the inverse transformation matrix is the transpose of the fundamental rotation matrix.

Figure 6 shows a schematic with geocentric coordinates at the top, true 3-D coordinate differences on the right, and pseudo 3-D state plane (map projection) coordinates on the left. Conventional terrestrial observations are shown on the lower-right side as feeding into either model. Algorithms for transforming between blocks are given in Appendix B and keyed to the circled numbers.

1. Geographic coordinates (plus height) to geocentric coordinates, both forward and inverse.
2. Geocentric coordinates to geocentric coordinate differences, both forward and inverse.
3. Geocentric coordinate differrences to local geodetic coordinate differences, both forward and inverse.
4. Local geodetic coordinate differences from corrected terrestrial observations of directions and distance.

Implementation

The following steps are listed as a suggestion for implementing the 3-D Geodetic Model:

1. Input existing geodetic control points into the data base. Ideally, these points will be the high-precision state network points. In any case, they must be on the same datum and have true 3-D values, either (X, Y, Z) or latitude, longitude and height. If geographic coordintes are entered, they are converted to geocentric coordinates (X,Y,Z) before being stored in the data base. The standard deviation of each coordinate component should also be entered.
2. Measurements are used to define new points added to the data base. Measurements and standard deviations can come from:
   a. GPS observations which give ΔX, ΔY, & ΔZ directly, or
   b. terrestrial total station observations of Δe, Δn, & up converted to ΔX, ΔY, and ΔZ.
3. Network adjustments are performed in the geocentric coordinate system using coordinate differences and a linear adjustment model. Standard deviations of the original control and each added component provide an efficient mechanism for in-

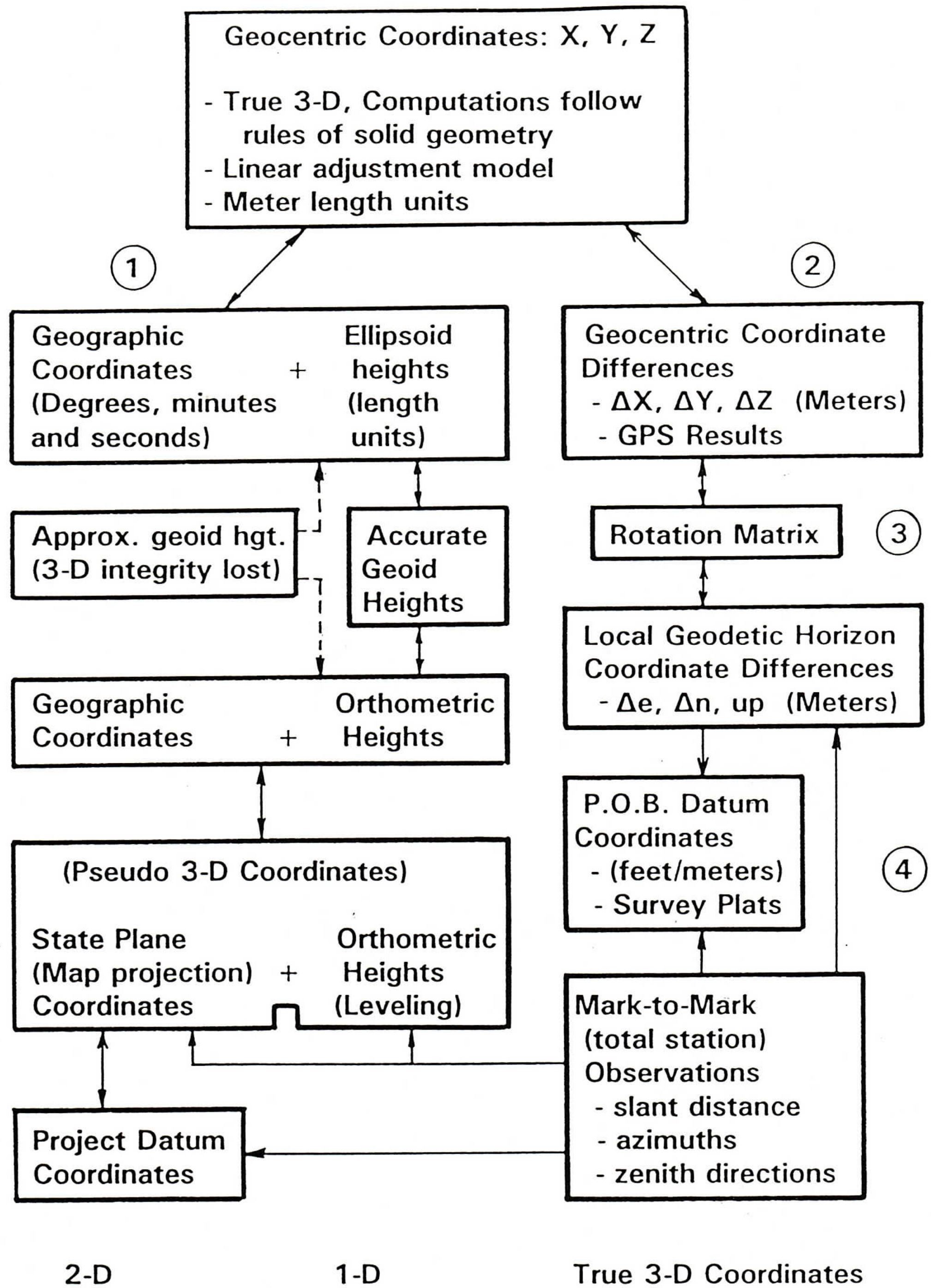

Figure 6, Schematic Showing Coordinate Systems and Transformations suring (and documenting) the quality of each point added. This system will be successful to the extent the integrity of the data base is established and preserved (Burkholder, 1990). Is this a function of each state DOT office or should responsibility be assigned to a State Survey Office?

4. After network adjustments are completed, final audited values are added to the data base and released for user consumption. Although the data base contains only X/Y/Z coordinates, the user can ask for values in any of the derivative forms, latitude/longitude/height, state plane coordinates, or local datum coordinates (referred to in Figure 6 as P.O.B. Datum Coordinates). Query routines can instantaneously convert geocentric coordinates to whatever form requested. Many users should have "read rights" to the data base, but only data passing rigorous quality control measures are added.

Implementation Issues:

Discussions among surveying and mapping professionals are required to resolved certain issues with respect to implementing the 3-D Geodetic Model. While the 3-D Geodetic Model is very efficient with respect to spatial relationships, it does not address the level surface. Many surveying activities are related to grades, elevations, and leveling. Another part of the same issue is lack of accurate information on the local geoid. As more 3-D GPS points are established in common with leveled bench marks a better picture of the geoid is emerging. What will be the best way to integrate accurate geoid heights with the 3-D Geodetic Model? Is it possible that survey control will be integrated into one data base but that "horizontal" and "vertical" surveys will continue to be used indefinitely. A query to the data base should give coordinates (geocentric, geographic, state plane), ellipsoid height, geoid height, deflection-of-the vertical components and standard deviations for each. Other relational attributes, such as a "to reach" description and/or use history, should also be part of the data base.

Another issue requiring clarification is the P.O.B. Datum Coordinates shown in Figure 6. The 3-D Geodetic Model can be interrogated for a single point position in geocentric, geographic, or map projection coordinates. It can also be querried for relative position of TWO points in the LGH coordinate system. But what about listing coordinates for numerous points in a given project area? Should an answer be "sequence dependent" such that listing inverse courses for a large tract (a township for example) will give one answer for a clockwise traverse and slightly different one for a counter-clockwise traverse. Certainly that is undesirable. Another part of the same question is, what kind of misclosure one would expect from a loop traverse with many short courses and one long closing course. Remember, each point-pair inverse lies in a slightly different LGH plane than adjacent courses.

A suggestion is a practice termed "Point of Beginning" Datum Coordinates shown in Figure 6. One point is taken to be the master point (P.O.B.) for a given project and all other points in the project are brought out of the data base with respect to the

P.O.B. The LGH coordinate differences in effect serve as plane coordinates for the area. The P.O.B. must be listed (with its 3-D values) so the P.O.B. Datum Coordinates can be related to the larger world of survey control. Additional research is being conducted and input is solicited.

Another issue deserving more attention is that of datum shifts. The absolute geocentric coordinate values in the data base can be refined, readjusted, or up-dated as necessary, but it will make little or no difference to the LGH coordinate differences in an area unless actual ground shifts have occurred in the area. Given the internal consistency of the high-precision state networks, there should be little if any distortion remaining in the network at the local level. Global datum improvements in the future can be implemented with little local ramifications and the relative position of the boundary monument on the ground is preserved (earthquakes and actual movement excepted).

And, finally concensus is needed on questions such as:

1. When area is computed, what is the implied plane? Is state plane grid area really acceptable? If not, at what elevation should area be computed?
2. What 3-D coordinates are appropriate for projects covering large areas (highway projects) or precise layout such as bridge abutments or industrial alignment?
3. What standards and specifications should be adopted for:
   a. Control surveys added to the National Geodetic Reference System. At what point is (or should) responsibility for the state-wide network be shifted to a state agency?
   b. Surveys conducted to control information slated for inclusion in a GIS or LIS?

Conclusions

In conclusion, the benefits and advantages of using the 3-D Geodetic Model are:

1. The 3-D Geodetic Model can be implemented without disrupting current practice. The grid/ground distance difference dilemma can be resolved in a cost effective manner.
2. The 3-D environment enjoys proper geometrical integrity on a global scale and provides a true unique 3-D location for each point defined. No approximations are inherent in the formulation of the mathematical model.
3. Computational integrity is enhanced by working with coordinate differences.
4. The mathematical formulation is actually less complicated than working on the ellipsoid or with map projections. However, overall implementation and administration of the 3-D Geodetic Model requires oversight of a knowledgeable professional.

5. GPS observations and conventional terrestrial observations can be incorporated (with respective standard deviations) into the same integrated adjustment.
6. Future datum shifts and improvements can be accomplished with little or no local impact.

Acknowledgements

I am endebted to colleagues at the Oregon Institute of Technology for the opportunity to take a sabbatical leave during the 1990-91 academic year for study at the University of Maine. At Maine I interacted with faculty and students alike, but especially acknowledge the mettle developed as a result of discussions with Drs. Ray Hintz and Alfred Leick. And, finally, I thank Mr. T. Vincenty for providing an English translation of his article written in German.

APPENDIX A. NOTATION.

a = Semimajor axis of reference ellipsoid.
b = Semiminor axis of reference ellipsoid.
f = Ellipsoid flattening.
e^2 = Eccentricity squared of reference ellipsoid.
X =
Y = } { Earth-centered earth-fixed geocentric rectangular Cartesian coordinates.
Z =
ϕ = Geodetic latitude, north + and south -.
λ = Geodetic longitude, east 0° to 360°.
h = Ellipsoid height.
H = Orthometric height.
N = Ellipsoid normal. } { Unfortunate symbol duplication,
N = Geoid height. differentiate using context.
r = Distance from origin to point (X, Y, Z).
P = Component of r in equatorial plane.
h' = First approximation of ellipsoid height.
a' = Semimajor axis of auxiliary ellipsoid.
b' = Semiminor axis of auxiliary ellipsoid.
T = Intermediate computational value.
U = Intermediate computational value.
Δe = Change in easting with respect to local geodetic horizon (LGH).
Δn = Change in northing with respect to LGH.
up = Change in height with respect to LGH.
ΔX = $X_2 - X_1$, Geocentric coordinate difference.
ΔY = $Y_2 - Y_1$, Geocentric coordinate difference.
ΔZ = $Z_2 - Z_1$, Geocentric coordinate difference.
s = Slant distance (to preserve geometrical integrity, slope distance is reduced to mark-to-mark slant distance).
α = Azimuth from north corrected for polar motion and deflection-of-the-vertical (if applicable).
z = Zenith angle (90° - vertical) corrected for refraction and deflection-of-the-vertical (if applicable).

APPENDIX B. TRANSFORMATIONS

3-D Transformation Equations

Referenced to (1) in Figure 6:
Geographic Coordinates and Height to Geocentric Coordinates:

$$X = (N + h) \cos\phi \cos\lambda \tag{1}$$

$$Y = (N + h) \cos\phi \sin\lambda \tag{2}$$

$$Z = [N(1 - e^2) + h] \sin\phi \tag{3}$$

Geocentric Coordinates to Geographic Coordinates and Height (Vincenty, 1980):

$$b = a(1 - f) \tag{4}$$

$$P^2 = X^2 + Y^2, \qquad r^2 = P^2 + Z^2 \qquad (5) \;\&\; (6)$$

$$h' = r - a + \frac{(a - b)Z^2}{r^2} \tag{7}$$

$$a' = a + h', \qquad b' = b + h' \qquad (8) \;\&\; (9)$$

$$\tan\phi' = \left(\frac{a'}{b'}\right)^2 \left(\frac{Z}{P}\right) \left[1 + \frac{1}{4}\frac{e^4 h' a (Z^2 - P^2)}{a^4}\right] \tag{10}$$

$$\cos^2\phi' = \frac{1}{1 + \tan^2\phi'}, \qquad \sin\phi' = \cos\phi' \tan\phi' \qquad (11) \;\&\; (12)$$

$$T = \frac{(P - h'\cos\phi')^2}{a^2}, \qquad U = \frac{(Z - h'\sin\phi')^2}{b^2} \qquad (13) \;\&\; (14)$$

$$h = h' + \frac{1}{2}\left[\frac{T + U - 1}{\dfrac{T}{a} + \dfrac{U}{b}}\right] \qquad (15)$$

$$\phi = \tan^{-1}\left[\left(\frac{a}{b}\right)^2 \frac{(Z - e^2 h \sin\phi')}{P}\right] \qquad (16)$$

$$\lambda = \tan^{-1}\left(\frac{Y}{X}\right) \quad \text{Normalized } 0° - 360° \qquad (17)$$

Another method would be to iterate for an "exact" solution as described by Leick (1990) using equations 6.31 to 6.36.

Referenced to (2) in Figure 6:

Geocentric coordinate computation:

$$X_2 = X_1 + \Delta X \qquad (18)$$

$$Y_2 = Y_1 + \Delta Y \qquad (19)$$

$$Z_2 = Z_1 + \Delta Z \qquad (20)$$

Geocentric coordinate differences from geocentric coordinates:

$$\Delta X = X_2 - X_1 \qquad (21)$$

$$\Delta Y = Y_2 - Y_1 \qquad (22)$$

$$\Delta Z = Z_2 - Z_1 \qquad (23)$$

Referenced to (3) in Figure 6:

Geocentric Coordinate Differences to Local Geodetic Coordinate Differences (Leick, 1990, Equations 7.9 and 7.10):

$$\begin{bmatrix} \Delta e \\ \Delta n \\ up \end{bmatrix} = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \tag{24}$$

$$\Delta e = -\Delta X \sin\lambda + \Delta Y \cos\lambda \tag{25}$$

$$\Delta n = -\Delta X \sin\phi\cos\lambda - \Delta Y \sin\phi\sin\lambda + \Delta Z \cos\phi \tag{26}$$

$$up = \Delta X \cos\phi\cos\lambda + \Delta Y \cos\phi\sin\lambda + \Delta Z \sin\phi \tag{27}$$

Local Geodetic Coordinate Differences to Geocentric Coordinate Differences (Vanicek and Krakiwsky, 1982, page 38):

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta n \\ up \end{bmatrix} \tag{28}$$

$$\Delta X = -\Delta e \sin\lambda - \Delta n \sin\phi\cos\lambda + up \cos\phi\cos\lambda \tag{29}$$

$$\Delta Y = \Delta e \cos\lambda - \Delta n \sin\phi\sin\lambda + up \cos\phi\sin\lambda \tag{30}$$

$$\Delta Z = \Delta n \cos\phi + up \sin\phi \tag{31}$$

Reference to (4) in Figure 6:

Local Geodetic Coordinate Differences Computed From Terrestial Observations (Corrected for Polar Motion and Local Deflection of the Vertical),(Leick, 1990, Equations 7.1, 7.2, and 7.3):

$$\Delta e = s \sin z \sin \alpha \qquad (32)$$

$$\Delta n = s \sin z \cos \alpha \qquad (33)$$

$$up = s \cos z \qquad (34)$$

Corrections to Terrestial Observations:

s: The EDM slope distance should be corrected for:

1. Geometrical configuration of set-up to accommodate reflector off-set and electrical center of EDMI, see section 4.34, (Davis et al, 1981).
2. Second term velocity and curvature of path, see page 24, (Fronczek, 1980).
3. Delay of signal for atmospheric conditions of temperature and pressure, see pages 1-8, (Fronczek, 1980).
4. Mark-to-mark slant distance, i.e. equation (6) (Burkholder, 1991b).

α: An observed astronomic azimuth should be corrected, using the first two terms of equation 3.85 by Leick (1980), for:

1. Polar motion and
2. Local deflection of the vertical.

z: The observed zenith (vertical) angle should be corrected for:

1. Refraction of line-of-sight in vertical plane, see equation 13b, (Burkholder, 1991b).
2. Local deflection of the vertical, see equation 3.86, (Leick, 1980).

2-D Transformation Equations

State plane coordinates and Universal Transverse Mercator (UTM) coordinates are derived from geographical coordinates via a conformal mapping of the ellipsoid to a developable surface, i.e. a plane, cone or cylinder. The equations and transformations are well known and widely used so there is no need to list them here other than for reference. The following publications are recommended for use of the Lambert conic conformal, transverse Mercator (including UTM), and oblique Mercator projections. Stem (1989) is the primary reference for use on the NAD 1983 and Claire (1968) contains information on how state plane coordinates were computed on the NAD 1927. Burkholder (1985) contains a listing of a FORTRAN 77 program along with flowchart and algorithm for performing conformal mapping transformations using any ellipsoid (or datum) for any of the following projections; Lambert, transverse Mercator, UTM, and oblique Mercator.

APPENDIX C. NOTES ON CONVENTIONS.

1. Leick (1990) uses X, Y, & Z for a coordinate system fixed in space. X & Y are in the equatorial plane with X directed toward the First Point of Aries. Z points toward the instantaneous north pole. Consistent with other GPS literature, X, Y, & Z in this article describes earth-fixed geocentric coordinates. This note of caution is offered to help prevent confusion from nomenclature. See also Solar & Hothem (1988).

2. The local geodetic coordinate system described by Leick, (1990) is a left-handed system (north is listed before east). A right-handed system is preserved by listing the coordinates as, east, north, and up.

3. The "h" component in the local geodetic coordinate system is the perpendicular distance from the forepoint to the tangent plane through the standpoint. It is not the same as h_2 - h_1. Therefore the symbol "up" was chosen for the local vertical component.

4. Vincenty (1980) uses an auxiliary ellipsoid obtained by adding approximate ellipsoid height to the reference ellipsoid semimajor and semiminor axis to avoid iteration on latitude and height. Results using Vincenty's method were compared to answers obtained by iterating and the maximum discrepancy encountered was less than 0.15 mm, even at satellite heights.

APPENDIX D.

Questionnaire on Coordinate Systems Used for Highway (& Related) Projects

This questionnaire was sent to all 50 state DOT's and to the FHWA. Questionnaires were returned by 44 DOT's accompanied by letter responses from numerous DOT's and the FHWA. The tabulation below shows the number of responses in two categories, "has" and "has not." The totals do not match because many spaces were left blank.

Our state DOT has _____ (has not) _____ used Global Positioning System (GPS) equipment and/or techniques for:

has	has not	
27	12	Establishing geodetic control points for use by DOT.
24	13	Determining coordinates of photo control points for mapping.
4	13	Resource (or feature) inventory location.
0	13	Navigation purposes for Department vehicles.

Whether GPS has been used or not, our DOT does ___ (does not) ___ use state plane coordinates for:

does	does not	
38	4	Survey control for DOT projects.
32	5	Defining location of highway (project) centerlines/control.
24	5	Surveying the location of DOT Right-of-Ways.
20	5	Centerline stationing of big, medium and/or small projects.

How is the problem of grid/ground distance differences handled by your organization?

13	The grid/ground difference is not a problem.
1	State plane coordinates are not used for survey control.
12	Datum coordinates are determined by multiplying state plane coordinates by an elevation factor for the project. See note 1 below.
12	State plane coordinates are used for survey control, but not for project centerline stationing.
27	Other: SEE COMMENTS LISTED ON SEPARATE SHEET.

What suggestions do you have for solving the problem of distance distortion inherent in the use of state plane coordinates?

SEE COMMENTS LISTED ON SEPARATE SHEET.

What suggestions do you have for points to be made in the paper I have proposed to present at the GPS '91 Conference?

SEE COMMENTS LISTED ON SEPARATE SHEET.

Note 1: Several respondents noted a combined factor for elevation and scale both is used. Users are reminded the scale factor changes (perpendicular to axis) with location just as the elevation factor changes with elevation. Use of a single factor throughout a project will not necessarily preserve geometrical integrity of the coordinates.

Summary of Responses to DOT Questionnaire

Comments answering the question, "How is the problem of grid/ground distance differences handled by your organization?"

1. We have recently started determining grid/ground differences in the field on each project by use of EDM distance measured between GPS points set in the field.
2. We use a system of "project datum" coordinates obtained by expanding SPC by dividing them by a mean grid scale and elevation factor. Large X & Y reductions are then made to eliminate mistaken use as SPC.
3. Survey control is established by the GPS ties to NAD 83 Coordinate System. These coordinates are then converted to local coordinates.
4. We have developed a County Coordinate System (ground) based on the Lambert Conformal Conic Projection which is directly related to NAD 83 Geodetic and/or State Plane Coordinates. The computer program runs on an IBM PC and is available to any requester (MN DOT).
5. At this point we plan to put a c/f factor on each plan sheet.
6. Please refer to enclosed paper presented at Spring 1983 ACSM-ASP Annual Convention. WisDOT has produced several internal papers leading to and expanding on issue. Presently, maps are produced in grid which are then converted with a Combined Factor to project for design, R/W, and surveys.
7. We will tie to a triangulation station if there is one close by. We use the coordinates with a geodetic bearing; angular adjustments are made, and horizontal closure is checked.
8. We have successfully utilized second order class II surveys (state plane coordinates) to control major projects and stationing on major projects, especially new locations or major relocations. We are duty bound to meet state minimum technical standards.
9. Projects are filed using our state plane coordinate system; therefore, each distance is reduced accordingly and does not create a problem.
10. We modify the state plane coordinates by a Datum Adjustment Factor, (combined factor) which is determined for each project to keep the grid/ground differences within acceptable tolerances, essentially using a local datum.
11. We use an elevated reference surface for different zones of the state.
12. A datum adjustment factor is determined for various areas throughout the state and is used for all projects in that area. We try to maintain (at least) a 1:20,000 relationship.
13. Datum coordinates are computed from state plane system by applying a combined factor (sea level * scale factor) to the grid distances.
14. Enclosed an example of combined factor computed for each county in state.
15. a). Use skelton network showing grid coordinates/project coordinates. Construction can then be performed on ground distances without corrections.
    b). Project coordinates using a scale factor of 1.0000 for construction layout (note highway segment).
16. The state plane coordinates are used for both centerline and survey of R/W. The centerline coordinate points will remain the same and any error is placed in the R/W survey. The land surveyors will use the lasered distance and true angles on the plat that will be filed in the public records.
17. After initial control is run & proved, we pick a centrally located even grid value, then use the combined factor for that point to obtain ground based coordinates. We then translate that point to the even grid value of the original, but minus the leading digits to keep values below 100,000.

18. Grid distances are converted back to ground distances (dividing grid distance by CSF).
19. Unfortunately, the problem is handled differently in each of our eleven Region Offices.
20. We convert to local datum plane.
21. State plane coordinates are adjusted by a combined adjustment factor.
22. Coordinates are multiplied by combination factors which have been developed for specific areas within the state. There is some problem when projects go from one defined area to another.
23. We solve the problem of grid/ground distances by applying a "Combination Factor" to each project. The "Combination Factor" is calculated by dividing the "Elevation Factor" by the "Scale Factor".
24. State plane coordinates are stored in our Survey Monument Data Base with elevation for each location so that the factors can be used for the project conversions. This data base is the responsibility of the Survey Units.
25. Presently IDOT is doing an internal analysis for determining the best way to handle the grid/ground differences. At this time, IDOT has no set policy. Various methods are used.
26. Grid factor is input into the total stations and used when on the project.
27. The grid/ground distance differences are handled by each District Office. This issue will be addressed and standardized in the surveying procedural manual.

Suggestions for solving the problem of distance distortion inherent in the use of state plane coordinates are:

1. Use a method of project datum coordinates. Ours are unique project by project. All factors & reduction constants are mandatorily recorded.
2. Use ground-grid conversion factor to scale NAD 83 (NAD 27) coordinates to local plane and save/record this factor.
3. I feel that an accurate ground coordinate system directly related to geodetic and state plane coordinaes is needed at the user level (local government and private surveyors/engineers). The fact that in Minnesota we have developed and encouraged the use of a ground coordinate system the past 20 years, has really helped our efforts in developing a state-wide GIS/LIS.
4. Not a problem; Caltrans survyors efficiently and accurately convert grid/ground - and have for many years.
5. I have been actively investigating and promoting use of county datums for WisDOT over the past several months similar to Mn DOT's system. I have discussed creation of such with university and consultants.
6. We believe non needed - by proper use of the existing state plane coordinate system no significant problems have been encountered.
7. This is a tool that can be used successfully by competent, experienced Location Surveyors. Often we can prorate horizontal distances between control points on alignments to fit the grid.
8. The entire project can be solved on local 3-D model datum.
9. Multiply ground distance by the grid factor. Elevation factor * scale factor = grid factor.
10. Keep the project size small or go to spherical coordinates.
11. We are in the process of purchasing 5 GPS receivers for geodetic control points and survey centerlines. This will increase problems with distortion and we will then begin to address this problem.

12. We are considering the use of an average scale factor on a project basis.
13. We have relatively little distortion in the state of XXXX. the new GPS network will improve the geoid model for future GPS derived orthometric heights in the state.
14. We have not decided.
15. If a more accurate grid/ground distance relationship is desired, a datum adjustment factor can be computed for each project.
16. The "distance distortion" is not a problem on small projects. It becomes a problem due to the lack of understanding and training in the use of the state plane coordinate system. With the proper background, converting between grid distances and horizontal ground distances is only a matter of applying one combined factor. The state plane coordinates should be used for control on preliminary mapping while horizontal ground distances should be used for project centerline stationing and plan sheet development.
17. A project "grid scale factor" is computed to scale (multiply) the starting NGS monument coordinates and the traverse is run and closed on a PC COGO system using ground distances. The resulting coordinates are referred to as "modified state plan [sic] coordinates" and the required CAF (combined adjustment factor) needed to reduce back to "near actual" state plane coordinates is listed on each design sheet.
18. Educate our personnel of the basic physical structure of the coordinate system, where they are in it, and the simple resolution of a problem.
19. All projects use state plane coordinates and average correlation factor shown on plans.
20. Keep projects short enough that the difference is negligible.
21. By using surface coordinates derived from state plane coordinates through combined factor, we have not encountered any problems to this date.
22. Use project datum coordinates when feasible.
23. Densification of NGRS with GPS will require universal access to NAD 83 (91) and factor conversions to local datum.
24. Each project has a combined adjustment factor that eliminates this problem for the project.
25. We are considering the use of an average factor which would be computed and used with each project (and shown on plans).
26. There be a central location for the coordinate base and that the Survey Unit be the only entity to hand out which will be used on individual projects. This is the only way to achieve uniformity.
27. a). After initial control survey is finished - convert coordinates to ground datum by applying the reciprocal of the grid factor. b). Use state plane coordinates through all phases of a highway project. Adjust grid distances to ground when staking construction centerline.

Suggestions for points to be made in presentation of this paper.

1. How do you address the commercial software available today to perform the proposed projections on automatic and productive levels?
2. I see the need for a "user friendly" 3-D least squares adjustment program available for the local surveyor to make full use of GPS and total stations.
3. Today, the conversion is made easier with powerful hand-held calculators and computers. With electronic data collection and stake-out, the conversion can be part of the "system."

4. a). Original SPC systems were set up to maintain accuracy at sea level - not at elevation of topography. Hence, "grid" cannot be used as "ground" to the same accuracy levels. b). CADD's, roadway design, automated surveys, & GIS all "need/demand" coordinate systems which accurately define ground values. c). One system universally accepted within the DOT is needed to avoid datum conversion costs.
5. Our major projects, Interstate & Freeway, often traverse through several counties so we will continue to utilize existing state plane zones.
6. Instead of creating a cartographical projection system, I would recommend utilizing one of many transformation programs to transform coordinates from one system to another.
7. Stay with NGS and state plane coordinates. Any other local coordinate system turns into a problem, especially going to geodetic coordinates. Any factor you apply to the coordinates causes errors on control stations.
8. I would recommend that you also investigate the 3-D concept with mapping. Maybe we could collect data using latitude & longitude since CADD systems can use real space to store points. Computer programs could then translate the grid and scale factors for the map that you need. In either event, computers will provide the solution.
9. Do all survey computations on state plane coordinate system and then multiply by the datum adjustment factor.
10. Control mapping or published coordinates should be on the state plane coordinate system. Conversion should be made to ground data for centerline stationing and plan sheets.
11. a). All projects should have a listing on the control network monuments. The listing should also include elevation scale factor, convergence angle, (grid) scale factor at monument, and identify NAD 27 vs NAD 83. b). The elevation datum for the project should be clearly noted. c). NAD 27 & NAD 83 coordinates should be presented in the paper as both are still in use.
12. In Vermont most projects are below 1000 feet and small in length. A combined factor would be recommended.
13. (Emphasize) ease of using grid based systems. (Show how) to resolve conflict of grid distances in R.O.W. documents.
14. (Describe) how the benefits of the system justify the committment of resources to plan, implement, document, and support the system. (Show) how essential administrative support is to make the system successful.
15. Statewide coordination is necessary for consistency for work overlapping into several counties. Local zone parameters may cause confusion.
16. We do the same type of adjustment on a project basis. We believe that on a county basis you introduce too much inaccuracies.
17. There is an immediate need for education in this area. What ever method is used, appropriate documentation is necessary on plans. Standards and specifications should be addressed by some national agency (NGS, AASHTO, FHWA, etc.).
18. (Recommend) that we completely get away from the use of NAD 27 and go entirely with NAD 83, for uniformity and consistency. So many problems have to do with the conversions bewteen old coordinates and those derived with GPS. If we are to use the latter let's use it exclusively. Start reobserving the old classical network with GPS and create a complete new basis for mapping and GIS/LIS needs.
19. Unable to reply - we have not received a copy of your paper you are presenting at the GPS '91 Conference. Please forward a copy. We are seriously looking into this problem and are interested in how some one else is viewing the best ways to handle it.
20. We are not experienced enough at this point to offer suggestions.

APPENDIX E, REFERENCES

Burkholder, E. F. (1985). "State Plane Coordinates on the NAD 1983," presented at ASCE Spring Convention, April 30 - May 3, Denver, CO. (Unpublished, available from author).

Burkholder, E. F. (1990). "Using Error Propagation Tools to Protect an Investment in GPS Technology," Proceedings ACSM Annual Meeting, March 19 - 23, Denver, Colorado.

Burkholder, E. F. (1991a). "Design of a Local Coordinate System for Surveying, Engineering & LIS/GIS," presented at ACSM Annual Meeting, March 25-29, Baltimore, MD, submitted for possible publication in Journal of Surveying & Land Information Systems.

Burkholder, E. F. (1991b). "Computation of Horizontal/Level Distances," Journal of Surveying Engineering, Vol 117, No. 3, pp 104-116.

Claire, C. N. (1968). "State Plane Coordinates by Automatic Data Processing," USC&GS Publication 62-4, National Geodetic Information Center, Rockville, MD.

Davis, R.E., Foote, F.S., Anderson, J.M., Mikhail, E.M. (1981). Surveying: Theory and Practice, McGraw-Hill Book Company.

Fronczek, C. J. (1977). "Use of Calibration Base Lines," NOAA Technical Memorandum NOS NGS-10, National Geodetic Information Center, Rockville, MD.

Leick, A. (1990). Satellite Surveying, John Wiley & Sons Inc.

Leick, A. (1980). Geometric Geodesy, 3-D Geodesy, Conformal Mapping, Report No. 19, Department of Surveying Engineering, University of Maine, Orono, ME.

Soler, T., and Hothem, L. (1988). "Coördinate Systems Used in Geodesy: Basic Definitions and Concepts," Journal of Surveying Engineering, Vol. 114, No. 2, pp 84-97.

Spofford, P. (1991). Geodesist, National Geodetic Survey, Rockville, MD., Personal Communication.

Stem, J. E. (1989). "State Plane Coordinate System of 1983," NOAA Manual NOS NGS 5, National Geodetic Information Center, Rockville, MD.

---, 1990. "Trimnet: Survey Network Software User's Manual," Trimble Navigation, Sunnyvale, CA., page 3-116.

Vanicek, P., and Krakiwsky, E. (1982). Geodesy: The Concepts, North-Holland Publishing Company.

Vincenty, T. and Schwarz, C. R. (1989). Chapter 12, "Mathematical Models," North American Datum of 1983, NOAA Professional Paper NOS 2, National Geodetic Information Center, Rockville, MD.

Vincenty, T. (1980). "Zur raeumlich-ellipsoidischen Koordinaten-Transformation (On the transformation from three-dimensional to ellipsoidal coordinates)," ZfV, 11/1980.