Fundamentals of Spatial Data Accuracy and The Global Spatial Data Model (GSDM)

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Three-dimensional (3-D) spatial data accuracy is described by the standard deviation of each component in the context of a global spatial data model (GSDM) that simultaneously accommodates both local and global perspectives, both high-level scientific and local "flat-earth" applications, and both activities that generate spatial data and activities that use spatial data.

Abstract:

The goal in this paper is to identify and build on fundamental concepts of spatial data and error propagation to promote a better understanding of spatial data accuracy. Starting with a definition of the spatial data primitive and associated conventions, spatial data of various types are added to beginning control point values to build a 3-D database. Using the global spatial data model (GSDM), the unique location of each point is stored as geocentric X/Y/Z coordinates and the stochastic information for each point is stored in a covariance matrix. Conventional latitude/ longitude, UTM, and map projection coordinates (plus other geometrical elements such as heights, areas, and volumes) are derived from the stored values. The standard deviation of each derived quantity is readily available from the stored stochastic information using conventional error propagation techniques. Regardless of discipline, spatial data users worldwide can enjoy the benefits of computing and comparing spatial uncertainties within a common standard system.

Introduction:

Spatial data accuracy is an umbrella term that encompasses concepts such as uncertainty, standard deviation, positional tolerance, confidence intervals, error ellipses, and others. When discussing spatial data accuracy, a question often overlooked is, "Accuracy with respect to what?" In some cases, the answer may be irrelevant but, in all cases, the answer should be unambiguous, correct, and readily available. The process of establishing spatial data accuracy relies upon the accuracy of the beginning control point values, the quality of the observations, the adequacy of the model(s) used for spatial data manipulations, and correctly identifying and tracking the accumulation of random error, component by component. This paper describes that process in the context of the global spatial data model (GSDM) that includes both a functional model of geometrical relationships and a stochastic model for tracking the uncertainty of any/all elements (Burkholder 1997). Not restricted to any one discipline, the GSDM facilitates collection, storage, manipulation, exchange, and use of spatial data worldwide because the same simple 3-D model accommodates both those activities that generate spatial data and those activities that use spatial data, whether in high-level scientific research or in local "flat earth" applications. And, regardless of application, questions regarding spatial data accuracy can be handled with a common set of stochastic model equations. The spatial data accuracy discriminator is the magnitude of the standard deviation, component by component, and each user's (or organization's) choice as to what is "good enough" and what isn't.

Definitions and conventions:

For purposes of this paper, the following definitions and conventions are used.

1. **Spatial data uncertainty** is given by its standard deviation in each of three dimensions. One standard deviation (1 sigma) provides a 68% confidence level. Many spatial data users routinely use a 95% (2 sigma) confidence level as the basis for making comparisons and/or inferences.

2. Euclid (Appendix 3, Pedoe 1970) used the following definitions: "A point is that which has no part." and "A line is breadthless length." In an attempt to be more practical, the **spatial data primitive** is taken to be the distance between endpoints of a line in Euclidean space. Even though a line is the path of a moving point, a distance (not a point) is viewed as the spatial data primitive because the location of a point is meaningless unless/until described with coordinates (distances). The endpoints of the line may be abstract entities (such as the origin or axes of a coordinate system) or physical objects (such as survey monuments, building corners, or the location of a GPS antenna – whether stationary or moving). Such a distance between endpoints may be curved (geodetic coordinates) or straight (rectangular coordinates) without violating the definition (Burkholder 2001).

3. Physical geodesists use a definition which also includes the gravity field (NIMA 1997) but, for purposes of describing location and spatial data accuracy, a **3-D geodetic datum** is defined as an earth-centered earth-fixed (ECEF) right-handed rectangular X/Y/Z coordinate system whose:
   a. origin is at the earth's center of mass.
   b. Z axis coincides with the earth's mean spin axis. That means X/Y coordinates are in the plane of the equator.
   c. X axis is coincident with zero degrees longitude (the Greenwich Meridian). That means the Y axis lies at 90° east longitude.
   d. distance unit is meters.
   e. ellipsoid is defined by two parameters that permit computation of equivalent latitude/longitude/height coordinates from geocentric X/Y/Z coordinates.

   The GSDM and error propagation concepts described herein work equally well with any well-defined 3-D datum. Three commonly used reference frames (datums) described by Snay and Soler (1999 – Part 1) are the North American Datum of 1983 (NAD83), the World Geodetic System of 1984 (WGS84), and the International Terrestrial Reference Frame (ITRF). Understandably, values in one system are best compared with other values in the same system.

4. Recognizing that the origin of any system is relative to some "larger" system (the center of mass of the earth is relative to the center of mass of our solar system etc), **absolute quantities** are expressed by a numerical value in a defined system. Units, such as meters, degrees, etc, must be associated with each numerical value. An exception is radian measure and other ratios in which the units cancel.
   a. Coordinates are often used as absolute values.
   b. An azimuth is an absolute quantity whose reference is either implied or defined explicitly.
   c. The numbers or values assigned to the origin of a well-defined system (zero or otherwise) may be viewed as absolute quantities.

   d. The accuracy of an absolute quantity is called **datum accuracy** and is often implied by the units and/or the context in which the number is used. Explicit accuracy statements are used to eliminate possible confusion.

5. A **relative value** is the difference between two absolute quantities expressed in the same system.
   a. Coordinate differences are relative.
   b. An angle, being the difference between two directions, is relative.
   c. It is possible for an absolute quantity to be treated as a relative quantity. This could happen if the origin has units of zero. If zero is subtracted from an absolute quantity, the result can be considered a relative value because it represents the difference of two absolute quantities.
   d. The accuracy of relative spatial data can be expressed in either of two ways. One expression, **network accuracy**, represents the uncertainty (standard deviation) of the difference between two values in the same system. Another expression, **local accuracy**, represents the uncertainty of one point with respect to another.

6. Elevations and time are similar in that each may look like an absolute value. But, in reality, both are used as relative values due to the ambiguity of their physical origins.
   a. Mean sea level, the geoid, enjoys a simple physical definition as the "zero" equipotential surface. But, as yet, that origin has not been precisely located worldwide. Therefore it can be said, precise absolute elevations do not exist.
   b. Other vertical datums are referenced to some arbitrary surface which implies those elevations are relative.
   c. Time is counted from the "big bang" (Hawking 1988), from the birth of Christ (B.C. and A.D.), from the vernal equinox (the instant of the sun's zero declination), from the daily transit of the sun over a stated meridian (A.M. or P.M.) or from some arbitrary zero computed from the readings of a group of atomic clocks. Whether in years, months, days, hours, or seconds, time is an interval between two specified events – a relative quantity.
   d. Time differences and elevation differences can each be measured quite precisely and that information can be quite useful. But, the accuracy of each is limited to relative accuracy statements. In terms of absolute accuracy, there is nothing to be gained from adding a precise interval to an absolute quantity of dubious value.
   e. Ellipsoid height is a derived quantity with respect to the ellipsoid (ultimately with respect to the earth's center of mass). Because the origin is well defined and measurable, ellipsoid height can be considered an absolute quantity. Ellipsoid height differences are relative quantities. This point can be added to the arguments in favor of redefining elevation to be ellipsoid height, see (Burkholder 2002).

Spatial Data Components and Their Accuracy:

As listed in Burkholder (2001), the following spatial data types are based upon the spatial data primitive in terms of the GSDM:

1. **Absolute X/Y/Z geocentric coordinates** (Figure 1, Box 1) are perpendicular distances in meter units from the respective axes of the ECEF coordinate system.

2. **Absolute geodetic coordinates** (Figure 1, Box 2) of latitude/longitude/height are computed from ECEF coordinates with respect to some named datum/ellipsoid.
3. **Relative geocentric coordinate differences** (Figure 1, Box 7) are obtained by differencing compatible geocentric X/Y/Z coordinate values or they can be obtained by rotating relative local coordinate differences into the X/Y/Z reference frame. Relative geocentric coordinate differences are also obtained directly as the $\Delta X/\Delta Y/\Delta Z$ components of a GPS vector.
4. **Relative geodetic coordinate differences**, $\Delta\phi/\Delta\lambda/\Delta h$, (not shown in Figure 1) are obtained as the difference of compatible (common datum) geodetic coordinates.
5. **Relative local coordinate differences**, (Figure 1, Box 8) are the local tangent plane components of conventional total station surveying measurements. If deflection-of-the-vertical is severe and if project requirements warrant same, the vertical based measurements of a total station instrument should be converted to normal based measurements before calling them local geodetic horizon components. Relative local coordinate differences are also components of a geocentric $\Delta X/\Delta Y/\Delta Z$ vector rotated into the local geodetic horizon.
6. **Local coordinates**, *e*/*n*/*u*, are distances from some origin whose local definition may be sufficient in three dimensions, two dimensions, or one dimension. Burkholder (2001) calls these absolute coordinates but, depending upon how they are viewed, they could also be considered relative values. Elevations are particularly difficult to categorize. The real underlying issue is how the local system is defined. Examples include:
    a. **Point-of-Beginning (P.O.B.) datum coordinates** (Figure 1, Box 9) are defined by Burkholder (1997) as the local tangent plane components from any point (origin) selected by the user to any other point. These derived coordinates enjoy full mathematical definition in three dimensions and suffer no loss of geometrical integrity in the GSDM.
    b. **Map projection** (or state plane) coordinates (Figure 1, Box 5) are well defined in two dimensions with respect to some named origin and geodetic datum.
    c. **Ellipsoid heights** (Figure 1, Box 2) and **orthometric heights** (Figure 1, Boxes 3 and 5) are one-dimensional distances above or below some named surface. Ellipsoid heights can be considered absolute but other elevations are considered relative.
7. **Arbitrary local coordinates** (not shown in Figure 1) may be 1D, 2D, or 3D based upon some assumed origin. Although useful in some applications, arbitrary local coordinates are generally not compatible with other local coordinate systems and have limited value in the broader context of geo-referencing. Many computer graphics and data visualization programs use arbitrary local coordinates.

With regard to all spatial data components, both absolute and relative, each one can have a standard deviation associated with it. If the standard deviation of any component is zero, the quantity is either known very precisely and/or the value (e.g. a control point) is being used as a "fixed" quantity. Standard deviations of subsequently computed spatial data components are based upon propagation of the measurement error and standard deviations of the computed points are determined through the network adjustment process. Given a successful network adjustment and computation of coordinates, the implied accuracy statement is "with respect to the points held fixed by the user." Maybe the beginning point was a hub pounded in the ground. Maybe it was a section corner of the U.S. Public Land Survey System. Maybe it was a high-accuracy-reference-network (HARN) point. Or maybe it was the orbit parameters of the GPS satellites. Understandably, the value of a completed project is greatly enhanced if explicit accuracy statements are made. But, making or not making an explicit statement is not the real issue. The real issue is being able to make one of the following statements supported by appropriate statistics.

1. "The absolute datum (identified by user) accuracy of Point X in 3 dimensions is σ_X = ________, σ_Y = ________, and σ_Z = ________." An equivalent statement, derived from the first, gives the standard deviations in the local reference frame as σ_e = ________, σ_n = ________, and σ_u = ________. The absolute accuracy statement involves only one point and is with respect to the datum selected/named by the user. If the project were a 2 dimensional survey (i.e. state plane coordinates), only two components would be named.
2. "The relative network accuracy of the direction and distance from Point 1 to Point 2 is σ_{AZ} = ________ and σ_{DIST} = ________." Relative accuracy applies to the difference between two independent points having absolute accuracy values in the same datum.
3. From Point 1 to Point 2, the relative network accuracy of the height difference (Δh) or perpendicular distance from the local tangent plane (Δu) is $\sigma_{\Delta h}$ = ________ or $\sigma_{\Delta u}$ = ________.
4. "The relative local accuracy of Point 2 with respect to Point 1 is σ_{AZ} = ________, σ_{DIST} = ________, $\sigma_{\Delta h}$ = ________ or $\sigma_{\Delta u}$ = ________." Relative local accuracy exploits and is largely governed by the statistical correlation which exists between two directly connected points in the same datum.

A procedure for computing each of the listed accuracies is given in equation (9).

But Everything Moves:

Most spatial data activities involve using a database such as a geographic information system (GIS). The importance of the basic geodetic control in a GIS is well documented by the National Research Council (NRC 1983) and others. Ideally, the geodetic control information upon which the database is built should be of such quality that it could be held "fixed", i.e. having a zero standard deviation. Here again the question "With respect to what?" becomes relevant. A monumented point that is stable in one system (e.g. NAD83) may, in fact, be moving in another (WGS84 or ITRF). With the advent of GPS positioning, it is now possible to determine the location of control points much more accurately than before and the scientific community now has conclusive evidence that points once thought to be permanent are, in fact, moving – with respect to what? An over-simplified answer is that "everything moves." A better answer is required. More specifically, the administrators and users of a database (whether local, regional, national or global) deserve explicit information as to the stability and accuracy for the various categories of points in the database. And, if they are moving, what is the velocity vector of the point? This paper is primarily about 3-dimensional uncertainties but, given that points move, time must be added as the fourth dimension and the epoch must enjoy equal standing with the coordinates. Software for converting X/Y/Z coordinates from one epoch to another is called HTDP and is available gratis from the U.S. National Geodetic Survey (NGS) at www.ngs.noaa.gov. HTDP can also be used to convert X/Y/Z coordinates from one 3-D datum to another (Snay 1999).

With respect to movement, a simple question must be asked – perhaps as a side bar issue. "Are we standing on the train watching the station go by or are we standing at the station watching the train go by?" The earth's center of mass is the location reference for the entire globe. Points on the earth's surface, or anywhere within the earth may move with respect to the earth's center of mass but the reference is fixed by definition – it does not move. Admittedly, with respect to where we stand or with respect to monumented points, statements are made that the center of mass of the earth moves. The implied perspective is considered subordinate to the explicit statement, "the earth's center of mass does not move."

The ITRF is defined such that the net tectonic movement of all the earth's continental plates is zero (Snay and Soler, 1999, Part 3). But, points on the earth's surface still move with respect to the earth's center of mass and with respect to each other. Therefore the locations of the ITRF monuments are defined with both coordinates and velocities. Spatial data users in North America will be reassured to know that the NAD83 datum is the one to use because, except for areas of tectonic activity, points on the NAD83 remain "fixed" to the North American plate and move together. Such oversimplification is dangerous. The NAD83 monumented control points on the ground may be stable, but the GPS satellite orbits are defined in and the continuously operating reference stations (CORS) coordinates are published in the ITRF reference frame. (NGS also publishes NAD83 coordinates for the CORS stations). The issue of which to be aware is that the absolute coordinates (for points on the ground) may be in one reference frame and the relative coordinate differences (obtained from GPS) may be in a different reference frame. Since the NAD83 and ITRF relative coordinate differences are nearly identical, it generally permissible to attach ITRF relative coordinate differences to absolute NAD83 datum coordinates but mixing absolute datum coordinates in the same solution should be avoided. And, as noted by Strange (2000), the difference in datums is even more important since the removal of selectivity availability, especially when using GPS code observations.

The point here is that most spatial data users should be aware of three competing 3-D geodetic datums – NAD83, WGS84 and ITRF. Each has a reason for existing and each has a role to fill. At a gross level of accuracy, it does not matter which of the three datums is used. But, as the tolerance for uncertainty gets smaller and smaller, it does matter which datum is used. The GSDM can be used with each datum individually and provides a systematic method for identifying and tracking the uncertainties in a given datum – whatever they are. Comparing uncertainties (standard deviations) between datums is beyond the scope of this paper.

Observations, Measurements and Error Propagation:

This entire paper could be devoted to measurement issues, but only a summary is included here. In many ways, observations and measurements are very similar and the terms are used interchangeably. But, a mathematical distinction is that observations are always independent quantities and measurements may be either independent or correlated. Stated differently, any observation may be called a measurement but a measurement can be called an observation if and only if it is an independent quantity. As listed in Burkholder (2001) there are only a limited number of quantities that can be directly measured. But, whether the measurement is a length, time, voltage, temperature, etc, spatial data components are determined indirectly from those measurements using appropriate models and computations. The standard deviation of each component is determined by propagating the measurement uncertainty through the variance/covariance equation given by the following matrix formulation:

$$\sum_{YY} = J_{YX} \sum_{XX} J^{t}_{XY}; \qquad (1)$$

where:

$\boldsymbol{\Sigma}_{\mathbf{YY}}$ = Covariance matrix of computed result.
$\boldsymbol{J}_{\mathbf{YX}}$ = Jacobean matrix of partial derivatives of the result with respect to the variables (measurements).
$\boldsymbol{\Sigma}_{\mathbf{XX}}$ = Covariance matrix of variables (measurements) used in the computations.

To reiterate, the variables in the measurement covariance matrix are independent and considered to be observations if and only if there is no correlation in the measurement covariance matrix.

Finding the Uncertainty of Spatial Data Elements:

In the process of establishing the spatial data uncertainty of each point, the user must first decide which datum will be used. Mixing datum values is permissible only if the datum differences are smaller than the resolution of data added to the database. For example, if 10-meter data are being used and if the datum differences are at the 1-meter level, it makes no difference which datum is used. On the other hand, if 10-millimeter data are being used and datum differences are at the 1-meter level, the choice of datum does matter.

Second, each project should be based upon reliable control points having X/Y/Z geocentric coordinates in the appropriate datum. One control point may be sufficient to put a new project on the chosen datum, but making a connection to two or more points is standard practice. If the basic control points are assigned a zero standard deviation, that means subsequent accuracy statements should be made "with respect to the control points selected and held fixed by the user." Better statements regarding datum accuracy statement can be made if realistic standard deviations are assigned to the points used to control the project. The covariance matrix for each new point and the correlation between points in the network are a standard by-product of a least squares adjustment. When the network adjustment is done in terms of geocentric coordinates and coordinate differences, the resulting covariance matrix is in terms of the geocentric reference frame. The geocentric environment is more efficient for storage and computer operations but, because of the human perspective, the local covariance matrix is preferred as being more intuitive – giving sigma east, sigma north, and sigma up as the square root of the diagonal elements.

The GSDM includes both the geocentric and local covariance matrices for each point but, since one can be derived from the other, a BURKORD™ database (Burkholder 1997) stores only the geocentric covariance matrix. The local covariance matrix is computed upon demand. Both covariance matrices contain the same datum accuracy of each point component by component but, because of perspective, the numbers are different. Each of the two covariance matrices is a 3x3 symmetrical matrix containing the following elements:

Geocentric Covariance Matrix — Local Covariance Matrix

$$\Sigma_{X/Y/Z} = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} \\ \sigma_{XY} & \sigma_Y^2 & \sigma_{YZ} \\ \sigma_{XZ} & \sigma_{YZ} & \sigma_Z^2 \end{bmatrix} \text{ and } \Sigma_{e/n/u} = \begin{bmatrix} \sigma_e^2 & \sigma_{en} & \sigma_{eu} \\ \sigma_{en} & \sigma_n^2 & \sigma_{nu} \\ \sigma_{eu} & \sigma_{nu} & \sigma_u^2 \end{bmatrix} \qquad (2)\ \&\ (3)$$

where:

$\sigma_X^2, \sigma_Y^2, \sigma_Z^2$ = variances for geocentric coordinates for the point.
$\sigma_{XY}, \sigma_{XZ}, \sigma_{YZ}$ = covariance elements for geocentric coordinates.
$\sigma_e^2, \sigma_n^2, \sigma_u^2$ = local perspective variances for the point.
$\sigma_{en}, \sigma_{eu}, \sigma_{nu}$ = local perspective covariance elements for the point.

And, as shown in Burkholder (1999), the two covariance matrices are related by the following rotation matrix evaluated at the latitude/longitude of the standpoint (local origin).

$$\mathbf{R} = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix} \quad (4)$$

The matrix expression for the relationship between the two covariance matrices is:

$$\sum_{e/n/u} = R \sum_{X/Y/Z} R^t \quad (5)$$

$$\sum_{X/Y/Z} = R^t \sum_{e/n/u} R \quad (6)$$

Points that are part of a network adjustment enjoy an inter-relationship described by correlation. The correlation is especially significant for adjacent points that have been connected by a direct measurement. Correlation exists between points not directly connected but the influence drops rapidly as the number of courses between points increases (correlation is the reason cross-ties serve to strengthen a network). If the significant correlations between points are stored along with the covariance matrix for each point, the local accuracy of one point with respect to the other is readily computed along with the inverse direction and distance. If correlations are not stored (or if they are assumed to be zero), an inverse computation will readily provide the direction and distance between points and the two endpoint covariance matrices will provide the basis of the network accuracy associated with the relative differences.

Using Points Stored in the X/Y/Z Data Base:

Each stored X/Y/Z location is unique within the birdcage of orbiting GPS satellites. Three application modes for using the stored X/Y/Z locations include single point, (unique location for inventory tag etc.), point-pair (used to create lines, surfaces, and objects), and "cloud" (mapping). Even though stored as X/Y/Z, the location of any point can be readily expressed in latitude/ longitude, UTM, or state plane coordinates. The uncertainty of a single point is given by the datum accuracy as computed from the geocentric covariance matrix. These uncertainties (standard deviations, variances, and other covariance elements) can be viewed in either the geocentric reference frame or in the local reference frame. The geocentric reference frame is more efficient for data storage and computerized manipulation but the local reference frame is more convenient for viewing because horizontal and vertical is the human perspective. A benchmark will have a small standard deviation on the vertical component. By contrast, a horizontal control point will have small standard deviations on the east and/or north components. A 3-D control point will have small standard deviations on all three components.

The point-pair application provides the relative location of one point with respect to another. A map is generated by extensive successive use of the point-pair mode and an accuracy statement as applied to such a "cloud" of points is not addressed in this paper. Although appropriate at some point in the future, it is not intended here to initiate a discussion of National Map Accuracy standards.

Specifically, in the point-pair mode, Point 1 is defined by $X_1/Y_1/Z_1$ and Point 2 is defined by $X_2/Y_2/Z_2$. The matrix formulation of the 3-D geocentric inverse from Point 1 to Point 2 is:

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ X_2 \\ Y_2 \\ Z_2 \end{bmatrix} \tag{7}$$

The matrix of coefficients to the variables is called the Jacobian matrix and the general error propagation formulation in the form of equation (1) is:

$$\sum_{\Delta} = J \sum_{1\to 2} J^t \tag{8}$$

Using the Jacobian matrix of 1's and 0's from equation 7, having the geocentric covariance matrix of Point 1 and Point 2 both available, and using the correlation between Point 1 and Point 2, the covariance matrix of the inverse is computed using equation (8) as:

$$\sum_{\Delta} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \sigma^2_{X_1} & \sigma_{X_1Y_1} & \sigma_{X_1Z_1} \\ \sigma_{X_1Y_1} & \sigma^2_{Y_1} & \sigma_{Y_1Z_1} \\ \sigma_{X_1Z_1} & \sigma_{Y_1Z_1} & \sigma^2_{Z_1} \end{bmatrix} & \begin{bmatrix} \sigma_{X_1X_2} & \sigma_{X_1Y_2} & \sigma_{X_1Z_2} \\ \sigma_{Y_1X_2} & \sigma_{Y_1Y_2} & \sigma_{Y_1Z_2} \\ \sigma_{Z_1X_2} & \sigma_{Z_1Y_2} & \sigma_{Z_1Z_2} \end{bmatrix} \\ \begin{bmatrix} \sigma_{X_1X_2} & \sigma_{Y_1X_2} & \sigma_{Z_1X_2} \\ \sigma_{X_1Y_2} & \sigma_{Y_1Y_2} & \sigma_{Z_1Y_2} \\ \sigma_{X_1Z_2} & \sigma_{Y_1Z_2} & \sigma_{Z_1Z_2} \end{bmatrix} & \begin{bmatrix} \sigma^2_{X_2} & \sigma_{X_2Y_2} & \sigma_{X_2Z_2} \\ \sigma_{Y_2X_2} & \sigma^2_{Y_2} & \sigma_{Y_2Z_2} \\ \sigma_{Z_2X_2} & \sigma_{Z_2Y_2} & \sigma^2_{Z_2} \end{bmatrix} \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

The off-diagonal sub-matrices reflect the correlation between Point 1and Point 2. **Datum accuracy** of Point 1 and Point 2 is included in equation (9) as their respective covariance sub-matrices.

The following concise mathematical statements are the basis for the definitions of local accuracy and network accuracy given earlier.

Local accuracy of the inverse between Point 1 and Point 2 is obtained by using the full covariance matrix in equation (9). Correlation between Point 1 and Point 2 is included.

Network accuracy of the inverse between Point 1 and Point 2 is obtained if the correlation between Point 1 and Point 2 is either non-existent or taken to be zero.
Additional details for computing the inverse direction, distance, and standard deviations are provided in Burkholder (1999).

Example:

The following example is hypothetical. Figure 2 shows two HARN control points, a central observation station, and three points on the face of a dam. The goal is to monitor the location of points on the dam to determine their stability and to document movement - if it exists. Although other technology might be used, it is presumed static carrier phase GPS data were collected. Of the several ways to organize the sessions, the stations are occupied with 4 GPS receivers and non-trivial baselines are defined as noted by the circled session numbers in Figure 2. This arrangement provides non-trivial baselines for the network adjustment with two moves – Unit A goes to station F, then unit D goes to station B.

Session 1 - units at A, C, D, and E
Session 2 - units at C, D, E, and F
Session 3 - units at B, C, E, and F

In processing the GPS data, a minimally constrained network uses all the baselines but holds only 1 HARN station as the anchor. Given appropriate internal network consistency and no outlying residuals on the first adjustment, the network is readjusted holding both HARN stations. If the second constrained adjustment is acceptable and if the reference variance is near unity, the covariance matrix (in this case, a 12x12) of the newly computed positions (parameters) is the inverse of the normal equations coefficient matrix (Appendix B, Davis, et, al., 1981). The variances of the computed coordinates are on the diagonal, the point covariance elements are adjacent to the diagonal (3x3 sub-matrices), and the correlation between points is given by the remaining off-diagonal elements.

Notes regarding the network adjustment:

1. It is presumed X/Y/Z coordinates for the HARN stations are used. If the coordinates of each HARN station are held fixed, then the datum accuracy for the computed points is "with respect to the HARN stations." In reality, each HARN station also has a small standard deviation associated with its published position. Technically, such uncertainties should be included as a prerequisite to making a datum accuracy statement "with respect to NAD83."

2. Each GPS vector is defined by $\Delta X/\Delta Y/\Delta Z$ components as determined from baseline processing.

3. The covariance matrix of each baseline vector is used in developing the weights for the adjustment. An approximation would be to develop weights using only the standard deviations of the $\Delta X/\Delta Y/\Delta Z$ components. Some network adjustments are made using equal weights for all components. Project requirements should dictate which procedure is used.

4. If the computed reference variance is not close to unity, the covariance matrix of the parameters should be multiplied by the computed reference variance.

Conclusions:

The deformation example shows how datum accuracy, network accuracy, and local accuracy are all part of the same project. This paper identifies concepts associated with various types of spatial data and spatial data accuracies. It goes on to describes how those concepts fit together in terms of the GSDM. The fundamental concepts are not new but using the GSDM is revolutionary. The difference is building and using a 3-D database having a single origin for spatial data instead of building and using a database having disparate origins for horizontal and vertical data. Rules of solid geometry and vector algebra are globally applicable in the 3-D environment. That means one set of equations is equally applicable worldwide and the necessity of keeping track of mapping equations and projection constants is enormously reduced. An even greater benefit is being able to compute standard deviations for each derived quantity. But, best of all, each user has the information readily available with which to answer the question, "Accuracy with respect to what?" Datum accuracy is with respect to the control held by the user and equation (9) provides an unambiguous procedure for computing network accuracy and local accuracy.

In writing this paper, it was intended for the logical development to move forward and it has. Furthermore, the dam deformation example was included purposefully to raise the issue about local accuracy. In that case, the real question to be answered is, "Are the points stable or did the points move?" Local accuracy really should be used to answer those questions. What about datum accuracy or network accuracy? Are they needed? Not really. Now suppose the two HARN points are not HARN points at all, but well monumented points set in bedrock on ridges on opposite sides of the canyon. Instead of constraining the network to the NAD83, the code phase position of the Central Station is used to anchor the entire network (a single point with X/Y/Z coordinates and rather large standard deviations). The datum accuracy for points in the network will not be very good at all but the local accuracy between points is governed by the quality of the connecting measurements. And, the local accuracy between points "once removed" from a direct connection is still governed by correlation between the points.

The digital revolution (Burkholder 2003) has had an enormous impact on the way spatial data are generated, manipulated, stored, and used. The original goal was to show how the GSDM could be used to establish and make better statements about spatial data accuracy. Ironically, the argument has come full circle. After building the case so carefully to start with high-quality control points and to add quality components to obtain high quality datum accuracy points, it turns out that is not required at all – depending upon the required answer, "With respect to what?" It occurs to the author that many more similar applications are waiting to be discovered. What, for example, about describing the local accuracy of points generated with close-range photogrammetry? Or, what about optical tooling applications? Research on these and other issues should be interesting and fruitful.

Acknowledgements:

Many people deserve credit for developing the concepts brought together in this paper. The goal in formulating the GSDM was to begin with fundamental, almost self-evident, concepts and arrange them in ways that accommodate the use of new technology while remaining consistent with established practice. Although innumerable persons including historical figures, professional colleagues, and students have provided input, the following deserve specific recognition: Appendix C in Bomford (1971) is titled, "Cartesian Coordinates in Three Dimensions;" Leick (1990 and 1995) defines the 3-D Geodetic Model; Mikhail (1976) provides a comprehensive discussion of functional and stochastic models; and, when discussing models, Moritz (1978) comments on the simplicity of using the basic global rectangular X/Y/Z system without an ellipsoid. Neither is the concept of a GSDM new. Seeber (1993) states that H. Bruns proposed the concept of a global three-dimensional polyhedron network as early as 1878. The difference now is that GPS and other modern technologies have made a global network practical and the polyhedron need not be limited to earth-based points. When the aforementioned concepts are combined in a systematic way, with particular attention to the manner in which spatial data are used, the synergistic whole – the GSDM - appears to be greater than the sum of its parts. Additional contemporaries are recognized in the defining document (Burkholder 1997).

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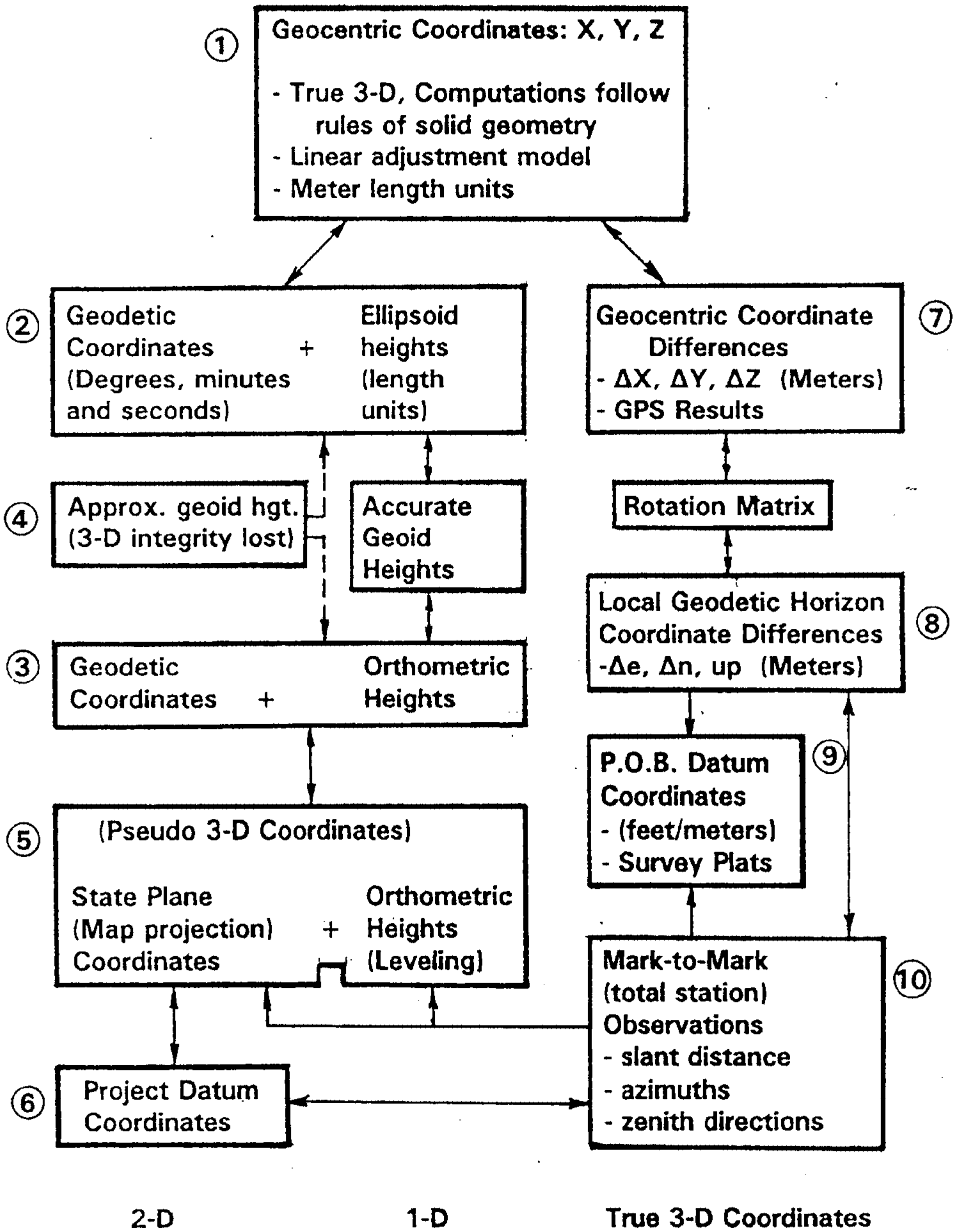

Figure 1 Schematic Showing Relationship of Spatial Data Elements

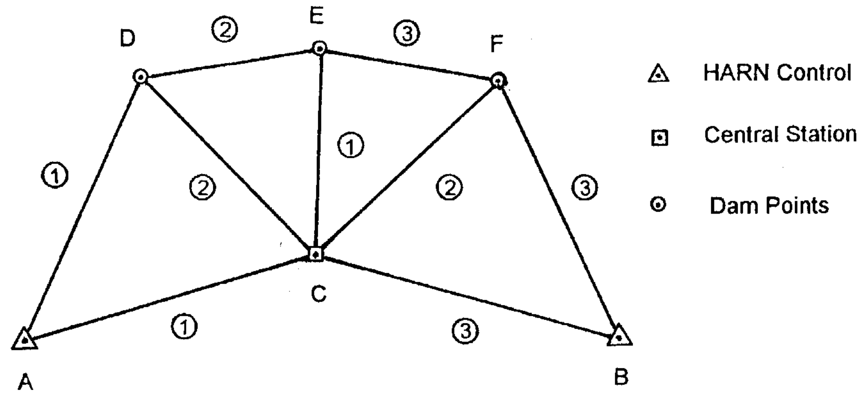

Figure 2, Example of Dam Deformation Network