Design of a Local Coordinate System for Surveying, Engineering, and LIS/GIS

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ABSTRACT. *State plane coordinate systems were designed to permit surveyors, engineers, and others to work with plane rectangular coordinates while enjoying the benefits of using the precisely surveyed National Geodetic Reference System (NGRS). The state plane systems are functional and useful, but lack complete acceptance because (1) benefits do not justify the extra data collection and computational effort, (2) people avoid using what they do not understand, and (3) grid distances differ from the horizontal ground distance. These obstacles are surmountable through education and specifications, but, with personal computer resources universally available, another solution has become increasingly attractive. By designing and implementing a local (county) coordinate system, benefits of using the NGRS and existing state plane coordinates can be retained, grid and ground distance differences are virtually eliminated, and users can continue to work with convenient rectangular coordinates. This paper is written to support ideas proposed by Nancy von Meyer in an article on county coordinate systems published in the June 1990* ACSM Bulletin, *and concludes by presenting algorithms that can be used to implement a local coordinate system.*

Introduction

Two-dimensional rectangular coordinate systems are widely used in surveying, engineering, and land information systems/geographic information systems (LIS/GIS) applications. If used, the third dimension is "up" or elevation. This practice is acceptable to the extent that a flat earth can be assumed without sacrificing geometrical integrity. The National Geodetic Reference System (NGRS), established and supported by the National Geodetic Survey (NGS), provides very precise latitude and longitude coordinates for thousands of control points scattered throughout the United States. The problem is that latitude/longitude positions have angular coordinates of degrees, minutes, and seconds. Most users find it more convenient to use two-dimensional plane coordinates, such as northings and eastings. A map projection "flattens" the earth, and permits use of latitude/longitude control points in a two-dimensional rectangular coordinate system. The geometrical integrity of the precisely surveyed NGRS is transferred to the two-dimensional system by using a properly designed and documented map projection. The state plane coordinate systems designed in the 1930s enable users of the NGRS to work with two-dimensional coordinates anywhere within a state or zone.

Drawbacks to using state plane coordinate control have included:

1. Lack of accessibility: Control stations were established at locations selected for intervisibility between points and strength of triangulation figures. Hence, many existing points are located on mountain tops and other hard-to-reach places.
2. Lack of proximity: Due to triangulation techniques used to establish many of the existing first- and second-order points, the points are often 6 to 15 miles or more apart, and not necessarily located in urban high-use areas.
3. Lack of quality: Many subsequent points tied to the existing first- and second-order network were established using third-order or undocumented procedures, and do not fulfill today's need for reliable control.
4. Lack of understanding: Many surveyors and engineers have not been required to use state plane coordinates and have not made the effort necessary to understand their use.
5. Distortion: Horizontal ground distance may differ significantly from the grid distance shown on a survey map or plat. This drawback is particularly important in route-location surveys, construction layout, and accurate area computations.

The first three drawbacks have been reduced dramatically with introduction of Global Positioning System (GPS) and total station surveying techniques. GPS is routinely used to establish high-order control stations at readily accessible locations throughout an urban area or for specific projects. With modern total stations, data collectors, and computers, it is easy and economical to determine precise coordinates for traverses tied to GPS control points. Tremendous progress also is being made in the level of understanding as more mapping professionals and technicians learn

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to use state plane coordinates. The last drawback, the difference between ground distance and grid distance, still is a problem and is the focus of this paper.

Definition of Terms

In an effort to be consistent with terminology used by NGS for state plane coordinates on the North American Datum of 1927 (NAD 27) and North American Datum of 1983 (NAD 83), the following definitions are used:

NAD 27 (ACSM/American Society of Civil Engineers [ASCE] 1978)

- *Scale factor* is grid distance divided by ellipsoid distance.
- *Sea-level factor* is sea-level distance divided by horizontal ground distance. It is identical to R/(R+H).
- *Grid factor* is the product of scale factor and sea-level factor. Grid distance = ground distance X grid factor.

NAD 83 (Stem 1989)

- *Grid-scale factor* is grid distance divided by ellipsoid distance.
- *Elevation factor* is ellipsoid distance divided by horizontal ground distance, and determined by R/(R+H+N).
- *Combined factor* is the product of grid-scale factor and elevation factor. Grid distance = ground distance X combined factor.

These changes were made in an attempt to be more specific with respect to using the ellipsoid, versus sea level, as an intermediate surface, and to avoid confusion with the generic use of "scale factor" as applied to mapping. For example, the 7.5-minute U.S. Geological Survey topographic map series has a scale factor of 1:24,000 (map scale, 1 in. = 2,000 ft.).

The Distance Distortion Tradeoff

Grid/Ground Distance

When using state plane coordinates, there is a difference between a horizontal ground distance at some elevation and the state plane coordinate inverse between the same two points. One is "ground distance," the other is "grid distance." As shown in Figure 1, the magnitude of the difference varies with elevation and location within a given zone. A 100.000-m horizontal distance at a ground elevation of 500 m near the center of a state plane zone having a minimum scale factor of 0.9999 gives a grid distance of 99.982 m, a relative difference of 1:5,600. The relative difference drops to 1:3,900 at 1,000-m elevation and degenerates further for higher elevations.

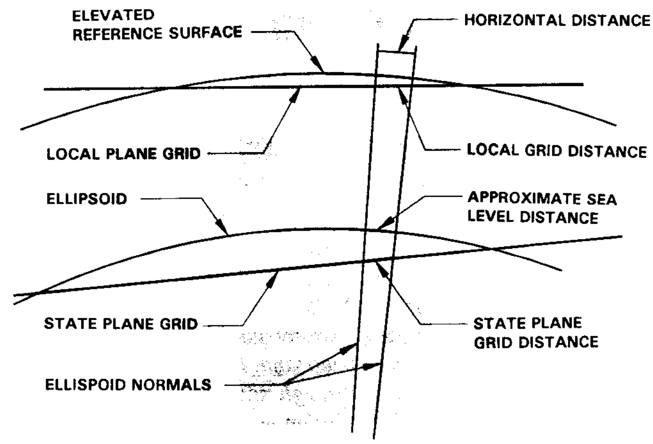

Figure 1. Relation of local grid to state plane grid.

This paper proposes designing a local projection to reduce the distance distortion between ground and grid without sacrificing the geometrical integrity of existing state plane coordinate systems and uses. The price of less distortion is being restricted to covering a smaller (local) area with a given projection.

Map Projections

A fundamental tenet of map projections is that a curved surface cannot be mapped to a plane without distorting angles, distances, or area. By choosing an appropriate map projection, it is possible to preserve one of the three. An equal-area projection preserves area, an equidistant projection preserves distance, and conformal projections used for the state plane systems preserve shape so that angles between lines on the curved surface are transformed without distortion to the map surface.

Distance distortions on a state plane system are minimized to the extent possible by limiting the width of the state plane coordinate zones to 158 miles. In being transformed to the state plane mapping surface, a distance on the ellipsoid near the center of the zone is compressed, and a distance near the edge of the zone is stretched. The amount of distortion is expressed numerically by the grid-scale factor for the line.

Distortion Components

A distance distortion of 1:10,000 corresponds to a grid-scale factor of 0.9999 or 1.0001. That means an ellipsoid (sea-level) distance of 100.000 m near the center of a zone is represented on the map as 99.990 m and the same 100.000 m distance near the edge of the zone shows up on the map as 100.010 m. At some point between the center and edge of the zone, the grid surface intersects the ellipsoid. In that area, the grid-scale factor is unity and a 100.000 m distance on the ellipsoid shows up as 100.000 m on the map.

Some mistakenly believe that state plane coordinates are good only to 1:10,000. It is true that a sea-level distance near the zone center or extreme edges may be distorted by 1:10,000, but if proper corrections are applied to precise field measurements, the geometrical integrity of state plane coordinates can be just as good as the latitude/longitude control points on which they are based.

Proper corrections include the elevation factor in addition to the grid-scale factor described above. Surveyors, engineers, and LIS/GIS professionals are concerned with the total distortion between ground distance and grid distance, not just with the grid-scale factor portion. The total distance distortion is given by the combined factor, which includes both the elevation and grid-scale factors.

Design Considerations

Transit/tape surveys were acceptable, normal practice in the 1930s, when the state plane coordinate systems were designed. Traverse accuracies of 1:5,000 were fairly routine, and a systematic distance distortion as large as 1:10,000 could be absorbed without detrimental impact on the quality of a survey. If greater computational accuracy were required, the grid-scale factor could be computed and applied as described above. Otherwise, a sea-level distance could be used as the grid distance.

The original state plane coordinate systems were designed to accommodate a scale factor up to 1:10,000, but did not include elevation as a design criterion. Michigan is an exception. Originally, the Coast and Geodetic Survey selected three transverse Mercator projections for Michigan but, when the state passed the Michigan Coordinate System law for the NAD 27 in 1964, the legislature adopted three Lambert projections designed by Ralph Moore Berry, then professor of geodetic engineering at the University of Michigan. Since most of the land surface in Michigan is within 200 ft. of 800 ft. above sea level, he designed NAD 27 projections at a reference surface 800 ft. above sea level. (At 200 ft. above datum, the elevation factor is 0.9999904 and the distortion is 1:100,000.) Therefore, except for areas exceeding 1,000 ft. in elevation, the elevation factor in Michigan is insignificant for NAD 27 coordinates, and the ground/grid distance difference does not exceed 1:10,000.

Modern Practice

With the introduction of electronic distance measuring instruments, the ratio of precision for many traverses improved dramatically. No longer could the grid/ground distance difference be ignored without degrading the quality of a survey (even in Michigan). As a result, routine practice is to compute and apply both the elevation and grid-scale factors when computing a survey. Geometrical integrity is preserved and the 1:10,000 grid-scale factor limitation is moot. Recognizing this, several states (Stem 1989) chose to eliminate separate NAD 27 zones in favor of a single NAD 83 zone covering the entire state. No integrity is lost, but in those states the grid/ground distance difference on NAD 83 is larger (depending on location within a zone) than it was on NAD 27.

Elevated Reference Surface

The difference between grid and ground distance can be minimized by using a map projection defined on an elevated reference surface for a local area. If the defining parameters are documented and incorporated into appropriate software, local users can work with grid/ground distances that differ by some insignificant amount, and the local coordinates can be converted into state plane (or geodetic) coordinates with established algorithms. Geometrical integrity and direct connection to state plane coordinates can be preserved.

The benefits and problems of working on an elevated reference surface (i.e., the Michigan Coordinate System) were investigated by the author (Burkholder 1980b) and documented in a master's thesis at Purdue University. It was concluded (perhaps incorrectly) that advantages of using a standard system were greater than the benefits of using a statewide elevated reference surface. The Michigan Coordinate System for NAD 83 uses standard Lambert conic projections on the GRS 1980 ellipsoid.

The Third-Wave Difference

Berry's foresight in proposing an elevated reference surface is validated by recent developments. The personal computer now provides any user the ability to use local coordinates within the framework of the NGRS. Toffler (1980) describes a society in which decentralization and individualism are enhanced by the information and computer revolution. In the past, choices were more limited. Henry Ford is often quoted as offering his customers any color car they wanted as long as it was black. Now there are many options available, and it is possible to buy a one-of-a-kind car built to order. And, today's consumer is encouraged to develop and express individual taste in many areas, including fashion, food, telephones, and computers.

The same concept applies to professional services. Cars express our individuality, but they are all built to certain safety standards, and almost all use unleaded gas. Telephone jacks have been standardized, but the consumer has a multitude of choices when selecting a telephone. The NGRS and established state plane coordinate systems are standards that support multiple uses and data interchange. But, with GPS tools, total stations, electronic data collectors, computers, and knowledgeable planning, it is possible for surveyors, engineers, and other mapping professionals to work in a local two-dimensional coordinate system in which ground and grid distance are essentially the same. And, if designed and implemented properly, the local coordinates can be fully compatible with the state plane systems and the NGRS.

Two conditions are required for successful implementation:

1. Knowledgeable planning is essential to identify and document appropriate design criteria. The county's physical configuration (length, breadth, and elevation differences) and distortion tolerances must be considered. Planning also includes formalizing the administrative structure responsible for implementing the local coordinate system, company policy manual, county ordinance, contract specifications, administrative rules, and state statutes.
2. Fully compatible means that anyone wishing to transform the local coordinates to state plane or geodetic coordinates will have convenient access to the design parameters, transformation algorithms and software that will perform the transformations without loss of geometrical integrity.

The Solution

A local coordinate system can be defined in several ways. Perhaps the most common way is to assign arbitrary coordinates to a point, and assume a reference bearing. That may fulfill the need of the plane surveyor for individual projects, but such a coordinate system usually lacks permanent definition and compatibility with other surveys in the region or area. By contrast, the state plane systems provide specificity and compatibility, but they often include more distance distortion (from both grid-scale factor and elevation) than is convenient or acceptable. A countywide coordinate system, as proposed by von Meyer (1990), can be used to bridge the gap by incorporating features of both.

Elevated Reference Surface

The conventional solution to unacceptable mapping distortion is to restrict the area covered by the projection and to use an elevated reference surface. The area covered is related to the grid-scale factor, while elevation is related to the sea-level factor. If both factors are controlled within specified limits, the grid/ground difference can be as small as desired. Table 1 shows the distance permitted on a tangent projection surface (be it plane, cone, or cylinder) from the point or line of contact to the zone edge, where the grid-scale factor limit is exceeded.

Greater area coverage for a given level of distortion can be achieved by using a secant projection. Table 2 shows the distance permitted on a secant projection surface from the middle of the zone to the edge, where the maximum grid-scale factor is exceeded. The minimum grid-scale factor occurs at the zone center.

Information in these tables can be used to design a map projection to a controlled level of distortion. The minimum grid-scale factor is a defining parameter on a transverse Mercator projection, while the spacing of the standard parallels is required for a Lambert conic projection. In either case, the total zone width is twice the distance from the zone center.

In addition to choosing the level of distortion permitted by the map projection, the designer of a local coordinate system must choose some reference elevation to which all horizontal distances are transformed (Burkholder 1991). Table 3 shows how nearly the elevated reference surface must match ground elevation before exceeding comparable distortion levels. Geometrical integrity can still be preserved (by applying corrections), but grid/ground differences will become excessive if the elevation limit is exceeded and corrections are not applied.

Steps for Designing a Local Coordinate System. The following steps can be used to define a local coordinate system having the geometrical integrity necessary to preserve data-sharing options with the NGRS:

1. Choose maximum grid-scale factor distortion permitted.
2. Choose projection type and locate it with design parameters as follows:
   Lambert Conic Conformal
   - North and south standard parallels
   - Longitude of central meridian
   - Latitude of false origin

Table 1. Distances and distortions for tangent projections.

Maximum grid scale factor	Maximum distortion	Approximate distance from point/line of contact
1.0001	1:10,000	90 km
1.00001	1:100,000	28 km
1.000002	1:500,000	13 km
1.000001	1:1,000,000	9 km

Table 2. Distances and distortions for secant projections.

Minimum grid scale factor at zone center	Maximum distance distortion	Approximate separation of standard parallels (Lambert)	Maximum distance from zone center
0.9999	1:10,000	1° 37′	127 km
0.99999	1:100,000	0° 31′	40 km
0.999998	1:500,000	0° 14′	18 km
0.999999	1:1,000,000	0° 10′	13 km

Table 3. Elevation factors and distortion levels.

Elevation factor	Elevation must be known within	Distortion if elevation ignored
0.9999	637 m	1:10,000
0.99999	64 m	1:100,000
0.999998	13 m	1:500,000
0.999999	6.4 m	1:1,000,000

- False easting/northing, meters

Transverse Mercator

- Minimum grid-scale factor
- Longitude of central meridian
- Latitude of false origin
- False easting/northing, meters

3. Choose projection surface height and define ellipsoid parameters. Start with GRS 1980 ellipsoid and modify.
   a. The ellipsoid semimajor axis equals 6,378,137.000 m plus chosen reference ellipsoid height, meters.
   b. Ellipsoid flattening or eccentricity is unchanged. For GRS 1980 $1/f = 298.25722210088$ and $e^2 = 0.0066943800229034$.
4. Compute projection constants (either Lambert or transverse Mercator) according to algorithm listed in Appendix A. Note that constants for any local coordinate system need be computed only once, as shown in the examples in Appendix A.

Steps for Using a Local Coordinate System. After zone constants have been computed, local coordinates for points having known latitude/longitude positions are computed using the transformation algorithms in Appendix A.

1. Use the "forward" transformation to compute local coordinates for known control points.
2. Each survey needs a place to start and a direction in which to go. Local coordinates on a known control point provide a place to start, and a reference azimuth is obtained from a coordinate inverse to another visible local control point.
3. Horizontal ground distances can be used as grid distances within tolerance of projection design. For maximum integrity, all horizontal distances in the local coordinate system area should be reduced to a common datum plane (Burkholder 1991).
4. When working with local coordinates, one should use local grid azimuth. The true mean bearing of any line can be obtained by applying local convergence computed at the midpoint of the line. If any line is so long (more than 5 km, for example) that the second term correction is needed, such correction can be applied as described by Stem (1989), pages 18-19 and 51-53.
5. Azimuths obtained from GPS measurements using equations 7.4 or 7.11 of Leick (1990) are geodetic azimuths and can be compared with local grid azimuths, if convergence at the point is used to convert the geodetic azimuth to local grid azimuth.
6. Surveys based on a local projection can be computed using plane Euclidean geometry, without grid-scale factor corrections to the state plane grid. After a local survey is completed, the local coordinates can be transformed to latitude and longitude using the local zone constants and the "inverse" algorithms listed in Appendix A.
7. Once latitude/longitude are known for any point, state plane coordinates can be computed using standard procedures (Stem 1989).
8. Given a project goes beyond the limits of a local coordinate system, another zone can and should be defined. A transition from one zone to another is handled by converting from one local coordinate system to latitude and longitude, then converting those values to local coordinates in the adjoining system. It is identical to moving from one state plane zone to another.

Conclusion

Using a properly designed local coordinate system, it is possible to enjoy the benefits of state plane coordinates while conducting local plane surveys using plane Euclidean geometry and coordinate geometry routines. Furthermore, there is no need to make sea-level or grid-scale factor corrections to the horizontal distances, because the local grid distance is very nearly identical to the horizontal ground distance. The maximum amount of distortion is determined by the designer of the local system.

The key to using a local coordinate system successfully is documenting the local zone parameters and being very specific about the elevated reference surface.

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Appendix A: Algorithms for Local Coordinate System Using Elevated Reference Surface

Except where noted otherwise, symbols are intended to be consistent with those used in NOAA Manual NOS NGS 5, *State Plane Coordinate System of 1983* (Manual 5).

Lambert Conic Conformal Projection

Input reference ellipsoid (GRS 1980 recommended):

a = semi-major axis = 6,378,137.000 m

$1/f$ = reciprocal flattening = 298.2572221008827

Input local coordinate system reference ellipsoid height:

h_{ref} = reference ellipsoid height in meters (not in Manual 5)

Compute semimajor axis of modified ellipsoid:

$a_{ref} = 6{,}378{,}137.000 + h_{ref}$ (used in place of "a" in Manual 5)

$1/f = 298.2572221008827$ (same as for standard ellipsoid, GRS 1980)

Compute ellipsoid constants:

$$e^2 = 2f - f^2 \;\&\; e = \sqrt{e^2}$$

$$\left.\begin{aligned} c_2 &= e^2/2 + 5e^4/24 + e^6/12 + 13e^8/360 + 3e^{10}/160 \\ c_4 &= 7e^4/48 + 29e^6/240 + 811e^8/11520 + 81e^{10}/2240 \\ c_6 &= 7e^6/120 + 81e^8/1120 + 3029e^{10}/53760 \\ c_8 &= 4279e^8/161280 + 883e^{10}/20160 \end{aligned}\right\} \text{Used once below.}$$

$$c_{10} = 2087e^{10}/161280$$

$$\left.\begin{aligned} F_0 &= 2(c_2 - 2c_4 + 3c_6 - 4c_8 + 5c_{10} \\ F_2 &= 8(c_4 - 4c_6 + 10c_8 - 20c_{10}) \\ F_4 &= 32(c_6 - 6c_8 + 21c_{10}) \\ F_6 &= 128(c_8 - 8c_{10}) \\ F_8 &= 512c_{10} \end{aligned}\right\} \text{Used in "Inverse" to compute geodetic latitude without iterating. See also page 42 in Manual 5.}$$

Input defining parameters for Lambert projection (user's choice):

ϕ_n = latitude of north standard parallel
ϕ_s = latitude of south standard parallel
ϕ_b = latitude of false origin (usually where northing = 0.0)
λ_0 = west longitude of central meridian
E_0 = false easting on central meridian, meters
N_b = northing at false origin, usually 0.0 m

Compute local (elevated) coordinate system projection constants:

(ln = natural logarithm and exp(x) = ϵ^x where ϵ = 2.718. . .)

$$Q_n = 0.5\left[\ln\frac{1+\sin\phi_n}{1-\sin\phi_n} - e\ln\frac{1+e\sin\phi_n}{1-e\sin\phi_n}\right]$$

$$Q_s = 0.5\left[\ln\frac{1+\sin\phi_s}{1-\sin\phi_s} - e\ln\frac{1+e\sin\phi_s}{1-e\sin\phi_s}\right]$$

$$Q_b = 0.5\left[\ln\frac{1+\sin\phi_b}{1-\sin\phi_b} - e\ln\frac{1+e\sin\phi_b}{1-e\sin\phi_b}\right]$$

Q_i = isometric latitude for corresponding geodetic latitude

$$W_n = \sqrt{1-e^2\sin^2\phi_n}$$

$$W_s = \sqrt{1-e^2\sin^2\phi_s}$$

$$\sin\phi_0 = \frac{\ln\left(\dfrac{W_n\cos\phi_s}{W_s\cos\phi_n}\right)}{Q_n - Q_s}; \qquad \phi_0 = \sin^{-1}(\sin\phi_0)$$

$$K = \frac{a_{ref}\cos\phi_s\exp(Q_s\sin\phi_0)}{W_s\sin\phi_0} = \frac{a_{ref}\cos\phi_n\exp(Q_n\sin\phi_0)}{W_n\sin\phi_0}$$

$$Q_0 = 0.5\left[\ln\frac{1+\sin\phi_0}{1-\sin\phi_0} - e\ln\frac{1+e\sin\phi_0}{1-e\sin\phi_0}\right]$$

$$R_b = \frac{K}{\exp(Q_b\sin\phi_0)} \qquad R_b = \text{mapping radius of latitude of origin}$$

$$R_0 = \frac{K}{\exp(Q_0\sin\phi_0)} \qquad R_0 = \text{mapping radius of central parallel}$$

$$N_0 = R_b + N_b - R_0 \qquad N_0 = \text{northing at central parallel}$$

$$k_0 = \frac{R_0\tan\phi_0\sqrt{1-e^2\sin^2\phi_0}}{a_{ref}} \qquad k_0 = \text{minimum grid-scale factor}$$

These zone constants need to be computed only once for each local coordinate system designed. They are used repeatedly in the "forward" and "inverse" transformations. For the state plane coordinate systems, the constants are tabulated in Appendix C of Manual 5.

Forward Transformation

Input:

ϕ = geodetic latitude
λ = geodetic longitude (west)

Compute:

$$Q_\phi = 0.5\left[\ln\frac{1+\sin\phi}{1-\sin\phi} - e\ln\frac{1+e\sin\phi}{1-e\sin\phi}\right]$$

$R_\phi = \dfrac{K}{\exp(Q_\phi \sin\phi_0)}$ R_ϕ = mapping radius at point (ϕ,λ)

$\gamma = (\lambda_0 - \lambda) \sin\phi_0$ γ = convergence at point (ϕ,λ)

$k = R_\phi \sin\phi_0 \dfrac{\sqrt{1 - e^2 \sin^2\phi}}{a_{ref} \cos\phi}$ k = grid-scale factor at point (ϕ,λ)

$N_L = R_b + N_b - R_\phi \cos\gamma$ N_L = local northing at point $(\phi\lambda)$

$E_L = E_0 + R_\phi \sin \gamma$ E_L = local easting at point $(\phi\lambda)$

Efficient manual computations can be performed if tables of values for R_ϕ, γ, and k are precomputed for every minute (or other interval) of latitude/longitude within the local zone area. Then manual transformation computations easily can be performed by interpolating appropriate values from the tables. It is the intent of using a local coordinate system that the grid-scale factor be close enough to 1.00000 as to be ignored. This can be verified as true or not as necessary by looking up the grid-scale factor for a given point covered by the grid-scale factor table.

Inverse Transformation

Input:

N_L = local northing of point (N in Manual 5)
E_L = local easting of point (E in Manual 5)

Compute:

$R' = R_b - N_L + N_b$
$E' = E_L - E_0$
$\gamma = \tan^{-1} (E'/R')$ γ = local convergence for point

$R_\phi = \sqrt{R'^2 + E'^2}$ R_ϕ = mapping radius at (N_L,E_L)

$Q_\phi = \dfrac{\ln\left(\dfrac{K}{R_\phi}\right)}{\sin\phi_0}$ Q_ϕ = isometric latitude at (N_L,E_L)

$\chi = 2 \tan^{-1}\left(\dfrac{\exp(Q_\phi) - 1}{\exp(Q_\phi) + 1}\right)$ χ = conformal latitude at (N_L,E_L)

$\phi = \chi + \sin\chi \cos\chi(F_0 + \cos^2\chi(F_2 + \cos^2\chi(F_4 + \cos^2\chi(F_6 + F_8 + \cos^2\chi))))$
(F coefficients are in radians.) ϕ = geodetic latitude at (N_L,E_L)

$\lambda = \lambda_0 - \dfrac{\gamma}{\sin\phi_0}$ λ = geodetic longitude at (N_L,E_L)

$k = R_\phi \sin\phi_0 \dfrac{\sqrt{1 - e^2 \sin^2\phi}}{a_{ref} \cos\phi}$ k = grid-scale factor at (N_L,E_L)

Transverse Mercator Projection

Input reference ellipsoid (GRS 1980 recommended):

a = semimajor axis = 6,378,137.000 m
1/f = reciprocal flattening = 298.2572221008827

Input local coordinate system reference ellipsoid height:

h_{ref} = reference ellipsoid height in meters (not in Manual 5)

Parameters of modified ellipsoid:

a_{ref} = 6,378,137.000 + h_{ref} (used in place of "a" in Manual 5)
1/f = 298.2572221008827 (same as for standard ellipsoid, GRS 1980)

Compute ellipsoid constants:

$$
\begin{aligned}
e^2 &= 2f - f^2 \; \& \; e = \sqrt{e^2} \\
n &= f/(2 - f) \\
r &= a_{ref}(1 - n)(1 - n^2)(1 + 9n^2/4 + 225n^4/64)
\end{aligned}
$$

$$
\left.
\begin{aligned}
u_2 &= -3n/2 + 9n^3/16 \\
u_4 &= 15n^2/16 - 15n^4/32 \\
u_6 &= -35n^3/48 \\
u_8 &= 315n^4/512
\end{aligned}
\right\} \text{Used once below.}
$$

$$
\left.
\begin{aligned}
U_0 &= 2(u_2 - 2u_4 + 3u_6 - 4u_8) \\
U_2 &= 8(u_4 - 4u_6 + 10u_8) \\
U_4 &= 32(u_6 - 6u_8) \\
U_6 &= 128u_8
\end{aligned}
\right\} \begin{array}{l}\text{Used to compute zone constants} \\ \text{and in ``forward'' transformation.}\end{array}
$$

$$
\left.
\begin{aligned}
v_2 &= 3n/2 - 27n^3/32 \\
v_4 &= 21n^2/16 - 55n^4/32 \\
v_6 &= 151n^3/96 \\
v_8 &= 1097n^4/512
\end{aligned}
\right\} \text{Used once below.}
$$

$$
\left.
\begin{aligned}
V_0 &= 2(v_2 - 2v_4 + 3v_6 - 4v_8) \\
V_2 &= 8(v_4 - 4v_6 + 10v_8) \\
V_4 &= 32(v_6 - 6v_8) \\
V_6 &= 128v_8
\end{aligned}
\right\} \text{Used in inverse.}
$$

Input defining parameters for transverse Mercator projection (user's choice):

λ_0 = west longitude of central meridian
E_0 = false easting on central meridian
k_0 = grid-scale factor on central meridian
ϕ_0 = latitude of false origin, usually where local northing = 0.0 m
N_0 = false northing at false origin, usually 0.0 m

Compute local (elevated) coordinate system projection constants:

$$
\begin{aligned}
\omega_0 &= \phi_0 + \sin\phi_0 \cos\phi_0(U_0 + \cos^2\phi_0[U_2 + \cos^2\phi_0(U_4 + U_6 + \cos^2\phi_0]) \\
S_0 &= rk_0\omega_0
\end{aligned}
$$

These constants are computed only once for each local coordinate system projection. After that, they are used in computing the "forward" and "inverse" transformations. The local zone width is determined by the grid-scale factor chosen for the central meridian. Suggested values are 0.999998 or 0.999999.

Forward Transformation

Input:

ϕ = geodetic latitude
λ = west geodetic longitude

Compute:

$$
\begin{aligned}
L &= (\lambda - \lambda_0)\cos\phi, \qquad \text{L in radian units} \\
t &= \tan\phi \\
\eta^2 &= e^2\cos^2\phi/(1 - e^2) \\
\omega &= \phi + \sin\phi\cos\phi(U_0 + \cos^2\phi[U_2 + \cos^2\phi(U_4 + U_6\cos^2\phi)]) \\
S &= rk_0\omega
\end{aligned}
$$

$$R = \frac{a_{ref}k_0}{\sqrt{1 - e^2 \sin^2\phi}}$$

$A_1 = -R$

$A_2 = 0.5\,Rt$

$A_3 = (1 - t^2 + \eta^2)/6$

$A_4 = [5 - t^2 + \eta^2(9 + 4\eta^2)]/12$

$A_5 = [5 - 18t^2 + t^4 + \eta^2(14 - 58t^2)]/120$

$A_6 = [61 - 58t^2 + t^4 + \eta^2(270 - 330t^2)]/360$

$A_7 = (61 - 479t^2 + 179t^4 - t^6)/5040$

$N_L = N_0 + S - S_0 + A_2L^2[1 + L^2(A_4 + A_6L^2)]$, local north coordinate

$E_L = E_0 + A_1L(1 + L^2[A_3 + L^2(A_5 + A_7L^2)])$ local east coordinate

$C_1 = -t$

$C_2 = (1 + \eta^2)/2$ — F_2 in Manual 5

$C_3 = (1 + 3\eta^2 + 2\eta^4)/3$

$C_4 = [5 - 4t^2 + \eta^2(9 - 24t^2)]/12$ — F_4 in Manual 5

$C_5 = (2 - t^2)/15$

$\gamma = C_1L[1 + L^2(C_3 + C_5L^2)]$, convergence at point (ϕ,λ)

$k = k_0[1 + C_2L^2(1 + C_4L^2)]$, grid-scale factor at point (ϕ,λ)

Inverse Transformation

Input:

N_L = local northing coordinate (N in Manual 5)

E_L = local easting coordinate (E in Manual 5)

Compute:

$\omega = (N_L - N_0 + S_0)/(k_0 r)$

$\phi_f = \omega + \sin\omega \cos\omega(V_0 + \cos^2\omega[V_2 + \cos^2\omega(V_4 + V_6 \cos^2\omega)])$

$\eta_f^2 = e^2 \cos^2\phi_f/(1 - e^2)$

$$R_f = \frac{a_{ref}k_0}{\sqrt{1 - e^2 \sin^2\phi_f}}$$

$Q = (E_L - E_0)/R_f$, radian units

$B_2 = -t_f(1 + \eta_f^2)/2$

$B_3 = -(1 + 2t_f^2 + \eta_f^2)/6$

$B_4 = -[5 + 3t_f^2 + \eta_f^2(1 - 9t_f^2) - 4\eta_f^4]/12$

$B_5 = [5 + 28t_f^2 + 24t_f^4 + \eta_f^2(6 + 8t_f^2)]/120$

$B_6 = [61 + 90t_f^2 + 45t_f^4 + \eta_f^2(46 - 252t_f^2 - 90t_f^4)]/360$

$B_7 = -(61 + 662t_f^2 + 1320t_f^4 + 720t_f^6)/5040$

$L = Q(1 + Q^2[B_3 + Q^2(B_5 + B_7Q^2)])$

$\phi = \phi_f + B_2Q^2[1 + Q^2(B_4 + B_6Q^2)]$ — geodetic latitude of point (N_L,E_L)

$\lambda = \lambda_f - L/\cos\phi_f$ — geodetic longitude of point (N_L,E_L)

$D_1 = t_f$

$D_2 = (1 + \eta_f^2)/2$ — G_2 in Manual 5

$D_3 = -(1 + t_f^2 - \eta_f^2 - 2\eta_f^4)/3$

$D_4 = (1 + 5\eta_f^2)/12$ — G_4 in Manual 5

$D_5 = (2 + 5t_f^2 + 3t_f^4)/15$

$\gamma = D_1Q[1 + Q^2(D_3 + D_5Q^2)]$ — convergence at point (N_L,E_L)

$k = k_0[1 + D_2Q^2(1 + D_4Q^2)]$ — grid-scale factor at (N_L,E_L)

The equations in this appendix are essentially the same as those in Manual 5, and have more than enough terms to preserve computational accuracy for local coordinate systems. By leaving the "extra" terms in the equations, one can use these same equations to compute state plane coordinate transformations, if one uses the GRS 1980 ellipsoid and the defining parameters for a given state and zone. As stated on page 35 of Manual 5, "The A_6, A_7, F_4, (C_4) and C_5 terms are negligible when computing within the approximate boundaries of the SPCS 83 zones. To use the SPCS 83 beyond the defined SPCS 83 boundaries and to compute UTM coordinates, the significance of these terms should be evaluated."Examples are shown in Figures 2 and 3.

```
USER: EARL F. BURKHOLDER
DATE: AUGUST 30, 1992


LAMBERT CONIC CONFORMAL COORDINATE TRANSFORMATIONS
PROJECTION NAME: OREGON TECH - CUSTOM PROJECTION

REFERENCE ELLIPSOID: GEODETIC REFERENCE SYSTEM 1980
                       A =  6378137.0000 METERS
                     1/F =  298.2572221008827

REFERENCE ELLIPSOID HEIGHT FOR PROJECTION =  1315.0000 METERS

MODIFIED ELLIPSOID FOR: OREGON TECH - CUSTOM PROJECTION
                         A =  6379452.0000 METERS
                       1/F =  298.2572221008827


ZONE PARAMETERS:
     NORTH STANDARD PARALLEL               42 18  0.000000
     SOUTH STANDARD PARALLEL               42 14  0.000000
     FALSE ORIGIN LATITUDE                 42 12  0.000000
     CENTRAL MERIDIAN (W)                 121 47  0.000000
     FALSE EASTING ON CM                        20000.0000 METERS
     NORTHING AT FALSE ORIGIN                       0.0000 METERS

ZONE CONSTANTS:
     CENTRAL PARALLEL                      42 16  0.010731
     SCALE FACTOR ON CENTRAL PARALLEL    0.999999831390399
     MAPPING RADIUS OF EQUATOR              12128718.29345 METERS
     MAPPING RADIUS OF FALSE ORIGIN          7037182.78425 METERS
     NORTHING OF CENTRAL PARALLEL ON CM         7407.04610 METERS
     CONFORMAL LATITUDE CONSTANTS:  F(0) = 0.006686920927
          F(2) = 0.000052014583     F(4) = 0.000000554458
          F(6) = 0.000000006718     F(8) = 0.000000000089


TRANSFORMATIONS:

NAME OF STATION: PUB - NAD 83  (BY NGS IN 1985)           FORWARD

     LATITUDE:     42 15 32.915660   NORTHING       6570.8535 METERS
     LONGITUDE:   121 46 54.802710   EASTING       20119.1490 METERS

     CONVERGENCE:        0  0  3.50  SCALE FACTOR:   0.999999839986


NAME OF STATION: PUB - NAD 83  (BY NGS IN 1985)           INVERSE

     LATITUDE:     42 15 32.915659   NORTHING       6570.8535 METERS
     LONGITUDE:   121 46 54.802708   EASTING       20119.1490 METERS

     CONVERGENCE:        0  0  3.50  SCALE FACTOR:   0.999999839986
```

Figure 2. Example of local lambert conic conformal projection.

```
USER: EARL F. BURKHOLDER
DATE: AUGUST 30, 1992


TRANSVERSE MERCATOR PROJECTION TRANSFORMATIONS
PROJECTION NAME: OREGON TECH - CUSTOM PROJECTION

REFERENCE ELLIPSOID: GEODETIC REFERENCE SYSTEM 1980
                           A = 6378137.0000 METERS
                         1/F = 298.2572221008827

REFERENCE ELLIPSOID HEIGHT FOR PROJECTION  =    1315.0000 METERS

MODIFIED ELLIPSOID FOR: OREGON TECH - CUSTOM PROJECTION
                              A = 6379452.0000 METERS
                            1/F = 298.2572221008827

ZONE PARAMETERS:

     CENTRAL MERIDIAN (W)                   121 47  0.000000
     LATITUDE OF FALSE ORIGIN                42 12  0.000000
     FALSE NORTHING AT FALSE ORIGIN              20000.0000 METERS
     FALSE EASTING ON CENTRAL MERIDIAN           50000.0000 METERS
     SCALE FACTOR ON CENTRAL MERIDIAN         0.999998000000

ZONE CONSTANTS:

     RECTIFYING SPHERE RADIUS                 6368761.9422 METERS
     RECTIFYING LATITUDE CONSTANTS:
          U(0) =  -0.005048250776    V(0) =    0.005022893948
          U(2) =   0.000021259204    V(2) =    0.000029370625
          U(4) =  -0.000000111423    V(4) =    0.000000235059
          U(6) =   0.000000000626    V(6) =    0.000000002181

     RECTIFYING LATITUDE OF FALSE ORIGIN      42   3 23.040620
     GRID MERIDIAN ARC TO FALSE ORIGIN     4674806.1973 METERS


TRANSFORMATIONS:

NAME OF STATION: PUB - NAD 83  (BY NGS IN 1985)          FORWARD

     LATITUDE:     42 15 32.915660    NORTHING     26570.8398 METERS
     LONGITUDE:   121 46 54.802710    EASTING      50119.1487 METERS

     CONVERGENCE:        0  0  3.50    SCALE FACTOR:   0.999998000175


NAME OF STATION: PUB - NAD 83  (BY NGS IN 1985)          INVERSE

     LATITUDE:     42 15 32.915659    NORTHING     26570.8398 METERS
     LONGITUDE:   121 46 54.802712    EASTING      50119.1487 METERS

     CONVERGENCE:        0  0  3.50    SCALE FACTOR:   0.999998000175
```

Figure 3. Example of local transverse Mercator projection.

A DOS-BASED MENU-DRIVEN PROGRAM, LOCALCOR.EXE, IS AVAILABLE FREE FROM EARL F. BURKHOLDER AT "globalcogo@zianet.com" 1/23/2006