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Computation of Constants for $\phi - \chi$ Series

Form as developed in Spec. Pub. #67 by Adams
And 10th power terms by Schmid:

$$\phi - \chi = \left[\frac{\varepsilon^2}{2} + \frac{5}{24}\varepsilon^4 + \frac{1}{12}\varepsilon^6 + \frac{13}{360}\varepsilon^8 + \frac{3}{160}\varepsilon^{10}\right]\sin 2\chi \; +$$

$$\left[\frac{7}{48}\varepsilon^4 + \frac{29}{240}\varepsilon^6 + \frac{811}{11520}\varepsilon^8 + \frac{81}{2240}\varepsilon^{10}\right]\sin 4\chi \; +$$

$$\left[\frac{7}{120}\varepsilon^6 + \frac{81}{1120}\varepsilon^8 + \frac{3029}{53760}\varepsilon^{10}\right]\sin 6\chi \; +$$

$$\left[\frac{4279}{161280}\varepsilon^8 + \frac{883}{20160}\varepsilon^{10}\right]\sin 8\chi \; + \left[\frac{2087}{161280}\varepsilon^{10}\right]\sin \frac{10\chi}{\cancel{8\chi}}$$

$$= A\sin 2\chi + B\sin 4\chi + C\sin 6\chi + D\sin 8\chi + E\sin 10\chi \qquad (1)$$

Make the following substitutions:

$$\sin 2\chi = 2\sin\chi\cos\chi$$

$$\sin 4\chi = 8\sin\chi\cos^3\chi - 4\sin\chi\cos\chi$$

$$\sin 6\chi = 32\sin\chi\cos^5\chi - 32\sin\chi\cos^3\chi + 6\sin\chi\cos\chi$$

$$\sin 8\chi = 128\sin\chi\cos^7\chi - 192\sin\chi\cos^5\chi + 80\sin\chi\cos^3\chi - 8\sin\chi\cos\chi$$

$$\sin 10\chi = 512\sin\chi\cos^9\chi - 1024\sin\chi\cos^7\chi + 672\sin\chi\cos^5\chi - 160\sin\chi\cos^3\chi + 10\sin\chi\cos\chi$$

General form $\sin(n\chi) = 2\sin(n-1)\chi\cos\chi - \sin(n-2)\chi$

$$\sin 12\chi = 2048\sin\chi\cos^{11}\chi - 5120\sin\chi\cos^9\chi + 4608\sin\chi\cos^7\chi - 1792\sin\chi\cos^5\chi + 280\sin\chi\cos^3\chi - 12\sin\chi\cos\chi$$

$$
\begin{aligned}
\phi - \chi = \;& 2A \sin x \cos x \\
& - 4B \sin x \cos x + 8B \sin x \cos^3 x \\
& + 6C \sin x \cos x - 32C \sin x \cos^3 x + 32C \sin x \cos^5 x \\
& - 8D \sin x \cos x + 80D \sin x \cos^3 x - 192D \sin x \cos^5 x + 128D \sin x \cos^7 x \\
& + 10E \sin x \cos x - 160E \sin x \cos^3 x + 672E \sin x \cos^5 x - 1024E \sin x \cos^7 x + 512E \sin x \cos^9 x
\end{aligned}
$$

$$\phi - \chi = P \sin x \cos x + Q \sin x \cos^3 x + R \sin x \cos^5 x + S \sin x \cos^7 x + T \sin x \cos^9 x \quad (2)$$

$P = 2A - 4B + 6C - 8D + 10E$

$Q = 8B - 32C + 80D - 160E$

$R = 32C - 192D + 672E$

$S = 128D - 1024E$

$T = 512E$

Coefficients for equation (1) & (2) were computed for specified spheroids in a program written by Earl Burkholder 11-7-1972

(Computed with & without 10th power term)

For ease of computation in Lilly II, $\phi - \chi$ reduces to:

$$\phi - \chi = \sin x \cos x (P + \cos^2 x (Q + \cos^2 x (R + \cos^2 x (S + T \cos^2 x))))$$