The Global Spatial Data Model
A Tool Designed for Surveyors
Part I of 3

Earl F. Burkholder, LS, PE

A global spatial data model (GSDM) has been designed which makes it easy for surveyors to combine GPS data with terrestrial survey data and to perform "geodetic type" computations using rules of solid geometry. Briefly, the GSDM is a collection of procedures by which 3D plane surveying measurements (all HIs must be measured) are converted into geocentric delta X/Y/Z components, and 3D coordinate computations are performed using rules of solid geometry in the geocentric X/Y/Z environment. From there, the user enjoys total flexibility and can continue to work with local plane surveying coordinate differences or convert the X/Y/Z values to conventional geodetic or state plane coordinates. The X/Y/Z coordinates are the underlying stored values which can be exchanged between data bases, users, or projects.

The Key Is Obtaining X/Y/Z Values

The key to using the GSDM is obtaining X/Y/Z values for each point. The National Geodetic Survey (NGS) publishes X/Y/Z values for all high-accuracy reference network (HARN) stations and delta X/Y/Z values are used to traverse from one point to another. GPS vectors are already defined by their delta X/Y/Z components and plane surveying delta e/n/u components are converted to delta X/Y/Z coordinates using simple equations. Everything else is derived from the stored X/Y/Z positions using existing procedures and standard equations. Those equations are included in the Technical Papers of the April 1994 ACSM Annual Meeting in Reno, Nevada, in my paper, "Exploiting the Connection Between Plane Surveying and GPS Vectors." The equations can also be found in geodesy books or obtained from your GPS vendor. Prototype software for performing 3D coordinate geometry computations is called BURKORD™ and is available from me. Finally, I have developed several one-day seminars to present the GSDM concept and plan to offer the seminars at various locations in the coming months. The first day covers basic geodesy while the second day focuses on integrating GPS and terrestrial survey data using the GSDM. For those with Internet access, the seminars are described in more detail under "seminars" at *http://www.lsrp.com*

Adapting To Changes In Equipment

The generation of surveyors whose practice covers any part of the past 30 years have many stories to tell about adapting to change. For example:

- With introduction of desktop computers in the 1960s (e.g. the Olivetti Programma 101), it was no longer necessary to look up trig functions in the tables or to use logarithms. That machine would compute a trig function in about 10 seconds.
- Affordable electronic distance measuring (EDM) instruments have greatly expanded the capability and productivity of many surveying businesses. Sadly, plumb bob taping has become a lost art.
- The HP-35 calculator was introduced about 1972. A hand-held electronic calculator, it had a square root key and capacity for 10 significant digits. Programmable calculators with ever-increasing storage capacity soon followed.
- Within the past 10 years, the electronic total station theodolite, coupled with a data collector, has become the standard for conventional surveying and, with the addition of robotics, the modern survey crew can be a one-person operation.
- GPS was invented during the 1970s, enjoyed continuing development during the 1980s, and initial operational capability was announced in December 1993. Although designed for navigation by the U.S. military, civilian use of GPS has blossomed and surveyors around the world have come to embrace GPS.

Adapting to Change In Concepts

The evolution of equipment is impressive, but what about more fundamental concepts? Consider:

- Coordinate geometry remains pretty much the same. René Descartes is credited with bringing order to the study of analytical geometry with publication of his discourse on geometry in 1637. Although curvilinear latitude and longitude coordinates are used by geographers, cartographers and geodesists, the rectangular cartesian coordinate system is still used extensively all over the world.
- When Gerardus Mercator designed his map projection in the middle 1500s, his goal was to create a map that could be used to sail a constant bearing from one port to another. Mercator's map, an array of parallels and meridians on the earth, was later formalized mathematically and became known as a conformal projection. Several kinds of conformal projections exist today and are used extensively in many part of the world, but they are strictly 2D and provide no mathematical basis for elevation as the third dimension.
- The state plane coordinate systems were designed in the 1930s to permit surveyors and engineers to use plane surveying methods on surveys tied to geodetic triangulation stations of the North American Datum of 1927 (NAD27). Although surveyors have used the systems for years, much of their current popularity is due to the emergence of geographic information systems and GPS data.
- Electronic storage of digital spatial data came into being with the mainframe computer. Early discussions on the challenges of efficient use of digital spatial data were held in the context of Modernization of Land Data Systems (MOLDS) and the concepts evolved into the present-day LISs and GISs.
- Geodetic datums are another concept with which surveyors have become more familiar. Reasons include the increasing use of GIS for storing spatial data, the readjustment of the NAD27, and the use of GPS receivers that collect data relative to the World Geodetic System of 1984 (WGS84). One could also suggest that the HARNs are a new datum to be

learned, even though the National Geodetic Survey (NGS) insists the HARNs are to be viewed as a refinement to the North American Datum of 1983 (NAD83) instead of being treated as a new datum. But, from the surveyors's perspective, use of HARN-based NAD83 coordinates is the same as working with a different datum, because there is a new (different) set of coordinates for the same monument.

• In order to determine elevations using GPS, the surveyor needs to deal with the concept of geoid heights. A 1967 geoid map of North America, published by the U.S. Army Map Service, shows one-meter geoid contours. Since then, geoid modeling has continued as an area of geodetic research, and programs such as GEOID93 and GEOID96 are available from the NGS for computing geoid heights. Accuracy claims for GEOID96 are approximately 3 cm (one sigma) in the absolute mode and 1 to 2 ppm for relative geoid height differences. Combined with appropriate GPS observations, it is possible to determine elevations with a relative accuracy approaching the guidelines established for conventional first-order leveling.

• When using map projection (state plane) coordinates, an inverse between two points yields a grid distance that may be different from the horizontal ground distance by an intolerable amount. For example, at an elevation of 2,000 feet near the center of a zone having a maximum scale distortion of 1:10,000, the difference between grid and ground distance is more than one foot per mile. Many state highway departments address the grid/ground distance issue by using "project datum" coordinate systems, which provide better agreement between grid and ground distances. In other cases, project datum coordinate systems are implemented on a county-wide basis.

The Need For Change

Is change a juggernaut not to be challenged? What are the consequences of dishonoring a sacred cow? Is pressure to conform justifiable or defensible as a basis for making decisions? Are new ways of handling spatial data necessarily better than old ones? Or are we guilty, as in the biblical example, of putting new wine into old wineskins when we use existing two-dimensional models for handling 3D data? These were some of the questions considered during preparation of a report entitled *Definition of a Three-Dimensional Spatial Data Model for Southeastern Wisconsin* (available from the SE Wisconsin Regional Planning Commission, Waukesha, Wisconsin). For surveyors, the most significant benefit identified in that report is:

> *By using an appropriate model, 3-D coordinate geometry computations can be accomplished at any level of precision using existing plane surveying procedures along with a bit of geodesy. The added effort for surveyors (other than becoming familiar with features of the model) is remembering to measure the height-of-instrument for each instrument/reflector setup!*

The appropriate model is the GSDM, which can be easily used by surveyors. As documented in the Wisconsin report, the model performs flawlessly using 3D control of existing NAD83 HARN stations. While the GSDM defines a simple, standard computational environment, it does not specify how larger issues, such as earth tides, subsidence/uplift and continental drift, should be handled. It is left for the NGS to address those issues in connection with a possible national readjustment of the HARN networks.

The GSDM:

• Is based upon the earth-centered, earth-fixed (ECEF) geocentric coordinate system defined by the Department of Defense for GPS use. The position of each point is defined by its X/Y/Z coordinates. Equivalent positions in other coordinate systems are derived using known relationships.

• Is designed to be used by non-geodesists. However, since the GSDM enjoys full mathematical rigor, there is no inherent reason why geodesists or other scientifically minded persons should not use it.

• Uses one set of solid geometry equations world-wide. There are no zones, projection constants, or tables needed to perform spatial data calculations anywhere within the "birdcage" of GPS satellites. While an understanding of coordinate geometry is adequate for using the GSDM, knowing some basic geodesy is helpful.

• Is compatible with, and makes efficient use of, modern digital electronic data collection, storage and manipulation technologies. The same model accommodates conventional survey data, GPS data and aerial mapping data with equal ease.

• Does not distort the physical measurement of distance, as does the map projection (state plane) model. Inverse distance in the GSDM is the same as horizontal ground distance. For those not wishing to give up state plane coordinates, be assured they remain as valid as ever; a listing of state plane coordinates can be readily obtained from stored GSDM points.

• Acknowledges that surveyors work primarily with local coordinate differences. Unless there is a change in local coordinate differences (due to earthquake or movement of a monument), the impact of future datum changes can be negligible for the local user.

• Gives the true azimuth from a standpoint to any forepoint with respect to the meridian through the standpoint. In the real world, meridians are not parallel, which means the forward and back azimuths of a common line are different. The GSDM provides correct forward and back azimuths for any given line.

• Treats elevation as a derived quantity, subject to the uncertainties of ellipsoid heights, geoid heights and vertical datum definition. As geoid modeling results become more accurate, it is possible to compute elevations easily with corresponding reliability.

Little Or No New Science Involved

The GSDM involves little or no new science but is a systematic arrangement of existing concepts. The geometrical relationships constitute what is called the functional component of the model. The GSDM also includes a stochastic component that defines the use of error propagation and positional tolerance for spatial data. The next two articles in this series will list the equations used in functional portion of the GSDM, will include examples of 3D coordinate computations, and will describe stochastic features of the GSDM and show how they can be used to compute the standard deviations of coordinates, distances, azimuths and other derived quantities such as area and volumes. ■

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Using the Global Spatial Data Model (GSDM) in Plane Surveying

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Editor's Note: This is the second in a series of three articles on the global spatial data model (GSDM). The articles are The Global Spatial Data Model—A Tool Designed for Surveyors, Using the Global Spatial Data Model (GSDM) in Plane Surveying *and* Positional Tolerance Made Easier with the GSDM.

The global spatial data model (GSDM) is an arrangement of equations that facilitates 3D coordinate geometry computations.This article describes using those equations in two broad categories—traversing and inversing in the context of the GSDM.

As shown at the top of Figure 1, the 3D position of a point is defined by its geocentric X/Y/Z coordinates. From there, geocentric coordinates can be converted to geodetic latitude/longitude/height or, more conveniently for surveyors, geocentric coordinate differences can be converted to local coordinate differences.This geocentric connection preserves the "big picture" advantages of working in a standard, universal system while providing users the luxury of working with local rectangular components. Figure 2 shows the rectangular X/Y/Z geocentric coordinate system superimposed upon the more conventional geodetic latitude/longitude/height coordinate system, and Figure 3 shows a local coordinate system whose origin is any "standpoint" selected by the user.

Equations and Computations

Equations for moving mathematically from one box to another in Figure 1 can be found in geodesy text books or obtained from your GPS vendor. Equations are also listed in the Technical Papers of the 1994 American Congress on Surveying and Mapping (ACSM) Convention in an article I wrote entitled, *Exploiting the Connection Between Plane Surveying and GPS Vectors*. As stated there and below, traversing and inversing in the geocentric coordinate system using rules of solid geometry is quite straight-forward.

- A 3-dimensional traverse is accomplished by:

$$X_2 = X_1 + \Delta X \quad (1)$$
$$Y_2 = Y_1 + \Delta Y \quad (2)$$
$$Z_2 = Z_1 + \Delta Z \quad (3)$$

- A 3-dimensional inverse is based upon:

$$\Delta X = X_2 - X_1 \quad (4)$$
$$\Delta Y = Y_2 - Y_1 \quad (5)$$
$$\Delta Z = Z_2 - Z_1 \quad (6)$$

X/Y/Z coordinates are an efficient way to store spatial information in a database, and use of coordinate differences enhances computational integrity. But human perception of spatial data dictates continued use of local horizontal and vertical spatial data components. Therefore, a conversion between local and geocentric rectangular components is needed. The conversions based upon the latitude/longitude position of Point 1 are:

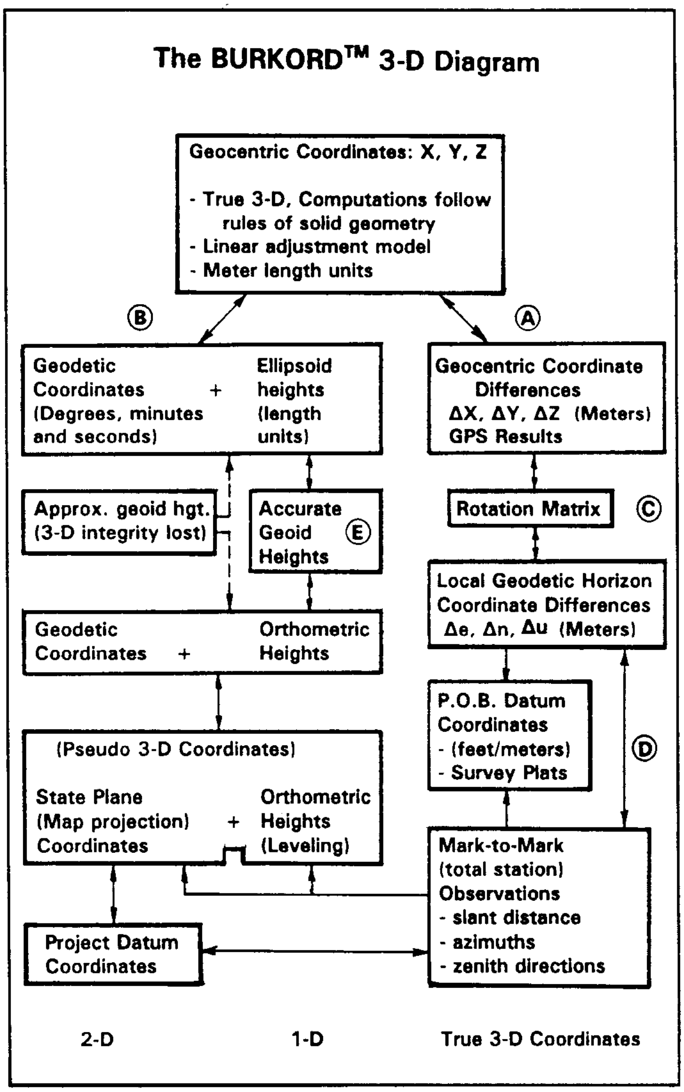

Figure 1. The 3D Diagram.

1. Local coordinate differences (used in 3D plane surveying) are obtained from geocentric components using:

$$\Delta e = -\Delta X \sin\lambda_1 + \Delta Y \cos\lambda_1 \quad (7)$$

$$\Delta n = -\Delta X \sin\phi_1 \cos\lambda_1 - \Delta Y \sin\phi_1 \sin\lambda_1 + \Delta Z \cos\phi_1 \quad (8)$$

$$\Delta u = \Delta X \cos\phi_1 \cos\lambda_1 + \Delta Y \cos\phi_1 \sin\lambda_1 + \Delta Z \sin\phi_1 \quad (9)$$

2. And, geocentric components are computed from 3D local

plane surveying components using:

$$\Delta X = -\Delta e \sin\lambda_1 - \Delta n \sin\phi_1 \cos\lambda_1 + \Delta u \cos\phi_1 \cos\lambda_1 \quad (10)$$

$$\Delta Y = \Delta e \cos\lambda_1 - \Delta n \sin\lambda_1 \sin\phi_1 + \Delta u \cos\phi_1 \sin\lambda_1 \quad (11)$$

$$\Delta Z = \Delta n \cos\phi_1 + \Delta u \sin\phi_1 \quad (12)$$

Traverse: Using total station measurements (mark-to-mark), local rectangular components from Point 1 (standpoint) to Point 2 (forepoint) are computed as:

$$\Delta e = \text{slope distance} * \sin(\text{zenith}) * \sin(\text{azimuth}) \quad (13)$$

$$\Delta n = \text{slope distance} * \sin(\text{zenith}) * \cos(\text{azimuth}) \quad (14)$$

$$\Delta u = \text{slope distance} * \cos(\text{zenith}) \quad (15)$$

These equations are readily recognized as 3D plane surveying computations for latitude, departure and vertical differences. (Note that without a curvature and refraction correction, the Δu component is not an elevation difference. Equation 15 gives the perpendicular distance from the forepoint to the tangent plane through the standpoint which is the correct quantity to use in3D traversing).

Plane surveying measurements are reduced to local rectangular components using equations 13-15. Those local components are converted to geocentric components using equations 10-12. Finally, X/Y/Z positions of Point 2 are computed using equations 1-3. Traversing with GPS vectors is done directly with equations 1-3 because GPS base lines are already defined by their geocentric $\Delta X/\Delta Y/\Delta Z$ components.

Inverse: With the 3D position of each point defined by its geocentric X/Y/Z coordinates, local rectangular components between points are readily obtained using equations 4-6 to obtain geocentric $\Delta X/\Delta Y/\Delta Z$ coordinate differences. Then, equations 7-9 are used to compute local plane surveying components of $\Delta e/\Delta n/\Delta u$. From there,

$$\text{horizontal distance (HD)} = \sqrt{(\Delta e^2 + \Delta n^2)} \quad (16)$$

$$\text{3D azimuth, standpoint to forepoint} = \arctan(\Delta e/\Delta n) \quad (17)$$

$$\text{elevation difference} = \Delta u + \text{curvature \& refraction correction} \quad (18)$$

where c&r (in meters) = $0.0675\,(HD/1{,}000)^2$

Notes about the inverse quantities:

1. The horizontal distance is a local tangent plane distance and is the same horizontal distance plane surveyors have been using for generations.

2. The 3D azimuth is often taken to be the same as the geodetic azimuth between points. And, because the meridians through the forepoint and standpoint are not parallel (unless they are on the same meridian), the azimuth from Point 1 to Point 2 differs from the reverse azimuth by 180° plus the convergence between points.

3. The elevation difference in equation 18 does not include geoid height differences and relies upon correctness of the curvature and refraction approximation. The approximation is generally adequate for distances less than 1000 meters. Precise elevation differences can be obtained (see left side of Figure 1) using ellipsoid heights and properly modeled geoid height differences.

Summary

These procedures bridge the gap between computer databases and surveyors' use of spatial data, accommodate either conventional survey data or GPS data, are equally applicable world-wide, are based upon rules of solid geometry (some geodesy is needed to understand equations 19 and 20), preserve true 3D geometrical integrity of spatial data, do not distort horizontal distance (as does the conformal mapping model), combine horizontal and vertical data into a single 3D database, allow computation of elevation differences using curvature and refraction corrections and direct computation of elevations using geoid modeling (they are different), and are compatible with error propagation techniques used in computing positional tolerances.

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See Example on page 42.

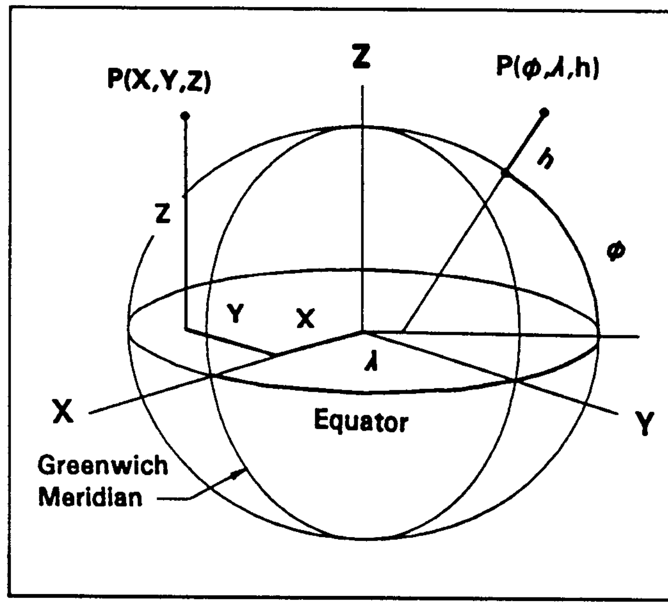

Figure 2. Rectangular and geodetic coordinate systems

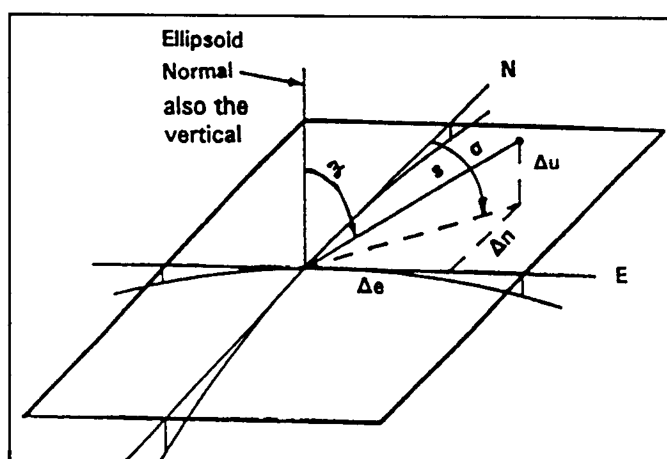

Figure 3. Local coordinate system.

Example

Given: The following points are all part of the GPS network located at the Oregon Institute of Technology (all length units are in meters).

Station K-785	X: -2,490,977.0490 Y: -4,019,738.1880 Z: 4,267,460.3830	Lat: 42° 15' 16.99289" Long: -121° 47' 09.35425"	

<u>Traverse - conventional:</u>

K-785 to Pub	Slope dist: 595.3002 Zenith dir: 86° 09' 50.2" Azimuth: 34° 11' 06.9"	Δe: 333.7322 Δn: 491.3440 Δu: 39.8267	ΔX: 442.1859 ΔY: 79.9921 ΔZ: 390.4551
Station Pub	X: -2,490,534.8631 Y: -4,019,658.1959 Z: 4,267,850.8381	Lat: 42° 15' 32.91354" Long: -121° 46' 54.79687"	

<u>Traverse - GPS:</u>

K-785 to Trimble	ΔX: 122.5471 ΔY: 56.9460 ΔZ: 131.0224		
Station Trimble	X: -2,490,854.5019 Y: -4,019,681.2420 Z: 4,267,591.4054	Lat: 42° 15' 22.59642" Long: -121° 47' 06.11902"	

<u>Inverses (Computed from X/Y/Z values listed above):</u>

K-785 to Trimble	ΔX: 122.5471 ΔY: 56.9460 ΔZ: 131.0224	Δe: 74.1716 Δn: 172.9331 Δu: 4.4968	c&r: 0.0024
	Horizontal distance Azimuth Difference in Elev.	= 188.1682 = 23° 12' 52.8" = 4.4992	
Trimble to Pub	ΔX: 319.6388 ΔY: 23.0461 ΔZ: 259.4327	Δe: 259.5635 Δn: 318.4072 Δu: 35.3415	c&r: 0.0114
	Horizontal distance Azimuth Difference in Elev.	= 410.7997 = 39° 11' 12.1" = 35.3529	
Pub to K-785	ΔX: -442.1859 ΔY: -79.9921 ΔZ: -390.4551	Δe: -333.7534 Δn: -491.3251 Δu: -39.8820	c&r: 0.0238
	Horizontal distance Azimuth Difference in Elev.	= 593.9627 = 214° 11' 16.7" = -39.8582	

Notes with regard to the example:

1. Geometrical integrity of the example is verified by the zero summation of ΔX/ΔY/ΔZ components around the loop. However, due to meridians not being parallel and due to Earth's curvature, the traditional latitudes, departures and elevation differences do not add up exactly to zero.

2. If the elevation of one station is known (either NGVD 29 or NAVD 88), the elevation differences obtained by applying the c&r corrections can be used to compute the elevation of other stations.

3. Computation of geodetic longitude from geocentric X/Y/Z coordinates is straight-forward (remember west longitude is negative):

$$\lambda = \arctan\left(\frac{Y}{X}\right) \tag{19}$$

4. Computation of latitude and height from geocentric coordinates involves a rather lengthy computation (see 1994 ACSM paper) or iteration of:

$$\tan\phi = \frac{Z}{\sqrt{X^2 + Y^2}}\left(1 + \frac{e^2 N \sin\phi}{Z}\right); \qquad (20)$$

$$h = \frac{\sqrt{X^2+Y^2}}{\cos\phi} - N; \qquad (20)$$

$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}} \qquad (20)$$

Where ϕ = geodetic latitude, X/Y/Z are the geocentric coordinates, N = ellipsoid normal, h = ellipsoid height, a = ellipsoid semi-major axis, and e^2 = ellipsoid eccentricity squared. Assume $\phi = 0$ for first iteration and continue using up-dated values of ϕ until the change of latitude from one computation to the next is inconsequential. ■

Positional Tolerance Made Easier With the GSDM

Earl F. Burkholder, LS, PE

The global spatial data model (GSDM) was described in the first article of this three-part series and 3D coordinate geometry equations for using the GSDM were in the second article. Now I will describe how positional tolerance and standard deviations can be computed in the context of the GSDM. A comprehensive description is not possible in this limited space, but we can explore the overall concept, describe the pieces, examine how positional tolerance computations can be made and identify sources of additional information. For interested readers, free prototype 3D coordinate geometry and error propagation software can be downloaded from *www.lsrp.com/burkind.html*

First let me review some of the issues related to positional tolerance computations:

- Equations for performing 3D coordinate geometry were included in the second article of this series and are called the **functional model** portion of the GSDM.
- Equations for performing error propagation and standard deviation (positional tc' ance) computations are called the **stochastic** model portion of the GSDM. The words "functional" and "stochastic" have specific meanings for mathematicians but they are used here in a simpler context to distinguish between coordinate geometry (COGO) computations and positional tolerance (standard deviation) computations.
- Equations for positional tolerance computations can be lengthy and complex unless stated in matrix form. Even so, the underlying procedures can be complicated and intimidating. The goal here is to present valuable, correct information as simply as possible by describing the essential pieces and by providing access to software that can be used to compute positional tolerances and standard deviations.
- For those interested in greater detail, the theory of error propagation is covered in Chapter 4 of Professor Edward Mikhail's book *Observations and Least Squares* and other surveying textbooks on adjustments. Details for applying the stochastic model to the GSDM are described in Appendix B-2 of "Definition of a Three-Dimensional Spatial Data Model for Southeastern Wisconsin" which is available from the Southeastern Wisconsin Regional Planning Commission, Waukesha, Wisconsin, 53187, telephone 414/547-6721.
- The GSDM stores both functional model data (geocentric X/Y/Z coordinates) and stochastic model information (the covariance matrix) for each point. This article describes use of the stochastic model for making standard deviation (positional tolerance) computations. The previous article described using the functional model for making 3D coordinate geometry computations.
- The GSDM really includes two covariance matrices for each point. Values for the geocentric covariance matrix are stored in the data base along with the X/Y/Z coordinates for each point. Computerized manipulation of digital spatial data is more efficient in the geocentric rectangular coordinate environment, but the local covariance values are also needed because humans perceive and intuitively relate to standard deviations in the horizontal/vertical modes. Local standard deviations are computed as needed using the stored X/Y/Z values and the geocentric covariance matrix. In symbols, the two matrices are:

Geocentric Covariance Matrix

$$\Sigma_{X/Y/Z} = \begin{bmatrix} \sigma_X^2 & \sigma_{X/Y} & \sigma_{X/Z} \\ \sigma_{X/Y} & \sigma_Y^2 & \sigma_{Y/Z} \\ \sigma_{X/Z} & \sigma_{Y/Z} & \sigma_Z^2 \end{bmatrix};$$

Local Covariance Matrix

$$\Sigma_{e/n/u} = \begin{bmatrix} \sigma_e^2 & \sigma_{e/n} & \sigma_{e/u} \\ \sigma_{e/n} & \sigma_n^2 & \sigma_{n/u} \\ \sigma_{e/u} & \sigma_{n/u} & \sigma_u^2 \end{bmatrix}$$

Notes about the covariance matrices:

- The covariance matrices are symmetric which means there will be no more than 6 unique elements in each one.
- In either case, a standard deviation is computed as the square root of the diagonal element in each matrix. In one case the standard deviations are perpendicular/parallel to the geocentric X/Y/Z coordinate system. In the other case, the standard deviations are perpendicular/parallel to the local e/n/u coordinate system.
- Correlation between coordinates is tracked by the off-diagonal elements. The coordinates are independent if, and only if, the off-diagonal elements are zero. Simultaneous independence in both systems is possible, but independence in the local system does not imply independence in the geocentric system and vice versa. (Programmers need this information more than the rest of us do.)
- The two covariance matrices are mathematically related to each other by a rotation matrix for the latitude/longitude position of a point as computed from its X/Y/Z coordinates (see previous article). The rotation matrix is:

$$R = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix}$$

and the matrix relationship between the covariance matrices is:

$$\Sigma_{X/Y/Z} = R\,\Sigma_{e/n/u}\,R^T$$

$$\Sigma_{e/n/u} = R^T\,\Sigma_{X/Y/Z}\,R$$

Notes about the rotation matrix:

Geodetic longitude in the rotation matrix is counted 0° to 360° east from the Greenwich Meridian. West longitude is a negative value.

Latitude is counted positive north of the equator and negative south of the equator.

- Storing the 3D geocentric covariance matrix for each point means horizontal and vertical survey data can be stored in the same 3D data base. The GSDM preserves the 3D geometrical integrity of the data and competently describes spatial data accuracy component by component.
- Standard deviations of individual components are the basis of error ellipse computations/plots.

Using the GSDM for Positional Tolerance Computations

Procedures for using the GSDM are listed for two categories: putting survey data, including standard deviations, into the data base, and using information stored in the data base to compute quantities such as bearings and distances. More important, standard deviations of local components, coordinate differences, azimuths and distances are routinely printed with each inverse. Existing prototype software called BURKORD™ can be used in both cases.

Following are the procedures for storing coordinates and standard deviations:

- The X/Y/Z coordinates or latitude/longitude/height values are entered to define the location of a point. The geocentric covariance matrix for the defined point is established by one of the following methods:
    - No standard deviations are available or used. The default covariance values are zero and the point is used as a "fixed" point.
    - The user inputs standard deviations component by component either in the local reference frame or in the geocentric reference frame.
    - The user inputs the entire point covariance matrix in either coordinate system.
- The X/Y/Z coordinates are computed as the result of a 3D traverse or network adjustment, using either local coordinate differences (total station survey data), geocentric coordinate differences (GPS vectors) or geodetic coordinate differences (such as traversing a parallel of latitude). Regardless of the computational mode used, the GSDM permits a user to input no standard deviations, standard deviations of the observations or, in the case of GPS data, the entire covariance matrix of the GPS vector. The covariance matrix of each new point is computed using the covariance matrix of the control point(s), the stochastic information provided by the user and formal error propagation procedures.

Positional Tolerances From GSDM Information

A GSDM database could be considered a digital terrain model (DTM), in that the location of each point in the database is defined by 3D coordinates. A GSDM database could be judged better than a DTM, in that it uses rectangular ECEF coordinates and stores the 3D spatial accuracy of each point in the geocentric covariance matrix. Solid geometry and error propagation equations for using GSDM data base information are universal world-wide.

Specific procedures used for computing the standard deviation of the direction and distance between two points stored in a GSDM data base are:

- The user selects two points stored in the data base. Points in a BURKORD™ data base are identified by point numbers.
- Geocentric coordinate differences and local coordinate differences between the points are computed using the equations given in the second article of this series. The inverse direction and distance are found using the e and n components.
- The covariance matrices of the geocentric coordinate differences and the local coordinate differences are found using error propagation equations which are listed in the Wisconsin 3D report.
- Standard deviations of the horizontal distance and the azimuth between selected points are computed from the covariance matrix of the local coordinate differences.
- Although the following information may not all be needed, the output of each inverse includes:
    - Geocentric coordinates of each point
    - Geodetic coordinates for each point
    - Local east/north/up standard deviations at each point
    - Geocentric coordinate differences and their standard deviations
    - Local coordinate differences and their standard deviations
    - Local tangent plane direction and distance between points along with the standard deviation of each.

Elevations Qualified

My goal is to provide correct information that is also complete. Given the limited space, completeness is sacrificed before correctness. Even so, I would be remiss to end this discussion without mentioning elevation standard deviations. Standard deviations of the local "up" components as defined by the GSDM are correct, but the "up" components differ slightly from elevation differences as described in equation (18) of the second article in this series. The uncertainty of the curvature and refraction correction needs to be combined with the "up" component uncertainty to obtain a standard deviation of the elevation difference.

Although the curvature and refraction method may be sufficient for localized use, a better method of obtaining elevations from the GSDM requires knowledge of geoid heights and/or geoid height differences. The GSDM defines elevation as the difference of ellipsoid height minus geoid height (see letter "E" in Figure 1 of the second article in the series). The challenge for users is to obtain ellipsoid heights and geoid heights with small standard deviations. GPS technology has made it possible to determine ellipsoid heights with small standard deviations and, as evidenced by publication of GEOID93 and GEOID96, enormous progress has been made in modeling geoid heights. The GSDM is valid and can be used even if the available data has large standard deviations. The quality of answers obtained using the GSDM depend upon the quality of data input. Ultimate benefits for the user community will be realized as better geoid models become available. ■

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