1	By Comparison – the GSDM is "Simple"
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0 7	<i>Value necessity may be the mother of invention</i> ,
8	Luziness is the futher of invention. unknown
9	Abstract
10	
11	The spatial data domain includes location as defined by the geometry of points, lines, planes, and sur-
12	faces. The term "spatial data" is generic. Geospatial data are those referenced to the Earth. Given the
13	hierarchy of reference systems, spatial data are taken to be a sub-set of geospatial data because spatial
14	data do not specifically include Earth's curvature. The mathematical ellipsoid is the underlying model for
15	geometrical geodesy and geospatial data referencing. Map projections are used to provide a connection
16	between geometrical geodesy and a flat-Earth perspective preferred for many applications. Consequently,
I7	ingenious map projections have been developed for accommodating both integrity and flat-Earth
18	convenience. The drawback, mitigated by software tools for handling complex equations, is that
19 20	norizontal and vertical are referenced to disparate origins. With advent of the digital revolution, spatial data are characterized as digital and 2 D. The 2 D global spatial data model (CSDM) uses a single origin for
20 21	data are characterized as digital and 3-D. The 3-D global spatial data model (GSDW) uses a single origin for
$\frac{21}{22}$	(ECEE) reference system for manipulating geospatial data. In using a 3-D model for 3-D geospatial data
$\frac{22}{23}$	geometrical integrity is preserved and by comparison 3-D equations for manipulating spatial data are
24	less complicated than those used in traditional methods.
25	
26	Keywords: Spatial data, geospatial data, Earth-centered Earth-fixed (ECEF), global spatial data model
27	(GSDM), map projection, conformal, and spatial data accuracy
28	
29	Stipulation
30	
31	To avoid being misinterpreted, laziness is taken to be the motivation to achieve better results with less
32	effort. Slothfulness is excluded, and no gender implication is intended. Throughout history, many talented
33	persons have devised methods to achieve some stated objective. Some of those methods involve rather
34 25	sophisticated mathematical concepts and, separately, some of the same methods include assumptions
35 36	that were needed to assure success. Specifically, concepts of geometry, location, and mapping have
37	and respect the contributions of our predecessors and to acknowledge input from current professionals
38	For the most part, the processes and procedures developed for handling spatial data have reached an
39	impressive level of sophistication and efficiency. They work! And they serve the needs of the spatial data
40	community as an integral part of the global economy which is measured in trillions of dollars! Question –
41	is it being lazy to compare traditional methods and results with those that can be achieved by starting
42	with the assumption of a single origin for 3-D data and building a spatial data model on rules of solid
43	geometry? Yes, demonstrated efficiencies of an integrated model (both conceptual and computational)
44	will ultimately justify and motivate transition to using a 3-D model for 3-D data worldwide.
45	
46	Conventions
47	
48	Instead of referring to a 3-D position as geodetic latitude, geodetic longitude, and ellipsoid height, this

49 paper uses geodetic latitude, geodetic longitude, and geodetic height. Humans have referred to the third

50 dimension in terms of elevation, altitude, orthometric height, ellipsoid height, and dynamic height - each 51 with good reason. Going forward, this convention completes the triplet of coordinates – geodetic 52 latitude/longitude/height. Mathematically well-defined, geodetic height is synonymous with ellipsoid 53 height as being the distance along the normal between the ellipsoid and a point. The Geodetic Glossary 54 (NGS 1986) and the Glossary of the Mapping Sciences (ASCE/ACSM/ASPRS 1994) each include a definition 55 for "height, geodetic" (Meyer 2021). 56 57 The following conventions are not new but are provided for clarity. Multiple designations, often used 58 interchangeably, are encountered when talking about the location of points. 59 60 Point 1, standpoint, and "here" refer to a station as occupied – physically or "in one's mind." 61 Point 2, forepoint, and "there" refer to a station at the other end of a line. 62 P.O.B. is a Point of Beginning as selected by the user, often taken to be Point 1. 63 64 Rectangular components of a vector are called "deltas" and computed as the difference of coordinate values - "there" minus "here." The length of a vector does not change when viewed from a different 65 66 perspective although the rectangular components do change with a change in orientation. 67 $\Delta X = X_2 - X_1 \quad \Delta Y = Y_2 - Y_1 \quad \Delta Z = Z_2 - Z_1$ $\Delta e = e_2 - e_1 \quad \Delta n = n_2 - n_1 \quad \Delta u = u_2 - u_1$ 68 Geocentric vectors: 69 Local perspective vectors: 70 3-D spatial distance = $SD = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2} = \sqrt{\Delta e^2 + \Delta n^2 + \Delta u^2}$ 71 (1) 72 73 Introduction

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75 Assertion – the 3-D global spatial data model (GSDM) defines computational processes for manipulating 76 spatial (and geospatial) data that are less complicated and more efficient than traditional methods 77 without sacrificing geometrical integrity. Geometrical geodesy equations utilize directions and distances 78 on the mathematical ellipsoid in terms of latitude and longitude. The third dimension is height above or 79 below the ellipsoid. One element of complexity is that latitude and longitude are expressed with 80 curvilinear units (radians or sexagesimal in degrees-minutes-seconds) while geodetic height is expressed 81 in meters. Map projections are used to "flatten the Earth" and enable spatial data users to work with 82 plane Euclidean geometry (length units) for horizontal position. Elevation describes the third dimension. 83 Although units associated with map projections are linear, 3-D mathematical compatibility for 84 manipulating spatial data suffers from the use of two separate origins – one for horizontal, another for 85 vertical. True, 2-D mathematical rigor for geometrical geodesy and map projection equations has been 86 preserved for horizontal positioning by promulgating high-quality bi-directional algorithms which have 87 been published, have been programmed, and are readily available in software tools for spatial data 88 conversions. To be consistent with conventions used by the National Geodetic Survey (NGS), "conversion" 89 describes computing plane coordinates from latitude and longitude and "transformation" describes 90 moving geospatial data from one datum to another. 91

The primary disadvantage of map projections is that they are strictly 2-D mathematical models while modern practice routinely works with 3-D spatial and geospatial data. A secondary disadvantage of map projections is that horizontal distances are distorted when projected to a map, giving an unsettling realization that "a meter is no longer a meter." In practice, distance distortions are controlled within some predetermined tolerance by using a low-distortion projection (LDP) which places the mapping surface near the average elevation of the area being mapped. The drawback to using LDPs is that, to stay within prescribed distortion tolerances, the effective horizontal area covered by a given projection is limited and

99 100	multi proje	ple zones are needed to map larger areas. Additionally, the geometrical integrity of any given map
101	estab	plished tolerance limits. Advantages of the 3-D GSDM over 2-D map projection procedures include:
102	0000	
103	1.	The GSDM preserves 3-D geometrical integrity – the model does not distort distances.
104	2.	The GSDM uses one set of public domain solid geometry equations worldwide.
105	3.	Subsequent geometrical elements are derived from stored X/Y/Z coordinate values.
106	4.	No zone constants, no grid scale factors, and no elevation factors are needed or used.
107	5.	GSDM equations are easily programmed with modest programming skills.
108	6.	A rotation matrix converts geocentric differences to local differences, $\Delta X / \Delta Y / \Delta Z \rightarrow \Delta e / \Delta n / \Delta u$.
109	7.	The GSDM provides local ground level horizontal distance computed as $HD = \sqrt{\Delta e^2 + \Delta n^2}$.
110	8.	The 3-D azimuth is true north with respect to the standpoint meridian, α_{3D} = arctan ($\Delta e/\Delta n$).
111	9.	A back azimuth is obtained by computing in reverse – from "there" to "here."
112	10.	Meridian convergence is found as the difference between forward and back azimuths - 180°.
113		
114	Chara	acteristics of the GSDM include (Burkholder 1997a, 2008, 2018):
115		
116	1.	The GSDM is prefaced on the assumption of a single origin at Earth's center of mass (CM).
117	2.	The GSDM is built on the geocentric Earth-centered Earth-fixed (ECEF) reference system.
118	3.	The functional model portion of the GSDM utilizes rules of solid geometry as formulated by René
119		Descartes in 1637 and supplemented by enhancements that include matrices and vector algebra.
120	4.	The GSDM enables the user to "view the world" (or a point cloud) from any X/Y/Z location and
121		provides local direction and distance from that user selected P.O.B. to any other point.
122	5.	A local plot of $\Delta e/\Delta n$ pixel locations with respect to a user-selected P.O.B. is an orthophoto map.
123	6.	The stochastic model portion of the GSDM embodies the standard error propagation procedure:
124		
125		$\Sigma_{YY} = J_{YX} \Sigma_{XX} J_{XY}^{L} \qquad \text{where} \qquad (2)$
126		
127		Σ_{YY} = Covariance matrix of computed result.
128		Σ_{XX} = Covariance matrix of variables used in computation.
129		J_{YX} = Jacobian matrix of partial derivatives of the result with respect to the variables.
130	~	
131	Com	DIEXITY
132	Com	
133	Some	a may justifiably take issue with describing existing geometrical concepts and computational
134	botto	edures as complex. The reader is reminded that the goal of this article is to show how similar (of
135	using	solid geometry equations in the restangular ECEE environment. The CSDM equations are not as
130	comr	solid geometry equations in the rectaingular ECEP environment. The GSDW equations are not as
137	comp	siex as those used to perform computations on the empsoid of using 2-D map projection equations.
130	Intog	rity of Algorithms
140	integ	
140	Integ	rity is a carefully laid foundation upon which a reputation for quality professional services is built
142	The i	ntegrity of existing algorithms stands as a tribute to the talented mathematicians and other
143	profe	essionals over the years who have developed and tested many procedures for manipulating spatial
144	data.	One consequence is that, of necessity, simplicity is sacrificed to achieve a required level of integrity.
145	Not t	o worry computers and software tools make it possible for sophisticated results to be obtained at
146	the p	ush of a button. Although integrity and simplicity can be mutually exclusive, the goal of this paper is
147	to sh	ow how the geometrical integrity of geospatial data can be preserved while using (simple) solid

geometry algorithms. Finding an appropriate balance between simplicity and integrity remains a challenge
 but, comparatively speaking, the geospatial data user can enjoy both when using the 3-D GSDM.

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- Plane Surveying: In plane surveying, a "forward" traverse computation uses direction and distance from a standpoint to compute coordinates of the forepoint. An "inverse" computation between forepoint and standpoint should return the same direction and distance as used in the "forward" computation. This example is trivial because the model for computations is a 2-D plane surface.
- 155
 2. Geodetic Surveying: In geodetic surveying, similar concepts and nomenclature are used in the
 "direct" and "inverse" computations but the process is less trivial the underlying computational
 model is the mathematical ellipsoid, not a plane surface. Methods of varying complexity have been
 devised over the years for performing geodetic "direct" and "inverse" computations. When
 choosing a method for computing a geodetic "direct" or "inverse," the reader should be alert to a
 possible trade-off between integrity and complexity embodied within a given algorithm. Several
 ellipsoid-based examples (there are others) include...
- 163
- a. Traditional methods for "direct" and "inverse" on the ellipsoid.
- 164 165
- i.) Thomas (1970) is a high-level comprehensive discussion of geodesics.
- ii.) Vincenty (1975) is also high-level and utilizes nested iterative algorithms.
- 166 167
- 167
- 168

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171

- iv.) Bowring (1981) presents a "simple" method for lines up to 150 km.v.) Pittman (1986) introduces a rigorous recursive method for any length line.
- vi.) Rollins (2010) adapts meridian arc length to a geodesic with a change in variable.

iii.) Jank and Kivioja (1980) is a rigorous iterative solution for programmable calculators.

- vii.) Meyer (2010) gives history of and describes trade-offs for various methods.
- 172 173 b. The 3-D solution (Burkholder 2016a) for geodetic "direct" and "inverse" problems is different 174 from ellipsoid-based methods in that computations are performed in 3-D space and not 175 restricted to the surface of the mathematical ellipsoid. Based on 3-D solid geometry equations, 176 there are no mathematical approximations in the GSDM algorithm. But there is an 177 approximation (by choice of user) if/when a chord or an arc length computed by the GSDM is 178 used rather than a more rigorously defined horizontal distance (Rollins and Meyer 2019). This 179 puts the user in control over the balance between simplicity and integrity. As Burkholder 180 (2016a) notes, comparison of an arc distance with a known geodesic length shows agreement 181 within 0.5 mm on a 50 km line. If needed, an improvement to a computed arc distance to find 182 the true geodesic distance can be realized using (more complicated) procedures as given in 183 Burkholder (2008, 2018). By comparison, the GSDM inverse is "simple." Additionally, the 184 GSDM supports computation of geometrical elements such slope, horizontal, or arc distance -185 at any specified elevation. The veracity of a GSDM inverse, say beyond 50 km or a user-186 selected threshold, can be found by comparing GSDM results with the results of a more 187 rigorous method. Note, a 3-D point is on the ellipsoid if its X/Y/Z coordinates are computed 188 using zero geodetic height (h = 0.000 m), i.e., ϕ , λ , $0.0 \rightarrow X/Y/Z$.
- 189
 3. Map Projections: When using a map projection, geodetic latitude and longitude are converted to plane coordinates using equations developed for the given projection and zone. Complexities of the bi-directional conversion equations are driven, in part, by making the projection "conformal." It is impossible to preserve all three geometrical elements of angles, distances, and area when projecting from a curved surface to a flat map. But when using a conformal projection, it is possible to constrain the conversion equations mathematically so that an angle on the ellipsoid is the same as the angle on the map. A conformal projection is an ideal candidate for surveying, engineering,

197	and mapping applications because angles are not distorted, and horizontal distance distortions are
198	kept within some predetermined tolerance. Mathematically, a projection is conformal if and only if
199	the bi-directional conversion equations satisfy the Cauchy-Reimann differential equations.
200	
201	This is where simplicity becomes secondary to rigor and complexity. According to Wikipedia (2021),
202	
203	"In the field of complex analysis in mathematics, the Cauchy–Riemann equations, named after
204	Augustin Cauchy and Bernhard Riemann, consist of a system of two partial differential
205	equations which, together with certain continuity and differentiability criteria, form a
206	necessary and sufficient condition for a complex function to be complex differentiable, that is,
207	holomorphic. This system of equations first appeared in the work of Jean le Rond d'Alembert
208	(d'Alembert 1752). Later, Leonhard Euler connected this system to the analytic functions (Euler
209	1797). Cauchy (1814) then used these equations to construct his theory of functions.
210	Riemann's dissertation (Riemann 1851) on the theory of functions appeared in 1851."
211	
212	a. Complexity notwithstanding, conformal projections for the U.S. state plane coordinate system
213	have a long successful history of mathematical development briefly summarized as:
214	
215	i.) Gerard Mercator (Crane 2002) and (Taylor 2004) is credited with publishing the first
216	conformal world map in 1569. The unique feature of his world map was that a navigator
217	could draw a line port-to-port on the map and sail the constant bearing of that line to a
218	distant port. It was not the shortest route port-to-port, but that simple procedure
219	provided a reliable method for reaching the intended destination.
220	
221	ii.) Johann Heinrich Lambert (1728-1777) is credited with inventing the transverse
222	Mercator projection and the Lambert conic conformal projection (Snyder 1989). The
223	conformal projection specifically enforces the condition that an angle on the globe is
224	projected without distortion to the map. Early justification for the conformal map
225	included the ability to sail port-to-port on a constant bearing, as obtained from the
226	map. Mathematicians such as Gauss (1777-1855), Krüger (1857-1923), and Hotine
227	(1898-1968) contributed to further development of conformal projections. Additional
228	sources can be found with a web search.
229	
230	iii.) In general, authors have used the following elements and symbols for the Cauchy–
231	Riemann equations and various latitudes.
232	
233	x = Easting on map projection.
234	v = Northing on map projection.
235	0 = geodetic latitude
236	$\hat{\lambda}$ = geodetic longitude east
230	$\gamma = conformal latitude$
237	χ = comotria latitude.
230	t = 100111etite latitude.
239	ω = rectrying latitude.
240	ϵ = ellipsoid eccentricity
241	
242 242	b. The Cauchy-Riemann equations for a conformal map projection (Snyder 1987 and others) are:
243	$\partial r = \partial v = \partial r = \partial v$
244	$\frac{\partial x}{\partial \lambda} = \frac{\partial y}{\partial \tau}$ and $\frac{\partial x}{\partial \tau} = -\frac{\partial y}{\partial \lambda}$ (3)

The conformal latitude and isometric latitude as used in mapping conversions are related as shown by Thomas (1952, page 86). The isometric latitude is computed using equation 4 and the conformal latitude by using equation 5. Note that the difference between the two equations is the natural log (In) function. Stem (1989, page 27) uses the isometric latitude in conversions for the Lambert conformal conic projection. The recommended solution is iterative (see Table 1), but a non-iterative solution can also be found using the conformal latitude and a series expansion of (φ - χ) as listed for conversions on the oblique Mercator projection (Stem 1989, page 42).

$$\tau = \ln \left[\tan \left(\frac{\pi}{4} + \frac{\emptyset}{2} \right) \left(\frac{1 - \epsilon \sin \emptyset}{1 + \epsilon \sin \emptyset} \right)^{\epsilon/2} \right]$$
(4)

$$\tan\left(\frac{\pi}{4} + \frac{\chi}{2}\right) = \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \left(\frac{1 - \epsilon \sin\phi}{1 + \epsilon \sin\phi}\right)^{\epsilon/2}$$
(5)

c. Among others, the (φ- χ) series expansion is part of the bi-directional conversions used by NGS (Stem 1989) for state plane coordinates. Adams (1921 reprint 1949) provides extensive development of various latitudes used in conformal mapping – given as an infinite series in terms of powers of ellipsoid eccentricity. For example, Adams (1949, pages 32 to 60) includes seven different derivations for the (φ- χ) infinite series in terms of the conformal latitude, χ. That series, shown in equation 7 below, is used in Stem (1989 page 39) as part of the oblique Mercator conversion equations.

$$\varphi - \chi = \left(\frac{\epsilon^2}{2} + \frac{5\epsilon^4}{24} + \frac{\epsilon^6}{12} + \frac{13\epsilon^8}{360}\right) \sin 2\chi + \left(\frac{7\epsilon^4}{48} + \frac{29\epsilon^6}{240} + \frac{811\epsilon^8}{11520}\right) \sin 4\chi + \left(\frac{7\epsilon^6}{120} + \frac{81\epsilon^8}{1120}\right) \sin 6\chi + \left(\frac{4279\epsilon^8}{161280}\right) \sin 8\chi$$
(6)

Assigning coefficients c₂, c₄, c₆, and c₈, equation 6 can be written more compactly as. . .

$$\varphi - \chi = c_2 \sin 2\chi + c_4 \sin 4\chi + c_6 \sin 6\chi + c_8 \sin 8\chi$$
(7)

To avoid working with trigonometric functions of multiple latitudes, the following substitutions were made and like terms collected to obtain equation 8.

277		
278	$\sin 2x = 2\sin x \cos x$	
279	$\sin 4x = 8 \sin x \cos^3 x - 4 \sin x \cos x$	
280	$\sin 6 x = 32 \sin x \cos^5 x - 32 \sin x \cos^3 x + 6 \sin x \cos x$	
281	$\sin 8 x = 128 \sin x \cos^7 x - 192 \sin x \cos^5 x + 80 \sin x \cos^3 x - 8 \sin x \cos x$	
282		
283	$\varphi - \chi = 2 c_2 \sin \chi \cos \chi$	
284	$-4c_4\sin\chi\cos\chi+8c_4\sin\chi\cos^3\chi$	
285	+ 6 $c_6 \sin \chi \cos \chi$ - 32 $c_6 \sin \chi \cos^3 \chi$ + 32 $c_6 \sin \chi \cos^5 \chi$	
286	$-8 c_8 \sin \chi \cos \chi + 80 c_8 \sin \chi \cos^3 \chi - 192 c_8 \sin \chi \cos^5 \chi + 128 c_8 \sin \chi \cos^7 \chi$	/
287		_
288	$\varphi - \chi = F_0 \sin \chi \cos \chi + F_2 \sin \chi \cos^3 \chi + F_4 \sin \chi \cos^5 \chi + F_6 \sin \chi \cos^7 \chi$ or nested.	
289		
290	$\varphi - \chi = \sin \chi \cos \chi \left(F_0 + \cos^2 \chi \left(F_2 + \cos^2 \chi \left(F_4 + F_6 \cos^2 \chi \right) \right) \right) $ (8)	

291			Where	Fo	=	$2(c_2-2c_4+3c_6-4c_8)$
292				F_2	=	$8(c_4 + 4c_6 + 10c_8)$
293				F_4	=	32 (c ₆ - 6 c ₈)
294				F_6	=	128 c ₈
295						
296		Stem	(1989, page 42) spe	cifically us	es e	quation 8 as part of the oblique Mercator inverse
297		algorit	thm. It is also offere	ed as an op	otion	ı (page 29) as part of the inverse on the Lambert
298		confo	rmal conic projectio	n. It is not	ed t	hat equation 5 can also be iterated to find geodetic
299		latituc	le from the conform	nal latitude	e. Th	ne accuracy of an iteration solution is determined by the
300		termir	nation criterion sele	cted by th	e us	er or (rarely) by the significant digit capacity of the
301		comp	uter being used. Iter	ration can	be c	quite efficient.
302						
303		Howe	ver, when using equ	uation 8, a	que	stion arises whether or not sufficient terms were
304		includ	ed in the infinite se	ries expres	ssed	in equation 6. Additional terms in the infinite series
305		will in	clude tenth powers	of ellipsoi	d ec	centricity. A class project performed for Professor
306		Ralph	Moore Berry docun	nented an	algo	prithm containing 10 th power terms in the (ω - χ) series –
307		see Bu	urkholder (1972). Ta	ble 1 inclu	ides	a summary of tests over a range of latitudes using the
308		eccen	tricity computed fro	om the rec	ipro	cal flattening of the GRS 80 ellipsoid. 1/f =
309		298.2	572221008827 Fou	r stens in t	the 1	tests include:
310		250.2	572221000027.100	i steps iii		
311		i)	Start with an "ever	n" value of	fger	odetic latitude and compute a value of conformal
312		,	latitude using the	closed for	n of	equation 5
313			latitude using the	closed for		
314		ii)	Use iteration to co	mnuto go	ndot	tic latitude from the conformal latitude as determined
315		,	in stop 1. The inter	arity of a d	ivon	method is readily apparent by noting any deviation
316			from the original y	yhon rotur	ning	to the starting value of geodetic latitude in seconds
317			nom the original v	viienretui	311118	
318		::: \	Lico oquation 8 to	computo t	ho c	readatic latitude and note any difference
310		···. <i>)</i>	Use equation a to	compute t	lie e	seduetic latitude and note any difference.
220		i)	Lico the equation i	n tha link	- h - ı	is to include the tenth newer term for accentricity
320		IV.)	Use the equation i	II the link	auuv	The to include the tenth power term for eccentricity.
321		T 1				(1, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3,
322		The va	alues in Table 1 Indi	cate that e	eignt	In powers of eccentricity (ε°) provide sufficiently
323		accura	ate conversions for	state plane	e co	ordinates. Although iteration is the "best possible"
324		metho	od, equation 8 provi	des result	s go	od to 0.0000004 seconds of arc or better (0.012 mm).
325		lable	1 also shows that ty	wo more n	nagn	litudes of accuracy can be achieved if additional terms
326		in the	infinite series and t	enth powe	ers c	of eccentricity (\mathcal{E}^{IO}) are included.
327						
328	d.	The pr	revious section look	ed at the I	Lam	bert conformal conic and the oblique Mercator
329		projec	ctions. This section l	ooks at th	e tra	insverse Mercator algorithm. The algorithm given by
330		Thom	as (1952) for the Me	ercator pro	oject	tion of the ellipsoid is straight forward and can readily
331		be sho	own to fulfill the Cau	uchy-Reim	ann	equations. But it is more of a challenge to verify that
332		the eq	luations given by Th	nomas (195	52) c	or those in Stem (1989) for projecting the ellipsoid using
333		a tran	sverse Mercator pro	ojection sa	tisfy	/ the Cauchy-Reimann equations. The equations given
334		by Ste	m (1989) for the tra	ansverse N	/lerc	ator projection and the Universal Transverse Mercator
335		(UTM)) projection have be	en thorou	ghly	tested and deliver excellent results. Comments on the
336		compl	lexity of those equa	tions are r	noot	t as conversion programs have been written (and are
337		readily	y available) for conv	version of g	geog	graphic latitude/longitude to plane coordinates
338		(forwa	ard) and plane coord	dinate con	vers	ion to latitude/longitude (inverse). For the reader

339 340

341

interested in pursuing the origins of the transverse Mercator projection, the equations included in Stem (1989) are built on the Gauss-Krüger algorithm.

Table 1 – Computing Geodetic Latitude from Conformal Latitude Comparing Results from Iteration and Powers of Eccentricity e⁸ and e¹⁰

PHI Deg	Given PHI in sec.	CHI Computed by Equation 4	PHI Computed by iteration	PHI Computed Using e 8th	PHI Computed Using e10th	e8th missed by	e10th missed by
5	18,000.000000000	17,880.1069220654	18,000.000000000	17,999.9999997728	17,999.9999999985	0.0000023	0.000000015
10	36,000.000000000	35,763.8270514390	36,000.000000000	35,999.9999996264	35,999.9999999978	0.0000037	0.000000022
15	54,000.000000000	53,654.6673609163	53,999.99999999999	53,999.9999996049	53,999.9999999979	0.0000040	0.0000000020
20	72,000.000000000	71,555.9251616317	71,999.9999999999	71,999.9999996988	71,999.9999999989	0.0000030	0.000000010
25	90,000.000000000	89,470.5902540013	90,000.000000000	89,999.999998538	90,000.0000000000	0.0000015	0.0000000000
30	108,000.000000000	107,401.2553499972	108,000.000000000	108,000.000000009	108,000.000000006	-0.00000000	-0.000000006
35	126,000.000000000	125,350.0373338391	126,000.000000000	126,000.000000896	126,000.000000008	-0.0000009	-0.000000007
40	144,000.000000000	143,318.5117432160	144,000.000000000	144,000.000001058	144,000.000000004	-0.0000011	-0.000000003
45	162,000.000000000	161,307.6625985220	162,000.000000000	162,000.000000703	162,000.0000000000	-0.0000007	0.000000000
50	180,000.000000000	179,317.8493738962	180,000.000000000	180,000.000000188	179,999.9999999997	-0.0000002	0.000000003
55	198,000.000000000	197,348.7924852802	198,000.000000000	197,999.9999999809	197,999.9999999997	0.0000002	0.000000003
60	216,000.000000000	215,399.5781671093	215,999.99999999999	215,999.9999999675	215,999.9999999998	0.0000003	0.000000002
65	234,000.000000000	233,468.6830278602	234,000.000000000	233,999.9999999726	233,999.99999999999	0.0000003	0.000000001
70	252,000.000000000	251,554.0179310941	252,000.000000000	251,999.9999999836	252,000.000000000	0.0000002	0.000000000
75	270,000.000000000	269,652.9901668963	270,000.000000001	269,999.9999999917	270,000.000000000	0.0000001	0.0000000001
80	288,000.000000000	287,762.5821902211	288,000.000000001	287,999.9999999955	288,000.000000000	0.00000000	0.000000001
	• • • • • • • • • • •			,		Note 3	Note 4

Note 2 - At 30.9 meters per second of latitude, 0.0000001 seconds is about 0.003 mm on the ground. Note 3 - This column shows sufficient accuracy of Special Publication #67 algorithm for state plane coordinate computations.

Note 4 - If needed, this column shows two magnitudes improvement over Special Publication #67 results.

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- e. Anticipating publication of the 2022 horizontal and vertical datums, NGS promotes continued use of map projections (with added LDP options) to be implemented as SPSC2022. Minimizing the difference between grid and ground distances on the projection is an overriding criterion for surveyors and engineers. Another conflicting criterion, attractive to the GIS community, is to adopt a map projection that provides a unique coordinate location for any point in a state. NGS is going to great lengths to serve both camps of users by designing and implementing map projections to include both criteria. Equations for performing mapping conversions and input parameters for the various zones are yet to be finalized but several possibilities include:
 - The conversion equations included in Stem (1989) for the NAD 83 can be modified to i.) accommodate input parameters proposed for the low-distortion projections. NGS is in the process of evaluating and adopting specific parameters to be used for SPCS2022 projections. Possible changes to existing NGS 5 algorithms are summarized in Burkholder (2020a). Those equations have been tested for integrity and can be shown to meet the conversion accuracy requirements established by NGS.
 - ii.) Rolling and Meyer (2019) focus on developing low-distortion projection equations for an elevated reference surface. A consequence of using an elevated reference surface with existing algorithms is that they fail to meet the strict definition of conformality. Rollins and Meyer (2019) include diagrams that show the amount of non-conformality for various values of constant-h surfaces for three of the four methods presented.
- 366 Question - does a LDP really need to be conformal? Or "What benefit does a conformal 367 projection provide?" For Mercator, the benefit of his 1569 World Map (later designated as 368 being "conformal") was that a sailor could plot a course port-to-port on a conformal map and 369 sail a constant bearing from one port to the other. That criterion is no longer critical as better 370 methods are readily available for navigation. Another consequence of a conformal projection is 371 that distance distortion at a point is azimuth independent – that is an angle on the ground is 372 portrayed without distortion on the map. When using the GSDM, there is no distance distortion 373 at a point meaning that the conformal criterion is moot. Investigation of the properties of a 3D

374 275		azimut	h (Burkholder 1997b) establishes the integrity of geodetic azimuths at a point and
276		subseq	uent angles between them.
370 277	£	Compo	ring the simplicity of the CCDM equations with more complex equations used in
270	١.	compa	ring the simplicity of the GSDM equations with more complex equations used in mal manning includes:
270		comon	na mapping includes.
319		: \	Coordinates on a conformal man projection are constrained to fulfill the conditions
381		1.)	coordinates on a comornal map projection are constrained to runni the conditions
387			supulated in equation 5. The bi-directional algorithms are mathematical, not graphical.
383		ii)	Mathematical reductions can be denicted graphically when using the GSDM
384		,	mathematical reductions can be depicted graphically when using the OSDM.
385		iii)	On the complexity scale, the GSDM geometrical procedures and equations are much
386			easier to follow than the rules imposed by the Cauchy-Reimann equations
387			casici to follow than the fules imposed by the eaderly Kelmann equations.
388		iv)	The integrity and adequacy of GSDM algorithms is well established Burkholder (2019b)
389		۱۷.)	The GSDM does not distort physical measurements and algorithms are easier to follow
390			The dobin does not distort physical measurements and algorithms are casier to follow.
391		V)	If appropriate software is used properly, the difference in complexity does not mean
392		•.,	that one system enjoys more integrity than the other.
393			that one system enjoys more integrity than the other
394		vi.)	For some, the ease of checking results of GSDM computations is preferred to placing
395		•,	blind trust in "button pushing" solutions. Admittedly, software that solves complex
396			problem exists and has been proven. Reputable software can be used with confidence.
397			
398	Reductio	n of Obs	ervations
399			
400	The equa	ations for	r map projection conversions discussed previously include some mathematical "heavy
401	lifting." A	Additiona	al issues more closely related to geometry of the survey are involved when using map
402	projectio	ns. Succ	essful use of state plane coordinates is predicated on using grid azimuths and grid
403	distances	s. The int	egrity issues of grid azimuths and distances as obtained from field observations are
404	separate	from the	e impact of assumptions (issues) built into the map projection algorithms.
405	-		
406	1. Grid	azimuth	A consequence of using a conformal projection is that angles on the map are the same
407	as th	ne observ	ved field angles - almost. On most conformal projections, no special steps are needed to
408	obta	in a grid	azimuth. The caveat is that when long distances are involved (say over 5 km), the
409	diffe	erence be	etween an angle and an azimuth could be an issue and a (t-T) correction may be needed
410	– se	e (Stem 2	1989) Figure 2.5 and Table 4.3a.
411			
412	Whe	en using t	the GSDM, the true north azimuth, called the 3D Azimuth by Burkholder (1997b), from
413	stan	dpoint to	o forepoint is computed directly as arctan ($\Delta e/\Delta n$). Computing the azimuth between
414	poin	ts is a sta	andard "simple" inverse computation. The angle at a standpoint, between two different
415	fore	points is	the same on the map as it is on the ground. The back azimuth from forepoint to
416	stan	dpoint is	computed the same way (in reverse) but gives a different result because the meridians
417	areı	not paral	lel at the standpoint and forepoint – an exception being two points on the same
418	mer	idian. Th	at means the local Δe and Δn components (as obtained from the rotation matrix) are not
419	thes	same fro	m standpoint to forepoint as from forepoint to standpoint. But note that forward and
420	inve	rse azim	uths are different by 180° when using Δe and Δn components from the P.O. B. as local
421	east	ings and	northings in the P.O. B. tangent plane.
422			

423		Added bonus: Due to Earth curvature, meridians are not parallel - convergence is the difference
424		between forward and back azimuth +/- 180°. Standard practice when using state plane coordinates is
425		to reference grid azimuths to the central meridian of the projection. Grid north lines on the
426		projection are parallel (an important feature for flat-Earth surveying) and, if a true-north azimuth is
427		required, convergence at a point is used to find the true (geodetic) azimuth. A similar practice can be
428		used with the GSDM in that the user chooses a P.O.B. – typically such that local Δe and Δn differences
429		are positive. Those local differences (whether positive or negative) can be used as local coordinates in
430		the tangent plane of the P.O.B. Just like plane surveying, an inverse between those local project
431		coordinates will provide a local tangent plane horizontal distance and a "local grid" azimuth. The
432		convergence concept is the same as when using a state plane (or LDP) projection. The only difference
433		when using the GSDM is that convergence (a measure of meridians not being parallel) is between the
434		local project point and the chosen P.O.B. – meaning that the user can choose between using true 3D
435		azimuths (as described in previous paragraph) or "local grid" azimuths on a plat. A detailed example
436		of using local grid azimuth is given in Burkholder (2007)
437		
438	2.	Grid distance: Conceptually, two separate operations are required to obtain a grid distance from a
439		horizontal ground distance: 1) reduction of the ground level horizontal distance to the mathematical
440		ellipsoid and 2) reduction of the ellipsoid distance to the mapping surface. Modern practice often
441		includes both steps in one operation (using the combined factor). Stem (1989) describes the steps
442		needed. A measured slope distance is reduced to horizontal, that horizontal distance is then reduced
443		to the ellipsoid, and finally, the ellipsoid distance is reduced to the mapping surface where it is known
444		as the grid distance. The reduction to ellipsoid is accomplished using the elevation factor and the
445		reduction from ellipsoid distance to grid distance is computed using the grid scale factor. The
446		elevation factor and the grid scale factor are often multiplied together and known as the combined
447		factor – making the reduction ground-to-grid a "one-step" process.
448		
449		Conventional computation of the combined factor – multiplying the elevation factor times the grid
450		scale factor – involves the following variables/questions:
451		
452		a. What is the appropriate elevation for computing the elevation factor?
453		b. What radius of Earth's curvature is to be used? This is generally not a problem.
454		c. Should the grid scale factor for the line to be computed at:
455		
456		i.) The standpoint?
457		ii.) The endpoint?
458		iii.) The midpoint?
459		iv.) Stem (1989) recommends Simpson's 1/6 Rule for long lines.
460		
461		An important goal in using a low-distortion projection is that the grid/ground difference is sufficiently
462		small as to be ignored for many applications. Yes, that is quite simple. But, for cases in which more
463		precision is needed, simplicity takes a back seat. Used by many and avoided by some, standard
464		procedures for computing appropriate combined and grid scale factors can be found in Stem (1989).
465		Burkholder (2004) provides additional information on the accuracy of a computed elevation factor.
466		
467	Cor	nments on the Combined Factor
468		
469	The	e integrity of a computed position (state plane or LDP coordinates) can be assured by careful
470	арр	plication of the procedures included in Stem (1989) and discussed above. Regretfully, the integrity of a
471	dist	tance reduction can be compromised by using a "defective" combined factor. A legitimate combined

- factor depends on choices of location and elevation. Sometimes inappropriate values are used. Of course,once decisions with respect to location and elevation are made, the definition of the combined factor is
- 474 unambiguous it is the product of the grid scale factor and the elevation factor.
- 475

With the advent of GNSS positioning, an alternate method for computing the combined factor avoids the
approximations and challenges of computing the correct combined factor between points. The difference
is that, using GNSS, latitude/longitude (and plane coordinates) are determined independently of a ground

- 479 traverse computation. The alternate (equivalent) definition of the combined factor is the ratio of grid
- 480 distance over horizontal ground distance. Horizontal ground distance is derived from GNSS vectors and
- 481 grid distance is computed from an inverse of grid (SPCS or LDP) coordinates. Approximations of grid scale
- 482 factors and elevation factors are thereby avoided. But it should be noted, computing the combined factor
- directly from the ratio, relies heavily on the user's choice of definition for horizontal distance and the
 elevation (geodetic height) at which the horizontal distance is computed. An example based on using the
 GSDM is provided in Burkholder (2019).
- 486
- 487 Overall Comparison GSDM and 2D/1D Methods
- 488489 The 3-D Diagram shown in Figure 1 first appeared in Burkholder (1993) and is described in more detail in
- 490 Burkholder (1997). A brief summary of the features in Figure 1 include:
- 491



492 493

- 494
 495
 1. Primary X/Y/Z coordinates are stored in Box 1 at the top of the diagram. Traditional practice using 496 geodetic latitude/longitude/height follows the left side of the diagram. Rectangular solid geometry 497 vector components are used on the right side of the diagram.
- 499
 2. Crossover between 3-D and traditional practice and measurements occurs at the bottom. Using total
 500 station observations, it is possible to "go up" either side to compute and store X/Y/Z positions.
- 501
 502 3. Geoid modeling is an essential part of traditional practice on the left side. Geodetic height is used for
 503 the third dimension on the right side of the diagram. That avoids geoid modeling and provides direct
 504 access for total station observations to be used efficiently.
- 505 506 Distance Options
- 507 508 The algorithms in the GSDM do not distort physical measurements (there is no grid scale factor or
- 509 elevation factor) but the user enjoys several options as to which distance to compute. In addition to being
- 510 able to compute a slope distance in 3-D space using equation 1, two fundamental choices for horizontal
- 511 distances are shown in Figure 2A and Figure 2B. In each case, the computation is "easy" to perform.
- 512
 - 513 Figure 2A: Distances are computed as chord distances using equation 1.
 - 514 Figure 2B: Flat-Earth horizontal distances are computed as $\sqrt{\Delta e^2 + \Delta n^2}$.

Figure 2A Chord Distances



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Figure 2B Right Triangle Components

A more comprehensive discussion of distance options is included in Burkholder (2019). Summary
 comments related to Figure 2A and Figure 2B include:

- X/Y/Z coordinates for Points A and B are computed with latitude/longitude of standpoint and
 forepoint but with zero geodetic height (*h* = 0.000 m) in each case.
- 524 2. X/Y/Z coordinates for Point E are computed with latitude/longitude of standpoint, but *h* at forepoint.
- 526 3. X/Y/Z coordinates for Point D are computed with latitude/longitude of forepoint, but *h* at standpoint.
- 528 4. Equation 1 can be used to compute the 3-D spatial distance between any two X/Y/Z points.
- 5. "Best" approximation for horizontal distance between C and F is $HD = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2 \Delta h^2}$. 531 This is very nearly identical to the mean of distances EF and CD (dotted line) in Figure 2A,

532				
533	6.	A chord di	stance between two points by equation 1 (approximates arc distance for lines < 5	0 km).
534		a. X/Y	/Zs of two points on ellipsoid – both points have zero geodetic height.	
535		b. X/Y	/Zs of two points at same geodetic height, $h_1 = h_2$.	
536				
537 538	7.	Local tang	ent plane horizontal distance from standpoint to forepoint, $HS_{1\rightarrow 2} = \sqrt{\Delta e^2 + \Delta e^2}$	n^2 .
539	8.	The local t	rangent plane horizontal distance $HD_{1\rightarrow 2}$ is close, but not exactly the same as $HD_{1\rightarrow 2}$	2-1.
540		a. Loca	al tangent plane distance is right triangle component of slope distance and assum	es plumb
541		line	is are parallel at both ends of the line – flat-Farth surveying.	
542		b. For	ward and reverse horizontal distances are not identical due to different tangent p	lanes.
543		c Ifus	sing P O B. local coordinates, the forward and reverse distances are the same bec	ause the
544		east	tings and northings (local P Ω B coordinates) lie in the same tangent plane as the	POB
545		d Ifus	sing P \cap B local coordinates the forward and back azimuths differ by 180°	
546		u. nu.	sing r.o. b. local coordinates, the forward and back azimatils differ by 100	
547	٥	The clone	distance between two points in 3-D space is easily computed using equation 1. In	cidentally
5/18	5.	the distant	co "here" to "there" in 3-D space is the same as the distance "there" to "here" " C	iven the
540			ce field to there in 5-D space is the same as the distance there to field. G	iven the
550		user rids se	elected the standpoint as there and forepoint as there, a fold for matrix is use	an matrix
551		compute i	ocal vector components from geocentric components. Stated unreferitiv, a rotation	
551			of the standardist equation 10. The transmost of the same retation metric is used	ae ana
552 552		iongitude	of the standpoint – equation 10. The transpose of the same rotation matrix is use	
333 551		compute g	geocentric components from local components – equation 11. The matrix form an	a
554		associated	a "long hand" form are given for each case as:	
333 556	1			
550 557	LOC	al vector co	omponents computed from geocentric vector components:	
221				
558			$ \begin{vmatrix} \Delta e \\ \Delta n \end{vmatrix} = \begin{vmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ -\sin\phi\sin\lambda & \cos\phi \end{vmatrix} \begin{vmatrix} \Delta X \\ \Delta Y \\ \Delta Y \end{vmatrix} $	(10)
559			$L\Delta u$ $L \cos \varphi \cos \lambda = \cos \varphi \sin \lambda = \sin \varphi \int L\Delta z \int$	
560			$\Lambda e = -\Lambda Y \sin \lambda + \Lambda V \cos \lambda$	(10_{2})
561			$\Delta t = \Delta X \sin \chi + \Delta I \cos \chi$ $\Delta n = -\Delta Y \sin \phi \cos \chi = \Delta V \sin \phi \sin \chi + \Delta Z \cos \phi$	(108) (10b)
562			$\Delta n = -\Delta X \sin \phi \cos \lambda + \Delta Y \cos \phi \sin \lambda + \Delta Z \cos \phi$	(100)
563			$\Delta u = \Delta x \cos \phi \cos x + \Delta t \cos \phi \sin x + \Delta z \sin \phi$	(100)
564	Goo	contric voc	tar companents computed from local vector companents:	
565	Get			
505			$[\Lambda Y] = [-\sin \lambda - \sin \theta \cos \lambda - \cos \theta \cos \lambda] [\Lambda a]$	
566			$ \begin{array}{c} \Delta X \\ \Delta Y \\ \Delta Z \end{array} = \begin{bmatrix} -\sin \lambda & -\sin \phi \cos \lambda & \cos \phi \cos \lambda \\ \cos \lambda & -\sin \phi \sin \lambda & \cos \phi \sin \lambda \\ 0 & \cos \phi & \sin \phi \end{bmatrix} \begin{array}{c} \Delta e \\ \Delta n \\ \Delta u \\ \Delta u \end{array} $	(11)
567				
568			$\Delta X = -\Delta e \sin \lambda - \Delta n \sin \phi \cos \lambda + \Delta u \sin \phi$	(11a)
569			$\Delta Y = \Delta e \cos \lambda - \Delta n \sin \phi \sin \lambda + \Delta u \sin \phi$	(11b)
570			$\Lambda Z = \Lambda n \cos \phi + \Lambda u \sin \phi$	(11c)
571				(110)
572	3-D	Direct and	Inverse	
573	50			
574	The	following	equations are included for completeness and to show simplicity. In Figure 1, the 3	-D
575	"dir	oct" is labo	side as RK3 and the 3-D "inverse" is labeled as RKA	5
576	un			
570				

577	3-D Direct (forward computation)	
578		
579	$X_2 = X_1 + \Delta X$	(12)
580	$Y_2 = Y_1 + \Delta Y$	(13)
581	$Z_2 = Z_1 + \Delta Z$	(14)
582		· · ·
583	3-D Inverse (reverse computation)	
584		
585	$\Delta X = X_2 - X_1$	(15)
586	$\Delta Y = Y_2 - Y_1$	(16)
587	$\Lambda Z = Z_2 - Z_1$	(17)
588		()
589	Geodesy Equations	
590		
591	The following information and equations are used to compute $X/Y/7$ geocentric rectangul	ar coordinates
592	from geodetic latitude/longitude/height and to compute geodetic latitude/longitude/height	x the from $X/Y/7$
593	X/V/7 coordinates in meters can be either positive or negative la	rge or small
591	geocentric coordinates. A 172 coordinates in meters can be either positive of megative, la	ge of small.
595	Name and parameters of ellipsoid – most generally the GRS 1980	
596	Semi-major axis a	
507	= 0,376,137 in exactly	
509	Eccentricity equared of ellipsoid = $2 f_{1} f_{2}^{2} = -0.006604280022$	
398 500	Eccentricity squared of empsoid = $2T - T^2$, $\mathcal{E}^2 = 0.006694380023$	
599	$C_{\rm employ}$ to a solution $V N / Z$ as and in stars the solution	
600	Compute geocentric X/Y/Z coordinates – input:	
601		
602	Geodetic latitude, north is positive, south is negative ϕ	
603	Geodetic longitude, east is positive, west is negative λ	
604	Geodetic height in meters h	
605	a	
606	N = ellipsoid normal for given latitude, $N = \frac{a}{\sqrt{1 - s^2 \sin^2 a}}$	(18)
607	$\sqrt{1-c}$ sin φ	
608	$X = (N + h) \cos \phi \cos \lambda$	(19)
609	$Y = (N + h) \cos \varphi \sin \lambda$	(20)
610	$Z = [N(1 - \varepsilon^2) + h] \sin \phi$	(21)
611		()
612	Compute geodetic latitude longitude and height from $X/Y/7$ coordinates – Input	x/y/7·
613		////_/
614	Note – this is the most complicated portion of the GSDM computations. Equation	s 19 20 and 21
615	can be algebraically inverted for a solution but the closed form of equation 23 ba	s geodetic
616	latitude on both sides of the "equals" sign, requiring a iteration to solve	Beoucie
617		
<i>c</i> 10	$2 \qquad -1 (Y)$	221
018	$\lambda = \tan^{-1}\left(\frac{1}{x}\right)$	22)
619		
620	$\varphi = tan^{-1} \left[\frac{Z}{\sqrt{\pi^2 - \pi^2}} \left(1 + \frac{\varepsilon^2 N \sin \varphi}{\pi} \right) \right] \tag{6}$	23)
621	$L_{V}X^{2}+Y^{2} \setminus Z J$	-
	$\sqrt{X^2 + Y^2}$	
622	$h = \frac{m + 1}{\cos \varphi} - N \tag{(1)}$	24)

623

624 Of various methods that could be employed to iterate equation 23, the following is recommended. Start 625 with an initial approximation of latitude (φ_0) by equation 25 and compute N_0 by equation 26. Use those 626 values as $\varphi_{I=0}$ and $N_{i=0}$ in equation 27 to find a better value of latitude ($\varphi_{I=1}$). With the better value of 627 latitude, compute a better value of $N_{i=1}$. Terminate iteration when the subsequent value of latitude is 628 sufficiently smaller than the previous one. Convergence should not require more than 3 or 4 iterations. 629

 $\varphi_0 = tan^{-1} \left(\frac{Z}{\sqrt{X^2 + Y^2} (1 - \varepsilon^2)} \right)$ and $N_0 = \frac{a}{\sqrt{1 - \varepsilon^2 \sin^2 \varphi_0}}$ (25) and (26)

635

 $\varphi_{i+1} = \tan^{-1} \left[\frac{Z}{\sqrt{X^2 + Y^2}} \left(1 + \frac{\varepsilon^2 N_i \sin \varphi_i}{Z} \right) \right]$ (27)

634 After equation 27 has been sufficiently iterated, equation 24 is used to find geodetic height.

636 Credible methods for computing geodetic latitude, longitude, height without iterating can be found in
637 Burkholder (1997, 2008, 2018), Vermeille (2009), Meyer (2010), or using a web search.

- 638
- 639 Stochastic Model
- 640

This article compares the complexity of traditional ellipsoid-based computations with the complexity of
 those equations used in the GSDM. In addition to preserving geometrical integrity of spatial and
 geospatial data using "simple" equations, the GSDM also includes a stochastic model that can be used to
 assess spatial data accuracy.

645

646 Use of the stochastic model is covered in Burkholder (1997a, 1999, 2008, 2016, 2017, and 2018).

647 648 Other lss

648 Other Issues 649

The case for integrity and simplicity has been made. But it would be naïve to ignore other factors germane
 to the discussion. The challenge cannot be met by a "one size fits all" solution. Very briefly, other factors
 deserving consideration include:

653

658

Surveying, as a part of the spatial data community, provides <u>career opportunities</u> for persons
 talented in math/geometry, astronomy/geodesy, computers/electronics, legal/cadastral, office/field,
 sole proprietorship/corporate/agency environments, research/data collection, finance/logistics, and
 running a business.

- From the perspective of pedagogy and learning styles people, draw on various talents such as logical mathematical intelligence, spatial intelligence, linguistic intelligence, kinesthetic intelligence, musical
 intelligence, interpersonal intelligence, intrapersonal intelligence, and naturalistic intelligence
 (Brown, et.al., 2014).
- Surveying (as part of the spatial data spectrum) <u>policy is established</u> on various levels by
 Surveying (as part of the spatial data spectrum) <u>policy is established</u> on various levels by
 manufacturers, software vendors, technicians, analysists, sole practitioners, business owners,
 professional associations, corporate managers, federal/state/local agencies, researchers, academics,
 and others. Again, "once size fits all" is not the ultimate goal, but the benefits of using a common
 "simple" spatial data model across the entire spectrum deserves careful consideration.
- 669

670		
671	Sur	nmary
672 673	1.	The spatial data community has achieved impressive results using ellipsoid-based methods for spatial
674 675		data manipulations. Admittedly some of those computations are rather complex but algorithms have been developed, tested, and programmed. Software is readily available for manipulating spatial (and
676 677		geospatial) data and the end user need not understand the processes or steps involved.
678	2.	Reliable software, fast computers, unlimited (cloud) storage capacity, and ubiquitous mobile devices
679 680		mean that problems can be reliably solved using existing methods and procedures. As stated before, they work . On the other hand, an over-emphasis rote can be dangerous. Brown et al. (2014) make
681 682		the point that "it is better to solve a problem than to memorize a solution." At some level of accountability, the licensed professional must take responsibility for the integrity of results and
683 684		services provided to a client. An evaluation of consequences and ethics should never go out of style
685		
686 687	3.	Evolving to use of an integrated 3-D model for spatial data for all applicable applications will take
688		the extent common goals can be established and visionary leadership (involving various talents) is
689		brought to bear on the challenge.
690		
691 692	4.	The challenge is enormous, and opportunities can be exciting. The banner is "Simple is Better!"
693		
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