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Underground (Well) Mapping Re-Visited

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ABSTRACT: Reliable underground mapping is an essential part of efficient energy production. The challenge is to determine reliable 3-D point positions in a well and/or the trajectory of a wellpath. Sensor data (e.g., measured wellbore depth (distance), wellbore inclination, and wellbore azimuth) are typically used to compute the location of discrete points in a well and a wellpath is determined using the method of minimum curvature. Part of the positioning challenge relates to specific identification of the origin, orientation of the coordinate system, knowing the quality of measurements, and knowledge of datums and map projections. Historically, survey maps showed horizontal and vertical data separately (primarily analog) but modern measurement systems produce digital 3-D spatial data. The global spatial data model (GSDM) will be described which accommodates modern measurements and 3-D digital spatial data. The GSDM uses proven rules of solid geometry to determine each 3-D position and optionally stores the standard deviation of each computed point. The subsequent relative position between any two points is easily computed and the underlying coordinate positions can be mathematically transformed to other commonly used coordinate systems. With the GSDM, all uses (site drawings, drilling, well development, production, and regulatory activities) of the position data share a common definition and the positional uncertainty of each computed quantity can be determined from stored covariance matrices.

INTRODUCTION

As a consequence of the digital revolution and development of more sophisticated measurement systems, spatial data (maps) are now characterized as being digital and three-dimensional (3-D). This paper explores methods for handling 3-D digital spatial data that exploit those characteristics rather than being encumbered by them.

Historically - and still prevalent in practice – many horizontal and vertical survey data are handled separately. That separation is driven, in part, by human experience of walking erect on a “flat-earth.” More specifically, spatial data users find themselves working with horizontal datums and vertical datums. The justification for two datums is that horizontal and vertical have different origins. Horizontal is referenced to latitude/longitude on the mathematical ellipsoid while vertical is referenced to sea level (more specifically, the geoid). Regretfully, there are no closed-form equations that relate the ellipsoid to the geoid. Those disparate origins make it difficult (awkward) to combine horizontal and vertical data into a single 3-D data base.

While the energy industry may be quite adept at determining and using “flat-earth” components to describe the relative location of surveyed points, the reality is that the earth is not flat and that location data need to be handled accordingly. The paper by Williamson and Wilson (2000) on “Directional Drilling and Earth Curvature,” discusses those issues and includes the following quotes:

- “A third option, representing the well in 3D geocentric Cartesian coordinates, offers computational advantages but these are outweighed by the awkward fact that, in general, none of the axes are parallel with the vertical.”
- “While use of the Flat Earth model persists, directional drilling companies planning extended-reach wells should estimate the physical error introduced by the model and compare it with target tolerances.”
- “Developers of directional and survey software should consider an improved methodology, such as that described in this (Williamson/Wilson) paper, for inclusion in their products. This level of precision will be particularly desirable in software which integrates drilling and subsurface data.”

The global spatial data model (GSDM) described herein accommodates each of those points. The GSDM has a single origin for 3-D geospatial data (earth’s center of mass), uses time-honored rules of solid geometry to describe the unique location of points worldwide, and includes a rotation matrix to reconcile the 3-D geocentric axes with the local perspective. In addition to tracking geometrical locations with the functional model portion of the GSDM, the stochastic model portion of the GSDM provides an efficient way to compute the uncertainty (standard deviations) of surveyed locations using proven error propagation techniques. The standard deviation of derived quantities (directions/distances/etc.) can also be computed using similar error propagation techniques. But, perhaps the best feature of the GSDM is that it provides an efficient reliable connection between “flat-earth” concepts used in practice and the larger curved-earth world of modern computer data bases.

A truism is that we are where we are because of where we came from. Extensive records have been accumulated over the years that document drilling sites, the location of wells, and the trajectories of wellbores within the earth. Those records,

for the most part, are based upon the best available (or other) practices at the time. Such practices include assumed coordinates, datums, map projections, zone constants, coordinate systems, and computational procedures consistent with (many analog) measurements and existing technology. The GSDM avoids many of those nuisance issues. But, while the GSDM can be beneficially used to exploit the characteristics of modern technology and 3-D digital spatial data, it is acknowledged that finding the best path forward with regard to management and use of spatial information within the constraints of historical use, productivity, and profitability remains a challenge. The caveat is that, with adoption and use of the GSDM, incremental advancements by individuals, companies, and agencies/regulators can be aggregated for the benefit of all users.

DEFINITION AND DESCRIPTION OF THE GSDM

Burkholder (1997) provides a formal definition of the GSDM. In reality, the GSDM contains no new mathematical concepts and all equations used in the GSDM are in the public domain. But, the GSDM is a new model in that it is built on the assumption of a single origin for 3-D geospatial data. Time-honored rules of solid geometry provide the basis for coordinate computation. Features and applications of the GSDM are described more extensively in Burkholder (2008).

With reference to Fig. 1 and Fig. 2, the GSDM embodies three different coordinate systems: geodetic/geocentric/local. The geodetic coordinate system includes latitude, longitude, and ellipsoid height. The geocentric coordinate system includes earth-centered earth-fixed (ECEF) rectangular X/Y/Z coordinates and is used as a reference for global navigation satellite systems (GNSS) – including GPS. The geodetic and geocentric systems share a common origin – earth’s center of mass. The origin for the local system is any point chosen by the user. Local coordinates portray a “flat-earth” view of all other points relative to the user-selected origin.

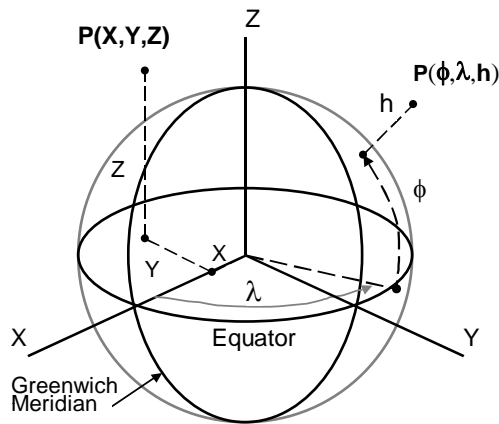


FIG. 1. Geodetic and Geocentric Coordinate Systems

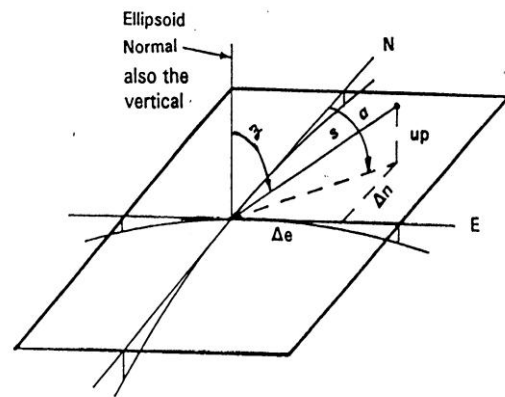


FIG. 2. Local (Flat-Earth) Coordinate System

Figure 3 provides an overview of the three coordinate systems and maps the various geometrical transformations encountered when using the GSDM.

Computation Diagram for the Global Spatial Data Model (GSDM)

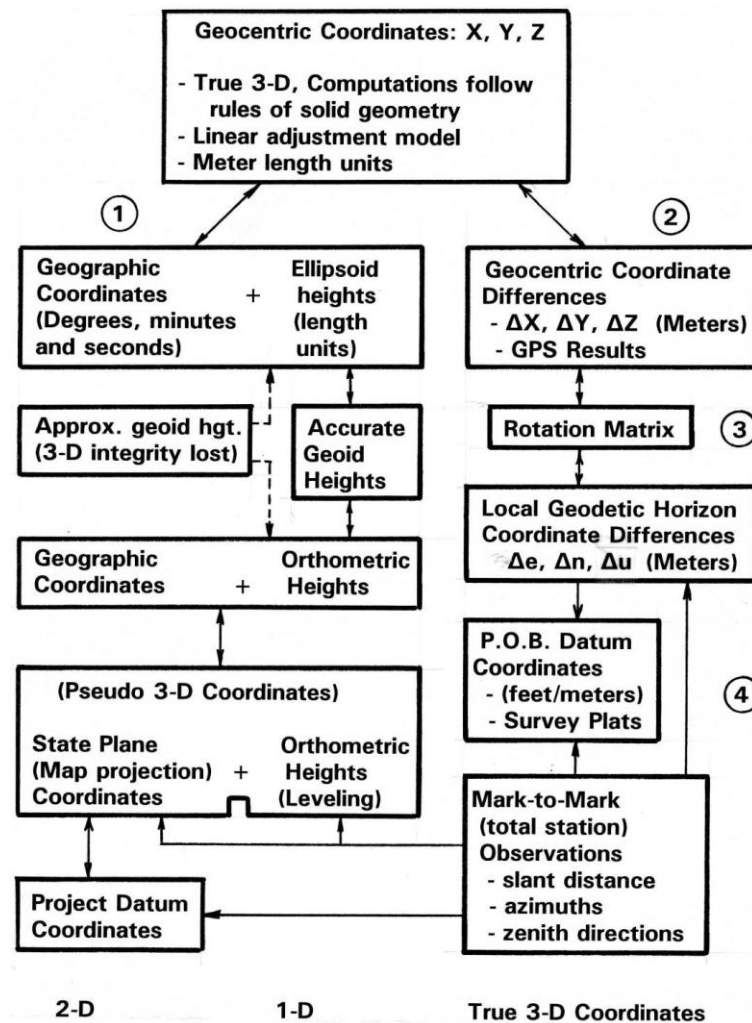


FIG. 3. Schematic Diagram of the Global Spatial Data Model (GSDM)

The top box in Fig. 3 stores the geocentric ECEF rectangular metric X/Y/Z coordinates that define the location of any/all points used in the GSDM. All other expressions of coordinates and positions are derived from the ECEF values. With regard to Figures 1, 2, and 3, additional items to note are:

- The X/Y axes lie in the plane of the equator and the Z axis very nearly coincides with the earth's spin axis. The X value is zero on the prime meridian.
- A vector between any two points is easily computed as geocentric coordinate differences. $\Delta X = X_2 - X_1$, $\Delta Y = Y_2 - Y_1$, and $\Delta Z = Z_2 - Z_1$.
- A rotation matrix (Burkholder 1997 or 2008) is used to convert ECEF coordinate differences into the local perspective $\Delta e/\Delta n/\Delta u$ components, thus addressing the Williamson/Wilson objection to using ECEF.
- Coordinate positions are computed in three-dimensional space, not on the ellipsoid. That avoids numerous measurement reductions to the ellipsoid.
- With a local origin selected, two operations are of interest:
 - a. Geocentric coordinate differences can be rotated to local "flat-earth" components. Those local horizontal components are the same as latitudes and departures historically used in plane surveying and can be added to the horizontal northing/easting values assigned to the local origin.
 - b. Local "flat-earth" components (derived from existing measurements) can be rotated to geocentric coordinate differences. Those $\Delta X/\Delta Y/\Delta Z$ values can be added to the ECEF values of the local origin to provide defining X/Y/Z values of the new point.

In both cases, 3-D geometrical integrity is preserved by reliance on the stored defining X/Y/Z values.

- The meter is the standard of length for all ECEF coordinates. Calculated offsets (if in non-metric units) need to be converted to meters before being rotated into the ECEF system. Conversely, after ECEF coordinate differences (meters) are rotated into the local reference frame, they can be converted to feet (or other units – see the P.O.B. Datum Box in Fig. 3).
- Compatible with existing practice, office/field users can both work with local "flat-earth" 3-D coordinate differences with no loss of geometrical integrity.
- There are no map projections, grid scale factors, or zone constants. Distance and angle measurements are not distorted to fit a map projection model.
- Azimuths are referenced to true north at a specified location. Grid north throughout a project is with respect to true north at the user-specified P.O.B.
- The GSDM is a single system equally applicable for spatial data users in all disciplines all over the world. That means distortions associated with crossing zone boundaries of traditional map projections are avoided

CONCEPTUAL CONSIDERATIONS

Functional and Stochastic Model Components

The functional model portion of the GSDM can be used without the stochastic model. If the stochastic model is not used, the positions of all control points are held fixed and all measurements are used as exact (standard deviations are all zero). Subsequently computed coordinate positions may be legitimate, but they have no standard deviation associated with them. On the other hand, the stochastic portion of the GSDM provides an opportunity for the user to assign realistic uncertainties (standard deviations) to all variables and to propagate the error. That prerogative enables the spatial data professional to make important decisions related to positional uncertainty with greater confidence. Of course, if bad information is used (or if good information is used inappropriately) unreliable answers can be obtained. The opposite case is the important one – the GSDM supports efficient use of spatial data by all disciplines.

Absolute and Relative Issues

Another concept involves the difference between “absolute” and “relative.” Absolute data are taken to be unique values expressed in a “fixed” reference system (datum) while relative is taken to be the difference of two absolute values within the same system. This is important because relative values are more closely associated with measurements and use in the field while absolute values are more closely associated with data archival and storage (data bases). Information management often focuses on absolute quantities while many practical applications work primarily with relative values. The two are inseparably related but often effort and resources are “wasted” by focusing too much attention on absolute values at the expense of relative considerations.

The value of most surveys is derived from the answer to one of two questions:

- What is the position of this point with respect to that point (and all other points in the survey)? The answer to this question is a relative quantity within the same datum. This illustrates the value of basing a survey on permanent control points.
- What is the position of this point with respect to where it was at some time (epoch) in the past? In other words, “did it move?” This too is a relative quantity within the same datum. This illustrates the importance of using permanent stable control points.

Since relative quantities are differences of absolute values in the same datum, it makes perfect sense to store absolute values and to compute relative values as needed. That practice is well established and certainly justified. But, upgrading or changing datums is a nuisance/problem faced by many spatial data users. That nuisance is exacerbated by the absence of closed-form equations for the difference between datums – old and new. Welcome to the real world of imperfect measurements and information management. As an aside, consider the consternation worldwide if, due to improved measurements/data, the scientific community decided it was necessary to update the value used for the speed-of-light every 5 years or so.

Issues of Spatial Data Accuracy

When working with spatial data accuracy, one of the most important questions is “accurate with respect to what?” Often the default answer is “accuracy with respect to the control on which the survey is based.” The stochastic model portion of the GSDM accommodates mathematical definitions of both network accuracy and local accuracy. As a reminder, network accuracy applies primarily to absolute data while local accuracy is more closely related to relative data. The manner in which the GSDM handles both network and local accuracy enables the spatial data user to make better choices as dictated by the circumstances and the available data. It puts the user in the driver’s seat. For example, the network accuracy of a collection of points is often stored in the covariance matrix for each point. Data representing the correlation between points are not stored. Storing only the point covariance is better than not storing any covariance information but, if appropriate covariance data (both point and point-pair) - say from a least squares adjustment – are stored, then both the network accuracy and local accuracy between points can be readily computed. Burkholder (2013) is an example of current research that investigates network/local accuracy as applied to a ground-based GPS network. Additional research focusing on local vertical accuracy (such as that in a wellbore) is needed.

Trade-off of Costs and Benefits

The point is made that the GSDM addresses the issues raised by Williamson and Wilson (2000). But, those benefits come at a price (in some cases, one that is already being paid). Conceptually, the issue is that measurements are made in a physical environment and that computations are performed according to an adopted model. That is already being done – for example, when using “flat earth” assumptions and ignoring earth curvature. The underlying question (raised in this paper) becomes one of selecting an appropriate model. Ideally, the “best” model for any operation is one that is both simple and appropriate – Occam’s Razor. It could be argued that the appropriateness of the curved-earth computations recommended by Williamson and Wilson (2000) justify the added complexity of working on the ellipsoid with geodesy equations. As a survey engineer, this author is fully sympathetic with that position. However, it is argued here, because of the features already listed, that the GSDM is even more appropriate for underground (wellbore) mapping and that the computational complexity of the GSDM is significantly less than performing

computations on the ellipsoid or using a map projection model. But there are other important issues that also need to be accommodated.

For example, it is presumed that the correction between magnetic north (physical) and true north (mathematical) is already being made. The gyroscope also responds to a physical stimulus, but the gyroscope detects and references a different north than does a magnetometer. Separate models (equations) are used to relate those physical data to the adopted “north” reference. Similarly, the difference between the ellipsoid normal (mathematical) and the vertical plumb line (physical) is a correction that should be made (if the difference rises to a level of positional significance). The GSDM includes strictly mathematical components but sensor data are derived from observations of physical phenomena. Physical observations and the associated mathematical model need to be reconciled before physical measurements can be used in coordinate position computations.

At a gross level, the difference between normal and vertical (deflection of the vertical) may be inconsequential for underground mapping. But as tolerances become tighter and as computed positions are known with greater certainty (smaller standard deviations), the difference will need to be accommodated. Part of that discussion needs to recognize that relative ellipsoid height differences (in a wellbore) are not significantly different than elevation differences. The point is that knowledge of the depth of a wellbore can be expressed in ellipsoid height (via the GSDM) more reliably than when using elevation. In short, the need for geoid modeling is largely mitigated. That point needs careful discussion and is too important to be decided here and now. Burkholder (2002) contains additional information that may be applicable to that discussion.

GSDM PROCEDURES FOR UNDERGROUND (WELL) MAPPING

For the most part, existing field procedures are valid and can continue to be used when implementing the GSDM. The differences lie primarily with the manner in which data are processed, managed, and used. First, it might be helpful to review the current map projection and “low distortion” models being used. The following are to be noted with regard to Fig. 4.

- In spite of the fact that map projections are strictly two-dimensional models, they have been used very beneficially with performing flat earth computations for many projects.
- Figure 4 illustrates how map projections have been used to minimize impact of curved-earth surveying. Low Distortion Projections have become popular with various city/county/state organizations.
- The underground mapping may begin on/near the earth’s surface but the vertical portion of a wellbore quickly extends beyond the effective range of a

map projection model. The reason is that a map projection is concerned primarily with horizontal relationships within a fairly narrow vertical window.

- The 3-D GSDM overcomes drawbacks of the 2-D map projection model and preserves true 3-D geometrical integrity throughout.

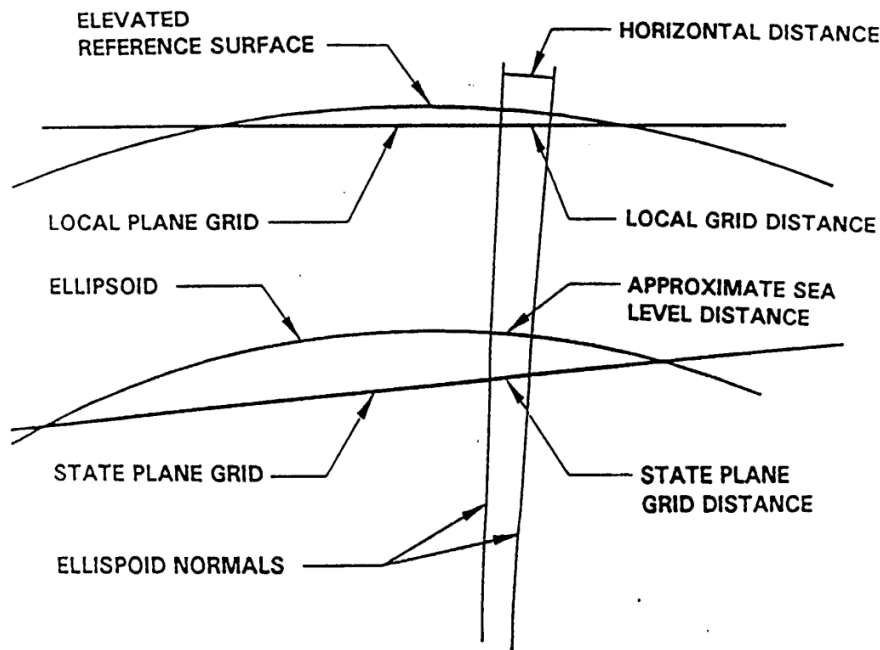


FIG. 4. Map Projections Have A Narrow Range of Vertical Validity

Implementation Procedures:

- The beginning reference point (well-head or nearby well monumented point) must have reliable 3-D ECEF coordinates (X/Y/Z) – typically determined using GPS. If it is not possible to occupy the point with GPS, those values can be determined using a total station side-shot from a known X/Y/Z reference point.
- Existing sensors and procedures are used to determine local 3-D spatial data components (calculated offsets). These local differences are converted to metric units, rotated into the ECEF environment, and added to beginning ECEF coordinate values of the known point.
- The computed ECEF coordinates are stored (top box in Fig. 3) and used as primary values. The stochastic portion of the GSDM accommodates storing the covariance values of each new point. In the case of a network adjustment, the covariance values representing point-pair correlation can also be stored.

- Using stored ECEF coordinates involves selecting a convenient point-of-beginning (P.O.B.) as a reference. The relative position from the P.O. B. to any other ECEF point is a simple subtraction to obtain geocentric coordinate differences. Those ECEF differences are transformed to the local perspective $\Delta e/\Delta n/\Delta u$ components using a rotation matrix at the P.O.B.
- Those local flat-earth components can be used to compute distances, azimuths, and other quantities in much the same manner as current practice.

$$HD = \sqrt{\Delta e^2 + \Delta n^2} \quad \text{and} \quad AZ = \text{atan}\left(\frac{\Delta e}{\Delta n}\right) \quad (1) \ \& \ (2)$$

- The GSDM stores ECEF values that are common to all disciplines worldwide. That means that all spatial data users are on the same page and can enjoy the luxury of transforming those ECEF values to latitude/longitude/ellipsoid height, state plane coordinates, or UTM coordinates – see Burkholder (2008).

CONCLUSIONS

Although core 3-D coordinate computations are performed in a standard geometrical environment in which measurements are not distorted to fit the model, each user has the option of transforming those values to a user/job specific system that permits continued use of familiar practices. Additionally, the GSDM defines specific procedures for handling spatial data accuracy. But, the real take-away from this paper relates to the issues raised by Williamson and Wilson (2008):

- The disadvantage of non-parallel coordinate systems is handled by using a rotation matrix to transform the ECEF coordinate differences into local differences.
- The GSDM accommodates continued use of flat earth practices in the field while preserving geometrical integrity of the measurements and providing rigorous connection to the physical curved-earth environment.
- Using the GSDM, drilling and subsurface data can be integrated into the same data base in a manner that also supports establishing and tracking spatial data accuracy.

Additional information related to the GSDM is posted on the author's web site and gratis prototype software that validates the concepts described herein is available at <http://www.globalcogo.com/WBK3D.html>. Significant work remains to be done to tailor software for the energy production industry. But, it is this author's opinion that the added effort needed to relate physical sensor data measurements to mathematical geometrical components is a small price to pay for enjoying the benefits offered by using the GSDM.

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